One-neutron removal reactions on neutron-rich psd-shell nuclei

E. Sauvan a, F. Carstoiu a,b, N.A. Orr a,*, J.C. Angélique a, W.N. Catford c, N.M. Clarke d, M. Mac Cormick e,1, N. Curtis e,2, M. Freer d, S. Grévy f,3, C. Le Brun a, M. Lewitowicz e, E. Liégard a, F.M. Marqués a, P. Roussel-Chomaz e, M.G. Saint Laurent e, M. Shawcross c, J.S. Winfield a,4

a Laboratoire de Physique Corpusculaire, IN2P3-CNRS, ISMRA et Université de Caen, F-14050 Caen cedex, France
b IFIN-HH, P.O. Box MG-6, 76900 Bucharest-Magurele, Romania
c Department of Physics, University of Surrey, Guildford, Surrey, GU2 5XH, UK
d School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, UK
e GANIL, CEA/DSM-CNRS/IN2P3, BP 5027, F-14076 Caen cedex, France
f Institut de Physique Nucléaire, IN2P3-CNRS, F-91406 Orsay cedex, France

Received 21 June 2000; accepted 24 August 2000

Abstract

A systematic study of high energy, one-neutron removal reactions on 23 neutron-rich, psd-shell nuclei (Z = 5–9, A = 12–25) has been carried out. The longitudinal momentum distributions of the core fragments and corresponding single-neutron removal cross sections are reported for reactions on a carbon target. Extended Glauber model calculations, weighted by the spectroscopic factors obtained from shell model calculations, are compared to the experimental results. Conclusions are drawn regarding the use of such reactions as a spectroscopic tool and spin-parity assignments are proposed for 15 B, 17 C, 19–21 N, 21 23 O, 23–25 F.

The nature of the weakly bound systems 14 B and 15,17 C is discussed.

0370-2693/00 $ – see front matter © 2000 Elsevier Science B.V. All rights reserved.

PACS: 25.60.-t; 25.60.Gc; 27.20.+n; 27.30.+t

Keywords: One-neutron removal; Momentum distributions; κ→1n; Glauber model

Fragment momentum distributions have long been recognised as signatures of the large spatial extent of the valence nucleons in halo nuclei [1]. Recently measurements of one-nucleon removal5 reactions on light targets have been proposed as a spectroscopic tool for high-energy radioactive beams [2,3]. This approach has arisen from the development of reaction calculations in which the strong absorption limit [4] and core excited states are accounted for [3]. More

5 The term “knockout”, which has been employed to refer to such reactions [2], is not adopted here as it has long been used for (p,2p) and (e,e′p) reactions, the description of which is very different from that of absorption and diffraction in one-nucleon removal.
specifically, the integrated cross sections are related to spectroscopic factors using an extended version [3, 5] of the spectator core model [6], whilst the momentum distributions are derived in the opaque limit of the Serber model [7, 8]. To date, this approach has been applied to a few near dripline and halo nuclei [2, 9–11].

In this letter the results of an investigation of high-energy one-neutron removal reactions over a broad range of light, neutron-rich psd-shell nuclei are reported. The goals of the work were twofold. Firstly, to explore the evolution in structure, and the manner in which it is manifested in the core fragment observables, from near stability to dripline and halo systems. Secondly, for many of the near stable nuclei the ground state structure is well established and, consequently, it has been possible to test the validity of one-neutron removal reactions as a spectroscopic tool.

In the following, measurements of the core fragment longitudinal momentum distributions and integrated cross sections resulting from reactions on a C target are presented. Comparison is made for both observables to the results of extended Glauber type calculations incorporating second order noneikonal corrections to the JLM parameterisation of the optical potential [12]. In the case of those systems with unknown, or poorly defined ground state structures, probable spin-parity assignments have been made.

The secondary beams were produced via the fragmentation on a 490 mg/cm$^2$ thick C target of an intense ($\sim 1 \, \mu$Ae) 70 MeV/nucleon$^{40}$Ar$^{17+}$ beam provided by the GANIL coupled cyclotron facility. The reaction products were collected and selected according to magnetic rigidity using the SISSI device coupled with the alpha-shaped beam analysis spectrometer. A mean rigidity of 2.880 Tm was selected to allow for the transmission of nuclei from $^{12}$B to $^{25}$F with energies in the range of 43–71 MeV/nucleon (Table 1). The energy spread in the secondary beams, as defined by the spectrometer acceptances, was $\Delta E/E = 2\%$.

The measurements of the momentum distributions and one-neutron removal cross sections were performed using the SPEG spectrometer [13]. Owing to the large energy spread in the secondary beam, SPEG was operated in a dispersion matched energy-loss mode [14] for which a resolution in the momentum measurements of $\delta p/p = 3.5 \times 10^{-3}$ was obtained. Importantly the large angular acceptances of the spectrometer ($4^\circ$ in the vertical and horizontal planes) provided for complete collection of the core fragments, obviating any ambiguities in the integrated cross sections and longitudinal momentum distributions that would arise from limited transverse momentum acceptances [15]. Furthermore, the broad momentum acceptance of the spectrometer ($\Delta p/p = 7\%$) allowed the momentum distributions for one-neutron removal on all the nuclei of interest to be obtained in a single setting ($B_{\text{SPEG}} = 2.551 \, \text{Tm}$). A secondary C reaction target of thickness 170 mg/cm$^2$ was employed for the measurements described here (the results obtained with a Ta target will be reported elsewhere [12]).

Ion identification at the focal plane of SPEG was achieved using the energy loss derived from a gas ionisation chamber and the time-of-flight between a thick plastic stopping detector and the cyclotron radio-frequency. Additional identification information was provided by the residual energy measurement furnished by the plastic detector and the time-of-flight with respect to a thin-foil microchannel plate detector located at the exit of the beam analysis spectrometer. Two large area drift chambers straddling the focal plane of SPEG were employed to determine the angles of entry of each ion and, consequently, allowed the focal plane position spectra to be reconstructed. The momentum of each particle was then derived from the reconstructed focal plane position. Calibration in momentum was achieved by removing the reaction target and stepping the mixed secondary beam of known rigidity along the focal plane. This procedure also facilitated a determination of the efficiency across the focal plane for the collection of the reaction products.

The intensities of the various components of the secondary beam were measured in runs taken with the secondary reaction target removed and the spectrometer set to the same rigidity as the beamline. These were calibrated in terms of the primary beam current, which was recorded continuously throughout the experiment using a non-interceptive beam monitor. Checks were also provided by the counting rates in the microchannel at the exit of the beam analysis spectrometer and a second located just upstream of the secondary reaction target. Typical secondary beam intensities ranged from $\sim 600 \, ^{12}\text{C}/\text{s}$ to $\sim 1 \, ^{25}\text{F}/\text{s}$. 
Fig. 1. Core fragment longitudinal momentum distributions for one-neutron removal on C. The solid lines correspond to the Glauber model calculations (see text for details).

The longitudinal momentum distributions for the core fragments arising from one-neutron removal are displayed in Fig. 1 and the extracted widths (FWHM in the projectile frame) are summarised in Table 1. The widths were derived from Gaussian fits to the central regions of each distribution. The effects arising from the target (straggling etc.), efficiency along the focal plane and instrumental resolution have been taken into account in deriving the final values. The corresponding one-neutron removal cross sections are listed in Table 1 and displayed in Fig. 2. The uncertainties quoted include the contributions from both the statistical uncertainty and that arising from the determination of the secondary beam intensity (~ 7%).

A number of features are immediately apparent on inspection of Figs. 1 and 2. Firstly, the crossing of the $N = 8$ shell and $N = 14$ sub-shell closures are associated with a marked reduction in the widths of the core momentum distributions (viz, $^{14,15}$B, $^{15}$C, $^{23}$O and $^{24,25}$F). Secondly, with respect to the neighbouring isotopes, $^{14}$B and $^{15}$C exhibit enhanced one-neutron removal cross sections. The former effects arise from the large $v2s_{1/2}$ admixtures expected in the ground
a combination of weak binding (14 B: $S_n = 0.97$ MeV; terms, the enhanced cross sections may be attributed to the momentum distributions may also be expected for valence neutrons occupy the low), which may also persist for $N$ states of the $Z = 4–6$, $N = 9$ isotones [24] (see below), which may also persist for $N = 10$, as suggested by recent studies of 14 Be [25,26]. A narrowing of the momentum distributions may also be expected for $N = 15$ and 16 as in a simple shell model picture the valence neutrons occupy the $\nu 2s1/2$ orbital. In general terms, the enhanced cross sections may be attributed to a combination of weak binding (14 B: $S_n = 0.97$ MeV;

Table 1
Summary of results for one-neutron removal

<table>
<thead>
<tr>
<th>$^A Z$</th>
<th>Energy [MeV/nucleon]</th>
<th>FWHM$_{cm}$ [MeV/c]</th>
<th>$\sigma_{-1n}$ [mb]</th>
<th>$\sigma_{Glauber}$ [mb]</th>
<th>$J^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 B</td>
<td>67</td>
<td>142±3.5</td>
<td>81±5</td>
<td>91</td>
<td>1$^+$</td>
</tr>
<tr>
<td>13 B</td>
<td>57</td>
<td>135±7</td>
<td>59±4</td>
<td>62</td>
<td>3/2$^-$</td>
</tr>
<tr>
<td>14 B</td>
<td>50</td>
<td>56.5±0.5</td>
<td>153±15</td>
<td>185</td>
<td>2$^-$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$^A Z$</th>
<th>Energy [MeV/nucleon]</th>
<th>FWHM$_{cm}$ [MeV/c]</th>
<th>$\sigma_{-1n}$ [mb]</th>
<th>$\sigma_{Glauber}$ [mb]</th>
<th>$J^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 B</td>
<td>43</td>
<td>73±2.5</td>
<td>108±13</td>
<td>89</td>
<td>3/2$^-$ c</td>
</tr>
<tr>
<td>14 C</td>
<td>71</td>
<td>180±5</td>
<td>65±4</td>
<td>89</td>
<td>0$^+$</td>
</tr>
<tr>
<td>15 C</td>
<td>62</td>
<td>63.5±0.7</td>
<td>159±15</td>
<td>168</td>
<td>1/2$^+$</td>
</tr>
<tr>
<td>16 C</td>
<td>55</td>
<td>108±2</td>
<td>65±6</td>
<td>75</td>
<td>0$^+$</td>
</tr>
<tr>
<td>17 C</td>
<td>49</td>
<td>111±3</td>
<td>84±9</td>
<td>71</td>
<td>3/2$^+$ c,a</td>
</tr>
<tr>
<td>84</td>
<td>145±5</td>
<td>26±3</td>
<td>96.8</td>
<td>41±4</td>
<td></td>
</tr>
<tr>
<td>904</td>
<td>141±6</td>
<td>129±22</td>
<td>126±5</td>
<td>115±18</td>
<td>119</td>
</tr>
<tr>
<td>18 C</td>
<td>65</td>
<td>141±4</td>
<td>55±5</td>
<td>67</td>
<td>1/2$^-$</td>
</tr>
<tr>
<td>19 N</td>
<td>53</td>
<td>177±3</td>
<td>86±9</td>
<td>83</td>
<td>1/2$^- c$g</td>
</tr>
<tr>
<td>20 N</td>
<td>48</td>
<td>162±4</td>
<td>98±13</td>
<td>101</td>
<td>2$^- c$</td>
</tr>
<tr>
<td>21 N</td>
<td>43</td>
<td>149±7</td>
<td>140±44</td>
<td>151</td>
<td>1/2$^- c$</td>
</tr>
<tr>
<td>19 O</td>
<td>68</td>
<td>190±8</td>
<td>104±12</td>
<td>80</td>
<td>5/2$^+$</td>
</tr>
<tr>
<td>20 O</td>
<td>62</td>
<td>219±5</td>
<td>112±11</td>
<td>96</td>
<td>0$^+$</td>
</tr>
<tr>
<td>23 O</td>
<td>56</td>
<td>210±6</td>
<td>134±14</td>
<td>123</td>
<td>5/2$^+ c$e</td>
</tr>
<tr>
<td>22 O</td>
<td>51</td>
<td>206±4</td>
<td>120±14</td>
<td>140</td>
<td>0$^+$</td>
</tr>
<tr>
<td>23 O</td>
<td>47</td>
<td>114±9</td>
<td>7</td>
<td>122</td>
<td>1/2$^+ c$</td>
</tr>
</tbody>
</table>

As noted in Table 1, the present measurements may be compared to those made for 14 B [10,16] and 15,17,18 C [16–18]. While agreement is found for the momentum distributions, the integrated cross sections are systematically some 3–5 times higher than those reported at similar energies using the A1200 fragment separator [16,17]. Analysis of the transverse momentum distributions obtained in the present experiment demonstrate that the rather limited acceptances of the A1200 are the origin of this discrepancy [12]. In the case of 14 B, the present results and those of Ref. [10], also obtained using a high acceptance spectrometer, are in good accord.

In order to make a more quantitative analysis of the measurements and examine the utility of such reactions as a spectroscopic tool, extended Glauber type calculations have been carried out. The calculations, the principal features of which follow Refs. [3,27,28], include absorption (or stripping) and diffractive...

15 C: $S_n = 1.22$ MeV) and the large $\nu 2s1/2$ admixtures in the ground state wavefunctions, which may be related to extended valence neutron density distributions, as discussed below.
(or elastic) one-nucleon breakup. An important feature is that the $S$-matrices describing these processes have been derived from the microscopic interaction of Jeukenne, Lejeune and Mahaux (JLM) [30] within an eikonal approximation employing noneikonal corrections [32,33]. As discussed by Bonaccorso and Carstoiu [31] and Tostevin [5], such microscopic potentials are much better adapted to the intermediate energy range than optical limit [3] or global parameterisations [27]. In addition to the cross sections, the core longitudinal momentum distributions have been computed within this framework, as opposed to the black disk approximation of Ref. [7]. A detailed description of the calculations, together with the results obtained for the transverse momentum distributions and with a Ta target, will be presented elsewhere [12].

In terms of structure, overlaps were calculated between the ground state wavefunctions of the projectiles ($J^P$) and the core states ($I^P_K$) coupled to a valence neutron ($nlj$). The single-particle wavefunctions were defined within a Woods–Saxon potential with fixed geometry ($r_0 = 1.15$ fm, $a_0 = 0.5$ fm for $Z = 5$ and 6; $r_0 = 1.2$ fm, $a_0 = 0.6$ fm for $Z = 7–9$) with the depth adjusted to reproduce the effective binding energy ($S_{c}^{nl}$) which was fixed as the sum of the single-neutron separation energy and the excitation energy of the core state. The cross section to populate a given core final state is then,

$$\sigma (I^P_K) = \sum_{nlj} C^2 S(I^P_K, nlj) \sigma_{sp}(nlj, S_{c}^{eff}). \tag{1}$$

where $C^2 S$ is the spectroscopic factor for the removed neutron with respect to the core state and $\sigma_{sp}$ is the cross section for removal of the neutron by absorption ($\sigma_{abs}$), diffraction ($\sigma_{diff}$) and Coulomb dissociation (only $\sim 7$ mb in the most favourable cases — $^{14}$B and $^{15}$C [12]). The total inclusive one-neutron removal cross section ($\sigma_{Glauber}$) is then the sum over the cross sections to all core states. Similarly, the inclusive core momentum distribution is the sum of all core state momentum distributions, weighted by the corresponding cross sections. Within the framework of the spectator core description used here, excitation of the core in the reaction and final-state interactions are neglected.

The spectroscopic factors employed here have been calculated with the shell model code OXBASH [34] using the WBP interaction [35] within the 1p–2s1d configuration space. Where known, the experimentally established spin-parity assignments and core excitation energies have been used. In all other cases the shell model predictions have been assumed. The resulting cross sections and momentum distributions are displayed in Table 1 and Figs. 1 and 2. The breakdown of the calculated cross sections over the core states for each nucleus is detailed in Ref. [12]; as examples, and to aid in the following discussion, the results are listed for $^{14}$B and $^{15,17}$C in Table 2. As the momentum distributions reflect the orbital angular momentum of the removed neutron, the calculated distributions have been normalised to the peak number of counts to facilitate the comparison (Fig. 1). For all the nuclei observed, including those with known structure, very good agreement is found between the calculated and measured distributions and cross sections, with the exception of $^{22}$F, where the cross section is underestimated. Consequently, spin-parity assignments, derived from the shell model predictions, have been proposed for $^{15}$B, $^{17}$C, $^{19–21}$N, $^{21,23}$O, $^{23–25}$F (Table 1). In the case of $^{24}$F, a $3^+$ or $1^+$ assignment appears possible based on the present data [12]. The decay study of Reed et al. suggests, however, that the former is the most likely [23], in line with the shell model predictions.

Of particular interest amongst the nuclei investigated here are $^{14}$B and $^{15,17}$C, which, based on the relatively weak binding of the valence neutrons and measurements of the core momentum distributions and one-neutron removal cross sections, have been suggested to be one-neutron halo systems [10,16,17]. As may be seen in Figs. 1 and 2 and Table 2, the momentum distributions and cross sections for $^{14}$B and $^{15}$C are well reproduced by the present calculations employing the spectroscopic factors derived from the shell model, in which the ground state wavefunctions are predominately a $2s_{1/2}$ valence neutron coupled to the core ($^{13}$B and $^{14}$C in the ground state, as suggested by decay studies [36] and single neutron-transfer experiments [37]. In the case of $^{17}$C, a spin-parity assignment of $3/2^+$ is favoured, whereby the ground state configuration is predominately a $1d_{5/2}$ valence neutron coupled to the $^{16}$C core $2^+_1$ state. This confirms the suggestion of Bazin et al. [17] and the calculations of Ren et al. [38], and is supported by the recent observation of the 1.76 MeV gamma-rays de-exciting the $2^+_1$ state in $^{16}$C following one-neutron removal on $^{17}$C [39]. Such a structure, with a high $S_{c}^{eff}$
(2.49 MeV) and a valence neutron angular momentum of \( l = 2 \), excludes the possibility of any halo structure developing as evidenced by measurements of the total reaction cross section [40–43].

Moderate enhancements, however, have been observed in the total reaction cross section measurements for \(^{14}\text{B}\) [40–42]. Together with the ground state structure deduced from the present experiment and Refs. [10,17], it seems probable that a spatially extended valence neutron density distribution does occur; although the one-neutron binding energy of nearly 1 MeV will suppress the development of a distribution as large as that found in the more weakly bound one-neutron halo nuclei \(^{11}\text{Be}\) and \(^{19}\text{C}\). Detailed measurements of the total reaction cross section over a range of energies would thus be of particular interest in mapping out the density distribution of \(^{14}\text{B}\).

In the case of \(^{15}\text{C}\) the situation is unclear, with measurements of the total reaction cross section exhibiting no effect [40,41,43] and small enhancements [42,44]. Despite the predominately \( l = 0 \) character of the valence neutron, the higher neutron binding energy of \(^{15}\text{C}\) (\( S_n = 1.22 \) MeV) should restrict further the spatial extent of the neutron density distribution. Interestingly, very recent measurements of the charge-changing cross sections for the C isotopes exhibit an increase for \(^{15}\text{C}\) [45].

In summary, a systematic investigation of one-neutron removal reactions has been carried out on a series of neutron-rich sd-shell nuclei. The longitudinal momentum distributions and corresponding single-neutron removal cross sections for the core fragments were measured using a high acceptance spectrometer. Extended Glauber model calculations, coupled with spectroscopic factors derived from shell model calculations employing the WBP interaction, reproduce well the momentum distributions and cross sections. On this basis spin-parity assignments have been proposed for \(^{15}\text{B}, \; ^{17}\text{C}, \; ^{19–21}\text{N}, \; ^{21}\text{O}, \; ^{23–25}\text{F}\). Given the ground state configurations deduced here and measurements of the total reaction cross section, it is suggested that \(^{14}\text{B}\) presents a moderately extended valence neutron density distribution. This does not appear to be the case for \(^{17}\text{C}\), whilst \(^{15}\text{C}\) exhibits contradictory behaviour.

In more general terms it is concluded that high energy one-nucleon removal reactions represent a powerful spectroscopic tool far from stability. Moreover it has been demonstrated that coupled with a high acceptance, broad range spectrograph, such reactions offer a means to survey structural evolution over a wide range of isospin in a single experiment. The current development of large area, highly segmented, multi-element Ge-arrays [46,47] should further enhance the sensitivity of such studies.

**Acknowledgements**

The support provided by the staffs of LPC and GANIL is gratefully acknowledged. Discussions with J.A. Tostevin are also acknowledged, as is the guidance provided by B.A. Brown into the intricacies of the shell model and the assistance provided by G. Martínez in preparing the experiment. This work was funded under the auspices of the IN2P3-CNRS (France) and EPSRC (United Kingdom). Additional support from the Human Capital and Mobility Programme of the European Community (contract n° CHGE-CT94-0056) and the GDR Noyaux Exotiques (CNRS) is also acknowledged.
References

[12] E. Sauvan et al. to be published;
[26] M. Labiche et al., Preprint LPCC 00-06;
   M. Labiche et al., nucl-ex/0006003.
Low-lying intruder $1^-$ state in $^{12}$Be and the melting of the $N = 8$ shell closure

H. Iwasaki $^{a,*}$, T. Motobayashi $^b$, H. Akiyoshi $^c$, Y. Ando $^b$, N. Fukuda $^a$, H. Fujiwara $^b$, Zs. Fülöp $^{c,1}$, K.I. Hahn $^{c,2}$, Y. Higurashi $^b$, M. Hirai $^a$, I. Hisanaga $^b$, N. Iwasa $^c$, T. Kijima $^b$, A. Mengoni $^{c,d}$, T. Minemura $^b$, T. Nakamura $^a$, M. Notani $^c$, S. Ozawa $^b$, H. Sagawa $^e$, H. Sakurai $^c$, S. Shimoura $^b$, S. Takeuchi $^b$, T. Teranishi $^c$, Y. Yanagisawa $^b$, M. Ishihara $^{a,c}$

$^a$ Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
$^b$ Department of Physics, Rikkyo University, 3-34-1 Nishi-Ikebukuro, Toshima, Tokyo 171-8501, Japan
$^c$ The Institute of Physical and Chemical Research (RIKEN), 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
$^d$ ENEA, Applied Physics Division, Via Don Fiammelli 2, I-40129 Bologna, Italy
$^e$ Center for Mathematical Sciences, the University of Aizu, Aizu-Wakamatsu, Fukushima 965-8580, Japan

Received 29 June 2000; received in revised form 28 August 2000; accepted 28 August 2000

Editor: J.P. Schiffer

Abstract

Inelastic scattering of the neutron-rich nucleus $^{12}$Be on lead and carbon targets has been studied by measuring de-excitation $\gamma$ rays in coincidence with scattered $^{12}$Be. The strong $\gamma$-ray transition from the state at $E_x = 2.68(3)$ MeV following E1 Coulomb excitation was observed for the lead target, leading to an assignment of $J^P = 1^-$ for the excited state. The low excitation energy of this intruder $1^-$ state and the deduced large $B(E1; 0^+_g \rightarrow 1^-)$ value of 0.051(13)$e^2$ fm$^2$ provide a consistent picture of the $N = 8$ shell melting in $^{12}$Be. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 25.70.De; 25.60.-t; 23.20.-g; 27.20.+n

The electric dipole (E1) strength in a nucleus is largely exhausted by a giant dipole resonance, which is constructed from a superposition of many particle–hole excitations, and essentially no E1 strength appears in low energy region below 5 MeV. It is not the case for some of light nuclei. In particular, a strong E1 strength is expected if the low-lying intruder positive parity state appears close to the negative parity states. The transition between the first excited $1^-_g$ state at $E_x = 0.32$ MeV and the $\frac{1}{2}^+$ ground state in $^{11}$Be is a well-known example of this anomaly, representing one of the strongest E1 transitions ever observed between nuclear bound levels [1]. Such a favoured E1 transition may be induced by a decoupled feature of the valence neutron and an extended single-particle wave function of one neutron halo in these two loosely

* Corresponding author.
E-mail address: iwasaki@rarfaxp.riken.go.jp (H. Iwasaki).

1 On leave from ATOMKI, Debrecen, Hungary.

2 Present address: Department of Science Education, Ewha Woman's University, Seoul 120-750, Korea.
bound states. The measurement of the low-lying E1 strength has been extended to the continuum excitation of loosely bound neutron-rich nuclei $^{11}$Li [2 – 4] and $^{11}$Be [5], where halo neutrons also play an essential role to increase the strength.

Recently, the $^{12}$Be nucleus obtains much attention because of the possible shell melting in the $N = 8$ isotones [6 – 8]. An evidence of disappearance of magicity in $^{12}$Be has been obtained by our earlier study of proton inelastic scattering on $^{12}$Be [7]. A knockout reaction of $^{12}$Be measured at MSU also showed a strong indication of the shell melting in its ground state [8]. It is expected that the resultant smaller gap between $p$-shell and $sd$-shell brings down the lowest $1^-$ state considerably. A unique feature of $^{12}$Be is possible strong correlations of two neutrons outside the core in the ground state. These correlations may cause a coherent effect in the transition amplitudes and help to enhance the low energy E1 strength substantially [9].

Experimentally, an attempt has been made to search for the low-lying $1^-$ state in $^{12}$Be and an upper limit has been obtained for the E1 strength in the energy range from 0.15 MeV to 2.00 MeV [10]. The candidate of such a state has been observed in the neighboring isotope $^{11}$Li [2 – 4, 11, 12], though the experimental accuracy and the theoretical interpretation of the E1 strength are still controversial because the observed E1 strength is above the threshold [13, 14]. Since $^{12}$Be has a higher neutron threshold energy at 3.17 MeV, it is probable that the $1^-$ state appears as a discrete bound state. Thus a further experiment to find the $1^-$ state is strongly encouraged.

In the present experiment, we have studied inelastic scattering of $^{12}$Be on lead ($Z = 82$) and carbon ($Z = 6$) targets at intermediate energies of about 50 MeV/u. Comparison of the cross section between the high-$Z$ and low-$Z$ targets provides a tool to distinguish between the E1 ($l = 1$) and E2 ($l = 2$) excitations. While the Coulomb excitation cross section sharply rises with increasing target Z, the relative importance between the Coulomb and nuclear contributions varies with the transition multipolarity. For an unhindered E1 transition, the Coulomb contribution dominates over the nuclear contribution for a high-Z target, whereas only a small cross section due to the nuclear interaction is left for a low-Z target. For the case of E2 excitation, the nuclear contribution becomes more significant and may become compatible with the Coulomb contribution even with a heavy target such as lead. By exploiting these features, the inelastic scattering incorporating a combination of heavy and light targets has provided a useful means to populate and identify the $1^-$ state in $^{12}$Be.

The experiment was carried out at the RIKEN Accelerator Research Facility using the same experimental arrangement described in Ref. [7]. We measured de-excitation $\gamma$ rays in coincidence with inelastically scattered particles. Angle-integrated cross sections were deduced from the observed $\gamma$-ray yields. A radioactive $^{12}$Be beam was produced by the fragment separator RIPS [15] via fragmentation reactions of a 100 MeV/u $^{18}$O primary beam on a 1.11 g/cm$^2$ $^9$Be target. Two 1 mm thick plastic scintillators placed 5.3 m apart along the beam line were used to identify secondary beam particles on an event-by-event basis. The resulting beam of $^{12}$Be had a typical intensity of $2 \times 10^4$ counts per second. The isotopic purity was found to be around 96%. The $^{12}$Be beam bombarded a secondary target placed at the final focal plane of RIPS. Two different targets (350.8 mg/cm$^2$ thick $^{208}$Pb and 89.8 mg/cm$^2$ thick $^{12}$C) were used to excite the projectiles. The beam energies in the middle of the targets were calculated to be 53.3 MeV/u and 54.0 MeV/u, respectively, for the lead and carbon targets. A measurement with the target removed was also made to evaluate background contributions.

After passing through the secondary target, scattered particles were stopped in a $\Delta E$–$E$ plastic scintillator hodoscope (details are given in Ref. [7]) located downstream of the secondary target. The isotopic identification of the scattered particles was performed by the time-of-flight(TOF)–$\Delta E$ and TOF–$E$ methods. The hodoscope with a total active area of $1 \times 1 \text{m}^2$ had a finite acceptance up to 6.8 degrees. This angle corresponds to $\theta_{cm} = 7.3^\circ$ for the lead target and $\theta_{cm} = 13.9^\circ$ for the carbon target. In the present study, we have deduced the angle-integrated cross sections, defined as $\sigma_p(\theta_{cm} \leq 7.3^\circ)$ and $\sigma_c(\theta_{cm} \leq 13.9^\circ)$ for the respective targets. The overall efficiencies of the hodoscope relevant to the angle-integrated cross sections were estimated by a Monte Carlo simulation, which took into account the finite size and angular spread of the incident beam, the multiple scattering in the secondary targets (0.5 degrees and 0.1 degrees in r.m.s., respectively, for the lead and carbon targets), and the detector geometry. Theoretical angular distri-
butions calculated with the ECIS79 code [16] were incorporated in this simulation to properly evaluate the effective angular acceptance of the hodoscope. The calculated overall efficiencies were about 80%.

Fifty-five NaI(Tl) detectors surrounding the target were used to detect the de-excitation $\gamma$ rays. The intrinsic energy resolution of each detector was typically 7.0% (FWHM) for the 1275-keV $\gamma$ ray. The absolute efficiency and the line shape of the $\gamma$-ray energy spectrum were simulated by means of a GEANT code [17]. The total photo-peak efficiencies were calculated to be 7.1% and 5.7%, respectively, for 2.11-MeV and 2.68-MeV $\gamma$ rays emitted from a $^{12}$Be nucleus moving with $v/c \approx 0.3$. The simulated spectral shape of a $\gamma$ ray was used as a fitting function in deducing a photo-peak yield from the experimental energy spectrum.

Fig. 1 shows the Doppler-corrected $\gamma$-ray energy spectra measured in coincidence with scattered $^{12}$Be isotopes. In the figure, the de-excitation $\gamma$ rays, corresponding to the previously known $2^+_1 \rightarrow 0^+_g.s.$ transition in $^{12}$Be, are evident at 2.11(2) MeV for both the lead and carbon targets. Another peak is clearly observed at 2.68(3) MeV for the lead target, whereas no significant peak is observed for the carbon target. This yield dependence on the target indicates the dominance of the Coulomb contribution for the 2.68-MeV $\gamma$ rays. Since only two bound states (2.10 MeV and 2.70 MeV [18]) have ever been known in $^{12}$Be, it is likely that the 2.68-MeV peak corresponds to the transition from the second excited state to the ground state. The second excited state of $^{12}$Be has been observed in the vicinity of 2.70 MeV in several reactions such as $^{14}$C($^{14}$C, $^{12}$Be)$^{16}$O [19] and $^{10}$Be($t$, $p$)$^{12}$Be [20], while no clear $J^\pi$ assignment is given. Occurrence of the $\gamma$ transition as observed in the present measurement excludes the possibility of the $J^\pi = 0^+$ assignment discussed in Ref. [19,20].

In the present study, the measurement was also performed with a 60.0 MeV/u $^{10}$Be secondary beam incident on the lead target. This was made to evaluate the contribution from the $^{208}$Pb excitation leading to the $3^+_1$ state at 2.61 MeV, which is rather close to the observed $\gamma$-ray energy of 2.68 MeV. The $^{10}$Be + $^{208}$Pb inelastic scattering provided useful information on the magnitude of the $^{208}$Pb excitation, since it is almost identical with that for the $^{12}$Be reaction, while the $\gamma$-ray peaks of $^{10}$Be (such as the 3.37-MeV peak of the $2^+_1$ state) are more separate from the $^{208}$Pb peak. Fig. 2 (right) shows $\gamma$-ray energy spectra measured in coincidence with scattered $^{10}$Be isotopes. In the observed $\gamma$-ray spectrum without any Doppler correction (Fig. 2(b)), a clear peak can be seen around 2.61 MeV, showing that the $^{208}$Pb excitation indeed occurred. In contrast, the relevant events are widely distributed in the Doppler-corrected energy spectrum obtained with respect to the projectile frame of $^{10}$Be (Fig. 2(d)). When the data from the NaI(Tl) detectors placed at around 90° were used, the Doppler-corrected spectrum yielded a peak around 2.76 MeV corresponding to the $^{208}$Pb $\gamma$ transition. On the other hand, the $^{208}$Pb peaks were apart from either of the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Doppler-corrected $\gamma$-ray energy spectra measured in the inelastic scattering of $^{12}$Be on the lead (top) and carbon (bottom) targets.}
\end{figure}
two peaks from the $^{12}$Be excitation as far as the data with the other NaI(Tl) detectors were used. In the final analysis for $^{12}$Be, we therefore excluded the data from the 90° detectors and obtained the spectra shown in Fig. 2 (left). The 2.61-MeV peak is also clearly seen in the laboratory-frame spectrum (Fig. 2(a)). As confirmed in the case of $^{10}$Be, the $^{208}$Pb peak should not make any significant contribution to the energy region around 2.68 MeV in the Doppler-corrected spectrum of $^{12}$Be. Nevertheless, the spectrum shown in Fig. 2(c) clearly exhibits a peak at 2.68 MeV, verifying the assignment that the $\gamma$ transition belongs to $^{12}$Be.

The angle-integrated cross sections for the inelastic scattering were obtained from measured $\gamma$-ray yields after correcting for the detection efficiencies of both $\gamma$ rays and scattered particles. The angular distribution of the inelastic scattering was calculated by the ECIS code to be incorporated in the efficiency simulation as well as to deduce the transition strength as discussed later. For the $^{12}$Be + $^{208}$Pb scattering, we took the optical potential parameters as obtained by the $^{17}$O + $^{208}$Pb scattering at 84 MeV/u [21], while for the $^{12}$Be + $^{12}$C scattering, we used the potential parameters determined by the $^{12}$Be + $^{12}$C scattering measured at 57 MeV/u [22]. The nuclear contribution was evaluated by assuming a simple collective vibration mode with $\delta = \delta^N = \delta^C$, where $\delta^N$ and $\delta^C$ denote the nuclear and Coulomb deformation lengths. The uncertainty of the calculated efficiency arising from the theoretical angular distribution is negligible, as far as the transition multipolarity was taken among $E1$ ($M1$) or $E2$. This was because of the large dimension of the hodoscope. Note that the excitation via higher multipolarity is expected to be negligibly weak. Thus, for the purpose of the efficiency simulation, we safely assumed $E1$ multipolarity of the transition to the second excited state, while $E2$ multipolarity was obviously employed for the first excited state of $^{12}$C.

Table 1 shows the deduced angle-integrated cross sections. In deducing these cross sections, the photopeak yields were extracted after subtracting the estimated contributions from the $^{208}$Pb excitation. This was made using the simulated spectra for the 2.61-MeV $^{208}$Pb transition as shown by dotted curves in Fig. 2, which were calculated to match the data on $^{10}$Be. The cross section $\sigma_{\text{Pb}}$ of 46.5(11.5) mb was thus obtained for the 2.68-MeV peak of $^{12}$Be. The quoted error includes ambiguities in the photo-peak yield (21%), the hodoscope efficiency (5%), and the $\gamma$-ray detection efficiency (10%). Though no significant peak of the second excited state was observed for the carbon target, we could place the $1\sigma$ upper limit of

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E_x$ [MeV]</th>
<th>$\sigma_{\text{Pb}}(\theta_{\text{lab}} \leq 7.3°)$ [mb]</th>
<th>$\sigma_{\text{C}}(\theta_{\text{lab}} \leq 13.9°)$ [mb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^-$</td>
<td>2.68(3)</td>
<td>46.5(11.5)</td>
<td>&lt; 4.9</td>
</tr>
<tr>
<td>$2^+$</td>
<td>2.11(2)</td>
<td>81.8(12.8)</td>
<td>54.9(7.1)</td>
</tr>
</tbody>
</table>
4.9 mbo. The results of the angle-integrated cross sections exciting the $2_1^+$ state are also summarized in Table 1.

We first discuss the spin and parity of the second excited state of $^{12}$Be, since they have not ever been clearly assigned. As noted before, the excitation to the second excited state is dominated by the Coulomb contribution. Consequently, the electromagnetic transition strength can be readily deduced from the experimental cross section if a certain multipolarity is assumed, and the spin and parity of the 2.68-MeV state are constrained as follows. Utilizing the equivalent photon method based on the first order perturbation theory [23], the corresponding $\gamma$-decay strength was extracted from the experimental cross section 45.6 mb to be 0.050 Weisskopf units (W.u.), 11 W.u., and 11 W.u., respectively, for the cases of E1, M1, and E2 transitions. From the compilation of the experimental data in this mass region [24], a value of 11 W.u. for a M1 transition is too large and hence the $J^D = 1^+$ assignment can be excluded. Thus the remaining possibility is $J^D = 1^-$ (E1) or $J^D = 2^+$ (E2).

For inelastic scattering with a heavy target like lead, the Coulomb dominance of the E1 excitation has been proved both experimentally [25] and theoretically [26, 27], while the dominance becomes much weaker in the case of E2 transition of light projectiles [28]. Thus the observed Coulomb dominance already suggests E1 nature of the 2.68-MeV transition and hence the $1^-$ assignment for the second excited state. This conclusion is further substantiated by the ECIS calculation. The deformation length $\delta$ was translated from the measured cross section with the lead target. In the calculation, the deformation length was taken to be the same for the Coulomb and nuclear potentials. The results of $\delta$ were 0.24 fm and 1.56 fm, respectively, for the E1 and E2 cases. Using these deformation lengths, we estimated the cross sections with the carbon target. The results were 1.1 mb for E1 and 33.8 mb for E2. The experimental upper limit of the cross section, 4.9 mb, can only be explained by the E1 transition. It should be noted that the E1 cross section for carbon may be even smaller than the above quoted value (1.1 mb) due to the iso-scalar nature of the nuclear excitation induced by $^{12}$C.

The significant nuclear contribution to the E2 transition as expected above can be quantitatively shown by the ECIS analysis of the $2_1^+$ excitation. By assuming $\delta = \delta^N = \delta^C$, the deformation lengths were deduced to be 2.04(16) fm and 1.93(11) fm, respectively, for the data with the lead and carbon targets. Note that both these values are consistent with the result of 2.00(23) fm obtained by our earlier study on the proton inelastic scattering [7]. In these calculations, the nuclear excitation contribution yields 80.3 mb and 57.3 mb, respectively, for $\sigma_B$ and $\sigma_C$, demonstrating that the observed cross sections for both of the targets (81.8 mb and 54.9 mb) are dominated by the nuclear excitation.

Based on the above discussion, we conclude that the excitation to the second excited state occurred via an E1 transition. The spin and parity of the state is then determined to be $1^-$ uniquely. The E1 strength is obtained to be $B(E1; ^{12}\text{Be}, 1^-_x \rightarrow 1^-_g) = 0.051(13)e^2 fm^2$ by the ECIS calculation.

The location of the lowest $1^-$ state provides a useful measure of the energy difference between the $1p_{1/2}$ and $2s_{1/2}$ orbitals, $\Delta \epsilon = \epsilon(\frac{1}{2}^+^+) - \epsilon(\frac{1}{2}^-^-)$, since the main configuration of the $1^-$ state is expected to be the excitation between $1p_{1/2}$ and $2s_{1/2}$ states in a naive single particle picture. Comparison of the excitation energies of the $1^-$ state, $E_x(1^-)$, among $N = 8$ isotones ($^{16}$O: 7.12 MeV, $^{14}$C: 6.09 MeV, $^{12}$Be: 2.68 MeV) depicts the sharp decrease of $E_x(1^-)$ at the Be.
isotope, indicating the drastic narrowing of the shell gap at $^{12}\text{Be}$. In Fig. 3, the relevant energy levels are compared between the Be and C isotopes with $N = 7$ and $N = 8$. When one moves from $^{13}\text{C}$ to $^{11}\text{Be}$, $\Delta \varepsilon$ drops by about 3.4 MeV. A near degeneracy of the $2s_{1/2}$ and $1p_{1/2}$ orbitals achieved at $^{11}\text{Be}$ represents the $N = 8$ shell melting [29]. Similarly, $E_1(1^-)$ decreases from $^{14}\text{C}$ to $^{12}\text{Be}$. Incidentally the magnitude of the lowering of $E_1(1^-)$ is almost identical with that of $\Delta \varepsilon$ in the $N = 7$ isotones. This observation strongly supports that the degeneracy of the two orbitals is promoted in $^{12}\text{Be}$ as well as in $^{11}\text{Be}$. Recently the same trend of the anomalous reduction of $\Delta \varepsilon$ was found for the $N = 9$ nucleus $^{14}\text{B}$ in the $\beta$-decay study of $^{14}\text{Be}$ [30]. Thus, one can conclude that the $N = 8$ shell melting is a general phenomenon which occurs widely in the neutron-rich nuclei around $^{12}\text{Be}$.

Finally, we discuss the $E1$ strength, $B(E1) = 0.051(13)e^2\text{fm}^2$, obtained for the $0^+ \rightarrow 1^-$ transition, which is the first example of strong low-lying $E1$ transition observed in even-even nuclei. In groping for a possible enhancement mechanism, we refer to the prescription of the $E_1$ doorway state [9]. Firstly, the $^{12}\text{Be}$ ground state is considered as a $^{10}\text{Be}$ core and correlated two neutrons moving outside the core,

$$|^{12}\text{Be} : 0^+ = \alpha|(1p_{1/2})^2\rangle + \beta|(2s_{1/2})^2\rangle.$$  \hspace{1cm} (1)

In the limit of the complete degeneracy of the two orbitals, we have the amplitudes $\alpha = \beta = 1/\sqrt{2}$. Then a coherent $1^-$ excitation of the correlated two neutrons [9] is written as a doorway state for the dipole operator

$$\hat{D}_\mu^{\lambda = 1} = \frac{Z}{A} r_i Y_{1\mu}(r_i) - \frac{N}{A} N_i Y_{1\mu}(r_i);$$

$$|^{12}\text{Be}:1^- = \frac{1}{N} \hat{D}_\mu^{\lambda = 1} |^{12}\text{Be} : 0^+\rangle$$

$$= 0.63 (2s_{1/2} 1p_{1/2}) + 0.63 (1p_{1/2} 2s_{1/2}) + 0.45 (2s_{1/2} 1p_{3/2}),$$ \hspace{1cm} (2)

where $N$ is a normalization constant, and the coefficients are proportional to the single particle matrix elements of the dipole operator $\hat{D}_\mu^{\lambda = 1}$. In this equation, the particle–hole excitations are limited to the configuration space of $p, s$ orbitals to highlight coherent contributions of these excitations. The coefficients are obtained taking into account the small separation energies of the $1p_{1/2}$ and $2s_{1/2}$ states in $^{12}\text{Be}$. The $B(E1)$ value between the two states (1) and (2) is expressed as

$$B(E1; 0^+ \rightarrow 1^-) = \left| (1^- \mid \hat{D}^{\lambda = 1} \mid 0^+) \right|^2$$

$$= 0.63 (2s_{1/2} \mid \hat{D}^{\lambda = 1} \mid 1p_{1/2}) + 0.63 (1p_{1/2} \mid \hat{D}^{\lambda = 1} \mid 2s_{1/2}) + 0.45 (2s_{1/2} \mid \hat{D}^{\lambda = 1} \mid 1p_{3/2})^2.$$ \hspace{1cm} (3)

We can see in Eq. (3) that the degeneracy of $1p_{1/2}$ and $2s_{1/2}$ states indeed enhances the $B(E1)$ value about twice more than the single-particle transition rate $|\langle 2s_{1/2} \mid \hat{D}^{\lambda = 1} \mid 1p_{1/2} \rangle|^2$.

To make a more quantitative study, a shell model calculation involving a larger configuration space is desirable.

There has been so far one theoretical attempt to describe the low-lying $1^-$ state in $^{12}\text{Be}$ [31]. This work, which is based on a two-neutron pairing model [32], has predicted the lowest $1^-$ state in $^{12}\text{Be}$ at 2.7 MeV with a somewhat larger $B(E1)$ value of $0.23e^2\text{fm}^2$. They also pointed out that the strong correlation between the $(1p_{1/2})^2$ and $(2s_{1/2})^2$ configurations enhances the $E1$ strength. The deviation of the calculated $B(E1)$ value from the present experimental result might be attributed to the extended wave function adopted for the calculation.

In conclusion, we have studied inelastic scattering of the $^{13}\text{Be}$ nucleus using lead and carbon targets. The spin-parity assignment $J^P = 1^-$ for the state at 2.68(3) MeV explains successfully the experimental cross sections of the two targets, excluding other possible spin-parity assignments. The large $B(E1; 0^+_g.s. \rightarrow 1^-) \sim 0.051(13)e^2\text{fm}^2$ for the 2.68-MeV state was deduced. The lowering of the intruder $1^-$ state accompanied with the large E1 strength represents the characteristic feature of degenerate $1p_{1/2}$ and $2s_{1/2}$ states, thus indicating the melting of $N = 8$ magicity in $^{12}\text{Be}$.

Acknowledgements

Sincere gratitude is extended to the staff members of the RIKEN Ring Cyclotron for their operation of
the accelerator during the experiment. The present work is supported in part by the Ministry of Education, Science, Sports and Culture by Grant-In-Aid for Scientific Research under the program number (B) 08454069.

References

Effect of the intermediate velocity emissions on the quasi-projectile properties for the Ar + Ni system at 95 AMeV

INDRA Collaboration

D. Doré a,d,h, Ph. Buchet a, J.L. Charvet a, R. Dayras a, L. Nalpas a, D. Cussol b, T. Lefort b, R. Legrain a, C. Volant a, G. Auger e, Ch.O. Bacri d, N. Bellaize b, F. Bocage b, R. Bougault b, B. Bouriquet c, R. Brou b, A. Chibihi c, J. Colin b, A. Demeyer a, D. Durand b, J.D. Frankland c, E. Galichet d, E. Genouin-Duhamel b, E. Gerlic e, D. Guinet e, S. Hudan c, P. Lautesse e, F. Lavaud d, J.L. Laville c, J.F. Lecolley b, C. Leduc e, N. Le Neindre b, O. Lopez b, M. Louvel b, A.M. Maskay e, J. Normand b, M. Parlog f, P. Pawlowski d, E. Plagnol d, M.F. Rivet d, E. Rosato g, F. Saint-Laurent c, J.C. Steckmeyer b, M. Stern e, G. Tabacaru d, B. Tamain b, L. Tassan-Got d, O. Tirel c, E. Vient b, J.P. Wieleczko c

a DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France
b LPC Caen (IN2P3-CNRS/ISMRA et Université), 14050 Caen Cedex, France
c GANIL (DSM-CEA/IN2P3-CNRS), B.P. 5027, 14076 Caen Cedex 5, France
d IPN Orsay (IN2P3-CNRS), 91406 Orsay Cedex, France
e IPN Lyon (IN2P3-CNRS/Université), 69622 Villeurbanne Cedex, France
f Nuclear Institute for Physics and Nuclear Engineering, Bucharest, Romania
g Dipartimento di Scienze Fisiche, Univ. di Napoli, 180126 Napoli, Italy
h Conservatoire National des Arts et Métiers, 75141 Paris Cedex 03, France

Received 20 April 2000; received in revised form 29 June 2000; accepted 25 August 2000
Editor: V. Metag

Abstract

The quasi-projectile (QP) properties are investigated in the Ar + Ni collisions at 95 AMeV taking into account the intermediate velocity emission. Indeed, in this reaction, between 52 and 95 AMeV bombarding energies, the number of particles emitted in the intermediate velocity region is related to the overlap volume between projectile and target. Mean transverse energies of these particles are found particularly high. In this context, the mass of the QP decreases linearly with the impact parameter from peripheral to central collisions whereas its excitation energy increases up to 8 AMeV. These results are compared to previous analyses assuming a pure binary scenario. © 2000 Elsevier Science B.V. All rights reserved.

* Corresponding author.

E-mail address: dore@in2p3.fr (D. Doré).

1 Present address: CEA, DRFC/STEP, CE Cadarache, 13108 Saint-Paul-lez-Durance, France.

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.

PIE: S0370-2693(00)01007-8
The quasi-projectile deexcitation has been studied through a wide variety of systems at intermediate energies [1–10]. In this energy domain a transition from a binary process, leading to two main excited fragments (the quasi-projectile (QP) and the quasi-target (QT)) in the exit channel, towards a participant-spectator [11] mechanism, is expected. From inclusive or semi exclusive measurements it has not been possible to distinguish between these two mechanisms. In some cases, the experimental data could be described equally well either assuming a pure binary mechanism or a geometrical process [3,12]. With the improvement of experimental setups, namely the advent of 4π multidetectors allowing fully exclusive measurements, it should become possible to reconstruct the QP and the QT from their decay products on an event by event basis. However, this reconstruction process depends greatly on our ability to identify unambiguously the origin of the detected products. Unfortunately, in the intermediate energy range, the various sources of emission strongly overlap in the velocity space. Thus one has to rely on some assumptions on the underlying mechanisms in order to unfold the various sources of emission. Although it was generally admitted that below 100 A MeV, heavy ion collisions have essentially a binary character, it has been shown since several years that the decay products could not be fully imputed to the decay of excited quasi-projectile and quasi-target [9,10,13–19]. Besides preequilibrium and direct emissions already observed at low energies, processes like neck emission and aligned fission had to be taken into account in order to explain the experimental data. Indeed an excess of particles and fragments, not explained by the statistical deexcitation of excited QP and QT, is observed at intermediate velocity with unusual kinematical properties.

From recent experimental data on the Ar + Ni reactions between 52 and 95 A MeV obtained at GANIL with the 4π multidetector INDRA it was shown [19] that it was not possible to reproduce the light particle rapidity spectra by assuming only statistical emissions from excited QP and QT and that there was an excess of high energy particles at mid-rapidity which increases as the impact parameter decreases. In the present paper, we will concentrate on the Ar + Ni reaction at 95 A MeV and we will show how the properties of the QP that one can extract from the data are strongly affected by particle emission around mid-rapidity. First the impact parameter classification and the event selection will be presented. Then, the QP properties, mass and excitation energy, will be established according to two basic assumptions: (i) Neglecting mid-rapidity emission, following previous analysis [20,21], a two source reconstruction will be performed in the frame of a purely binary scenario; (ii) In an attempt to take into account mid-rapidity emission (MRE) as evidenced in [19] we will unfold the experimental light charged particle rapidity spectra assuming three sources of emission, the QP, the QT and a third source of emission to simulate the mid-rapidity contribution. Thermal and shape equilibrium are assumed in each source. Then the properties of the QP are extracted from its decay products. In both cases, the mass and the excitation energy of the QP thus obtained will be presented as a function of an experimental impact parameter. Finally, results of both reconstruction methods will be compared and discussed.

The experiment was performed at the GANIL facility which provided an 36Ar beam of (3–4) × 107 pps at 95 A MeV. After collision with a 193 µg/cm² self-supporting 58Ni target, reaction products were detected with the 4π charged particle detector INDRA [22] with a minimum bias trigger requiring a four fold event. Charge identification is achieved up to the projectile charge in the forward hemisphere. Hydrogen and helium isotopes are separated for detection angles from 3° to 176° (rings 2 to 17).

Using the prescription of Ref. [23], an impact parameter scale \( b_{\text{exp}} \) is deduced from the total transverse energy distribution \( E_{\tau}^{\text{tot}} \) for all detected events as shown by the full line in Fig. 1(a). For the forthcoming analysis, we will retain only events for which, at least the remnant of the QP has been detected. This is done using the correlation between the total detected charge \( Z_{\text{tot}} \) and the pseudo total parallel momentum \( P_{\text{tot}} = \sum Z_i \times V_i \) presented in Fig. 1(b). Only events for which \( P_{\text{tot}} \geq 70\% \) \( P_{\text{proj}} \) are
kept. This condition selects events with $Z_{\text{tot}}$ around and larger than the projectile charge and represents $\approx 60\%$ of the estimated [24] total reaction cross section, $\sigma_{\text{th}}^{\text{tot}}$. whereas the total detected one amounts to $80\%$ of $\sigma_{\text{th}}^{\text{tot}}$. We remark that the selected events (dashed line in Fig. 1(a)) still cover the whole range of $E_{\text{tot}}^{\text{tr}}$.

Proton and alpha particle reduced rapidity spectra ($Y/Y_p$ where $Y_p$ is the projectile rapidity) show two components centered respectively around the target and the projectile velocities as shown in Fig. 2 for protons at $b_{\text{exp}} = 6$ fm. This strongly suggests evaporation from excited QP and QT. Then, assuming a binary scenario and neglecting any non equilibrated emissions [20,25], all particles and fragments are attributed to the QP or the QT event-by-event. The reconstruction is based on a simplified version of the thrust method [26]. Both procedures roughly allocate all particles and fragments with a parallel velocity smaller than the center-of-mass velocity to the QT and the others to the QP. Charges, masses and velocities of both sources are then calculated. Neutrons added in order to obtain the total mass of the system are distributed between the QP and the QT according to the $N/Z$ ratio ($= 1.04$) of the system. From simulations [27], the neutron kinetic energies are evaluated as the mean kinetic energy of the protons minus 2 MeV to take into account the absence of Coulomb barrier. Calorimetry is then used to calculate the excitation energy ($E^*$) of the QP. Event by event
we have \( E^* = \Sigma_i (m_i c^2 + E_i) - m_s c^2 \), where \( m_i \) is the mass of each particle/fragment, \( E_i \) their kinetic energy in the QP frame and \( m_s \) the mass of the source. The mass of the QP thus reconstructed (around 34) is almost independent of the impact parameter. In contrast, the excitation energy per nucleon increases almost linearly with decreasing impact parameter to reach 18 A MeV for central collisions. This value is in agreement with the one obtained in [21] for violent collisions. These results are shown by the full circles in Fig. 4 and will be further discussed in connection with the results of the second assumption.

It was shown in [19] that isotropic evaporation from excited QP and QT was not sufficient to explain the measured light products (\( Z \leq 6 \)) rapidity spectra. In particular, there is an excess of particles emitted around mid-rapidity which cannot be explained by a simple overlap of the QP and QT emission spheres. This mid-rapidity contribution increases with decreasing \( b_{\text{exp}} \). Furthermore, the average transverse energy \( \langle E_{\text{tr}} \rangle \) of these particles (Fig. 3(a) full circles) is much higher than expected from evaporation. The same behaviour is observed for all products of \( Z \leq 6 \), suggesting that the excess of particles at intermediate velocity has peculiar kinematical properties. It has to be noted that due to detection thresholds, the \( \langle E_{\text{tr}} \rangle \) values around the target rapidity are artificially increased.

In order to take into account this intermediate velocity component, besides the two evaporating sources, emission from a third source around mid-rapidity was assumed. A fit procedure, widely used to modelize differential cross sections [28–30], is performed supposing three thermalized sources. The laboratory energy spectra are fitted with the sum of three Maxwellian distributions assuming volume emission [31]:

\[
\frac{d^2\sigma}{dE \, d\Omega} = \sum_{i=1,3} N_i \sqrt{E_i} 	imes \exp\left[ -(E_i + E_{\text{si}} - 2\sqrt{E_i E_{\text{si}} \cos(\theta_i)}) / T_i \right].
\]

Due to statistics, only energy spectra of light particles (p, d, t, \(^3\)He, \(^4\)He) are fitted. For each particle type, the detection energy thresholds are adjusted in order to have the same value for all rings (independently of the experimental thresholds which may fluctuate slightly from one detector to the other). For a given impact parameter bin and a given particle type, all energy spectra from ring 2 to ring 17 are fitted simultaneously [19]. As ring 1 (2–3) does not provide isotopic separation, it is not included in the fit. Thus, a set of parameters is obtained for each light particle type and each impact parameter bin. As shown in Fig. 3(b), the overall quality of the fits is quite good. Distribution irregularities are due to experimental biases. Solid angles are different from one ring to the other. The average angle of a ring being used to calculate the rapidity, the distributions are slightly distorted. For protons, at \( b_{\text{exp}} = 3 \) fm, it is found that the mid-rapidity component contributes significantly to the total proton rapidity distribution and covers the whole rapidity range. Direct emissions evaluated with intranuclear cascade calculations give similar results...
Fig. 3. Data and fit results for $b_{\text{exp}} = 3$ fm. (a) Average transverse energy $\langle E_{\text{tr}} \rangle$ of protons vs reduced rapidity. The experimental data (full circles) are compared with the result of a three source fit (open circles). (b) Proton reduced rapidity distribution. The experimental data are indicated by the dark histogram and fit result by the grey line. The fit contributions of the QP, QT (grey lines) and the MR (dashed line) are drawn. The arrow labelled $Y_{\text{nn}}$ indicates the nucleon–nucleon reduced rapidity. For experimental and calculated spectra, an energy threshold of 2 MeV was imposed.

[33,34]. One notes also that the average transverse energies, $\langle E_{\text{tr}} \rangle$ as a function of $Y/Y_p$ are well reproduced (Fig. 3(a)).

The fit parameters for protons and alpha particles evolve rather smoothly with $b_{\text{exp}}$ from central to peripheral collisions (see Table 1). The proton source reduced rapidities are rather constant from central to peripheral collisions for the QP ($0.91 < Y_{\text{QP}}/Y_p < 0.93$) and the mid-rapidity source ($0.46 < Y_{\text{MRE}}/Y_p < 0.49$). For other particles, both rapidities increase with impact parameter. The apparent temperature of the QP increases significantly from peripheral to central collisions (Table 1). At a given impact parameter, different particle types yield different temperatures in contradiction with the equilibrium hypothesis. In [32], similar deviations were observed and their possible origin discussed. It has been shown [35] that introducing nucleon–alpha scattering could improve significantly the fit for the alpha particle rapidity spectra. Thus, nucleon-cluster collisions in the region of overlap between projectile and target may be in part responsible for the discrepancies between the temperature parameters and source rapidities obtained in our simple three source fits for different particle types. For all particles, the apparent temperatures of the mid-rapidity source are large (Table 1) and increase strongly from peripheral to central collisions where they reach $\simeq 25–30$ MeV. These variations with
Table 1
Fit parameters for protons and alpha particles in the quasi-projectile (QP) and the mid-rapidity component (MRE)

<table>
<thead>
<tr>
<th></th>
<th>QP</th>
<th>MRE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rapidity $Y/\gamma_p$</td>
<td>Temp. (MeV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b (fm)</td>
<td>0–1</td>
<td>7–8</td>
</tr>
<tr>
<td>proton</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td>alpha</td>
<td>0.82</td>
<td>0.95</td>
</tr>
</tbody>
</table>

The contribution of each source to the rapidity distribution depends upon the impact parameter and the particle type. The multiplicities of particles emitted by the QP and the QT follow the same evolution. The proton multiplicity for the QP (Table 1) stays constant around 1.5 from peripheral to mid-central collisions and then decreases to reach 0.5 in central collisions. For alpha particles, the multiplicity starts at a value of 0.5 in peripheral collisions to reach a maximum around 1.2 in mid-central collisions and then decreases to reach a value of 0.6 in central collisions. This behavior can be understood if the size of the source decreases with decreasing impact parameter while the temperature increases. For other particles the evolution is intermediate between that of protons and alpha particles. By contrast the multiplicity of particles emitted near mid-rapidity increases strongly as the impact parameter decreases whatever the particle type. This evolution suggests a geometrical effect as we will discuss later.

In order to evaluate the robustness of these results, several tests have been performed. Using different prescriptions to fit the data, constraining some of the parameters, adding Coulomb barriers, assuming surface emission instead of volume emission, lead essentially to the same evolution of the parameters (velocities, temperatures and multiplicities) with impact parameter. Assuming a surface emission for QP and QT, their source temperatures are found slightly lower but the fits are in poorer agreement with the experimental data. Using the heaviest fragment in the forward hemisphere, $Z_{\text{max}}$, as an indicator of the impact parameter (the closer to the charge of the projectile the fragment charge is, the larger is the impact parameter) avoids the correlation between the temperature and the impact parameter [35]. This procedure yields lower QP and QT temperatures but does not affect the relative contributions of each source. In [19], another global variable, related to the dissipated energy in the forward hemisphere, was used to select events according to the violence of the collision and emissions between 75° and 105° in the mid-rapidity frame were studied. In this case, the temperature parameters of the mid-rapidity evolved from 17 to 20 MeV for protons with the centrality of the collision. The event selection and the angular cut explain the differences between these results and those presented here. However, we remark that these values are located inside our limits (see Table 1).

The next step is the reconstruction of the QP as a function of impact parameter. The mean multiplicity and energy of each particle emitted by the QP at each impact parameter bin are used. Because the projectile has $N = Z$, neutrons are added assuming that neutron multiplicity is equal to proton multiplicity ($\langle \text{mult}_n \rangle = \langle \text{mult}_p \rangle$). The contribution of fragments at mid-rapidity being small, those are shared between the QP and the QT as in the two source analysis previously described. For a given impact parameter, the mass of the QP is calculated as,

$$
(A_{\text{QP}}) = \sum_i \langle \text{mult}_i \rangle \times A_i + \sum_f \langle \text{mult}_f \rangle \times A_f + \langle \text{mult}_n \rangle \times A_n
$$

(2)
and its excitation energy is estimated through calorimetry.

\[
\{E_{\text{QP}}\} = \sum_i \{\text{mult}_i\} \times \left( m_i c^2 + \frac{3}{2} T_i \right) \\
+ \sum_f \{\text{mult}_f\} \times \left( (m_f c^2 + \langle E_f \rangle) \right) \\
+ \{\text{mult}_n\} \times \left( m_n c^2 + \frac{3}{2} T_p \right) - \{m_{\text{QP}} c^2\},
\]

where \(i, n, f\) are the index for light charge particles, neutrons and fragments and QP refers to the emitting source. \(A\)'s are the atomic masses, \(m\)'s, the masses, \(\langle E_f \rangle\) are the fragment mean kinetic energies. Neutron temperature \(T_n\) is assumed to be equal to proton temperature \(T_p\). One can note that fragments have an important contribution in (2) due to their masses and a small one in (3) due to their low kinetic energies. All this reconstruction assumes that particles originate from the same source even if the velocities obtained with the fits are different. This difference being larger for small impact parameter, values below 3 fm are less significant.

The QP masses and excitation energies thus obtained, are presented (stars) in Fig. 4 as a function of the impact parameter, together with the results (circles) of the previous two source analysis. Whereas a two source analysis yielded QP masses independent of impact parameter, in contrast, for the three source approach, one notes in Fig. 4(a) a linear increase with impact parameter of the QP mass: from 10 for central collisions to 32 for peripheral ones. For peripheral collisions containing few mid-rapidity particles, both scenarii lead to nearly identical results. The linear mass increase with impact parameter suggests a geometrical dependence. In Fig. 4(a) the curve represents the QP mass predicted in a calculation [2], where the geometrical overlap of projectile and target is considered as the intermediate source and the non interacting volumes are taken as QP and QT. Although the general trend with impact parameter is similar to the three source result, the predicted QP mass decreases more rapidly with decreasing \(b_{\text{exp}}\) than obtained from the three source analysis. This discrepancy at low impact parameter may be imputed, in part, to the \(b_{\text{exp}}\) determination. One can also argue that at 95 A MeV the participant-spectator regime is not fully reached.

Dynamical calculations for small systems [9,36,37] present a similar relation between the QP mass and the impact parameter if particles emitted before the reseparation time of QP and QT are not included in the QP reconstruction. These calculations also show that these “early” particles are distributed over the whole range of parallel velocity as deduced from the three source fits. To obtain a realistic estimation of the QP emissions, it is important to subtract the mid-rapidity component over the whole rapidity range.

Excitation energies deduced from both analyses are compared in Fig. 4(b). The \(Z_{\text{max}}\) sorting (open symbols) has also been tested in order to roughly evaluate the effect of the event sorting. The \(Z_{\text{max}}\) value is the one corresponding to the more abundant QP residue in the considered \(b_{\text{exp}}\) bins. As observed, the obtained values are close to those of the three source fit method based on the \(b_{\text{exp}}\) sorting, indicating that the sorting has only small effect on the results. In all cases the excitation energies increase with decreasing impact parameter. However, except for the most peripheral collisions where mid-rapidity emission is negligible, the three source fit method yields excitation energies about a factor of two smaller than the two source analysis. As \(b_{\text{exp}}\) decreases, the mid-rapidity component carries an increasing amount
of the deposited energy, limiting the excitation energy imparted to the QP and the QT.

Preliminary analyzes between 32 and 95 A MeV [38] show that beyond 52 A MeV the yields of the different sources become independent of the bombarding energy. The mean transverse energy of the mid-rapidity component increases linearly with bombarding energy while it is constant for the QP and QT contributions. Above ~ 50 A MeV, most of the energy is deposited into the overlap region between projectile and target and is evacuated by the mid-rapidity particles.

Quasi-projectiles produced in the reaction 36Ar + 58Ni at 95 A MeV have been reconstructed from their decay products under two basic assumptions, (i) purely binary collisions, (ii) additional emission from the overlapping zone between projectile and target. The properties (mass and excitation energy) of the QP thus reconstructed depend strongly upon these assumptions. Indeed, for mid-central collisions, there is a factor of 1.71 between QP masses and 1.76 between excitation energies, the assumption (ii) leading to the lowest estimation. It has been shown that the properties of particles emitted at mid-rapidity are incompatible with an evaporation process from fully equilibrated quasi-projectiles and quasi-targets which made the assumption (i) unrealistic. Based upon results of (ii), these particles cover the whole rapidity range and mix in part with particles evaporated from the excited quasi-projectile and quasi-target. The unfolding procedure presented in this work is an important step in order to reconstruct sources with precision. The additional source in assumption (ii) includes many processes and it will be necessary to disentangle them to go further in the interpretation.

References

[34] D. Doré et al., in preparation.
Abstract

The breakup of radioactive $^{17}$F into a proton and $^{16}$O was measured by bombarding $^{208}$Pb with 170 MeV $^{17}$F. The angular correlations of the fragments and the energy distributions of the protons suggest that the dominant breakup mechanism is a direct process in which a proton is excited into the low energy continuum above the breakup threshold. The breakup cross section measured near the grazing angle is $6.0 \pm 0.7 \text{ mb/sr}$ for analysis with postacceleration considered and $3.9 \pm 0.4 \text{ mb/sr}$ without postacceleration. This cross section is small compared to the fusion cross section at energies near the Coulomb barrier, so the breakup can have little influence on the fusion process.

PACS: 25.60.-t; 25.60.Ge; 25.70.-z

Keywords: Radioactive ion beam; Breakup with weakly bound nuclei; Fusion

The identification of certain nuclei in which the valence nucleon(s) are distributed considerably outside the core (halo nuclei), has stimulated extensive discussions on fusion reactions involving these nuclei at energies near and below the Coulomb barrier [1 – 3]. For neutron halo nuclei, the extended radius reduces the barrier which the fusing nuclei have to overcome since the Coulomb barrier is inversely proportional to the separation of the two nuclei. Moreover, if a halo nucleus is excited to the low-lying soft dipole resonance, an isovector polarization of the nucleus can occur in which the charged core of the halo nucleus is displaced in the direction away from the reaction partner; this can also contribute to lowering the barrier [4]. In either case, the fusion cross section is enhanced. In contrast, proton halo nuclei are polarized such that the valence proton is repelled from the reaction partner. Therefore, the change in the barrier height and the fusion rate is expected to be small.

The nuclear halo is commonly associated with a very low binding energy of the valence nucleon(s). The weakly-bound valence nucleon(s) can be detached from the core nucleus when the halo nucleus impinges upon another nucleus. The breakup of the halo nucleus in the entrance channel removes some incident flux available to the fusion reaction which reduces the fusion probability [2,3]. However, the coupling of...
reaction channels which are open at subbarrier energies causes the Coulomb barrier to split into multiple barriers distributed around the uncoupled barrier [5]. The fusion cross sections are enhanced when the incident beam energy exceeds the lower-energy barriers. In this case, the breakup channel can lead to a fusion enhancement [1]. Whether the fusion rate will be enhanced or suppressed at subbarrier energies is still an open question [6].

Two recent experiments conducted with the two-neutron skin nucleus $^{6}\text{He}$, produced by low energy accelerators, on $^{209}\text{Bi}$ [7], and $^{238}\text{U}$ [8] showed a large enhancement of subbarrier fusion. Furthermore, a very large $^{6}\text{He}$ breakup cross section has been observed near the barrier [9]. In contrast, no evidence of fusion enhancement was observed in the bombardment of $^{208}\text{Pb}$ with the proton drip-line nucleus $^{17}\text{F}$ [10]. The $5/2^+$ ground state of $^{17}\text{F}$ has a small rms radius whereas the $1/2^+$ first excited state is expected to have an extended rms radius [11]. The fusion excitation function for the $^{17}\text{F}$-induced reaction behaves very similarly to that of the stable $^{19}\text{F}$-induced reaction and the authors of Ref. [10] speculated that the $^{17}\text{F}$ breakup cross section is very small.

The objective of this work was to explore the correlation between breakup and fusion. Because the intensity of the radioactive $^{17}\text{F}$ beam is low and larger breakup yields are expected at high energies [12], the measurements were performed at an energy much higher than the Coulomb barrier to make the experiment less difficult. Information concerning the breakup process near the barrier can be inferred from the high energy results.

The experiment was carried out at the Holifield Radioactive Ion Beam Facility (HRIBF) at Oak Ridge National Laboratory. The radioactive $^{17}\text{F}$ was produced by the $^{16}\text{O}(d,\alpha)$ reaction using a 42 MeV deuteron beam from the Oak Ridge Isochronous Cyclotron (ORIC) to bombard a fibrous hafnium oxide target [13]. The radioactive $^{17}\text{F}$ was made into a beam by the Isotope Separator On Line (ISOL) method using a kinetic ejection negative ion source [14]. Negatively charged $^{17}\text{F}$ ions were extracted from the ion source, mass analyzed and accelerated by the 25 MV tandem accelerator to 170 MeV. The accelerated beam was a mixture of $^{17}\text{O}$ and $^{17}\text{F}$ with an average ratio of $^{17}\text{O}$ to $^{17}\text{F}$ of 8 to 1. To obtain a pure $^{17}\text{F}$ beam, the beam was stripped to the 9+ charge state by an 80 $\mu$g/cm$^2$ carbon foil located at the exit of the accelerator but before the 90° analyzing magnet. The intensity of the $^{17}\text{F}^{9+}$ beam delivered to the target was $2.2 \times 10^5$ particles per second which was measured by a beam counter consisting of a 20 $\mu$g/cm$^2$ carbon foil and a microchannel plate mounted at the entrance of the target chamber [15]. The beam was focused onto a phosphor at the target position to a spot approximately 2 mm in diameter. During data taking, the beam counter was moved away from the beam path and the beam was monitored by a 200 mm$^2$ Si detector at 10° with respect to the beam axis and by the Enge split-pole magnetic spectrograph at 2.5° on the other side of the beam. The focal plane detector in the spectrograph was a position sensitive avalanche counter as described in Ref. [15]. The number of particles elastically scattered to the Si detector and into the Enge spectrograph was used to monitor the position of the beam. The elastic scattering measured by the Si detector was used for normalization.

A self-supporting $^{208}\text{Pb}$ target of 2 mg/cm$^2$ thickness was mounted in the center of the target chamber. The breakup proton and $^{16}\text{O}$ were measured by a $5 \times 5$ cm$^2$ silicon Double Sided Strip Detector (DSSD) placed 8.4 cm from the target and centered at 45° with respect to the beam. The DSSD, which has 16 vertical and 16 horizontal strips, provides information on the energy and position of the particles. The thickness of the DSSD is 300 $\mu$m which stops the breakup $^{16}\text{O}$ but allows some of the breakup protons to penetrate. A 100 $\mu$m-thick Si surface barrier detector (SBD) of diameter 3.5 cm was mounted behind the DSSD to detect the protons. The breakup events were identified by coincidences between the proton in the SBD and $^{16}\text{O}$ in the DSSD. Because the counting rate was very low, the contribution from random coincidence events was negligible.

Since at this bombarding energy the reaction products can originate from several reaction mechanisms such as breakup, nucleon transfer, fusion–fission, fusion–evaporation, and pre-equilibrium emission, the breakup events must be identified by a coincidence technique. Among the possible reactions, only breakup will produce a proton and an $^{16}\text{O}$ simultaneously. To verify this argument, a second experiment was performed by placing the SBD in front of the DSSD with the sole purpose of observing coincidences between the breakup proton and $^{16}\text{O}$. In this experiment, the
17F beam intensity was 9 × 10^5 particles per second. The 16O leaves approximately 60 MeV in the SBD; therefore both the 16O and proton can be identified by E−ΔE methods. However, in a coincident event, the energy measured by the SBD is the sum of the 16O and proton energies. The breakup protons were identified by requiring a coincidence in the DSSD with the 16O identified by energy loss in the SBD. Indeed, the coincidence between the breakup fragments was observed. This measurement demonstrated the validity of the results obtained with the detector system in the normal configuration. All discussion below is based on the data acquired with the DSSD in front of the SBD.

A very large dynamic range in particle energy measurement is required for detecting the breakup products. The energy of the breakup protons can be an order of magnitude smaller than that of the 16O. In order to be able to identify the protons, the pulse amplitude out of the amplifiers for the DSSD was saturated for the high energy heavy ions. If in a coincident event the proton and 16O strike the same DSSD strip, the pulse height recorded will be nearly equal to that of the 16O and will appear to be a singles event. Another kind of coincident event which will look like a singles event is when a breakup proton stops in the DSSD because there is no event recorded in the SBD. Furthermore, if breakup occurs at high excitation energies of 17F, which results in the products being produced under large relative energies, the geometric acceptance of the detectors allows for simultaneous detection of particles only if they are emitted almost parallel to the trajectory of 17F. For breakup particles emitted transverse to the direction of 17F, only one of the fragments can be detected if the relative energy is large. In order to obtain cross sections from the coincidence data, it is therefore necessary to perform Monte Carlo simulations to obtain the efficiency of the detector system.

In the Monte Carlo simulations, two types of breakup processes were considered, direct and sequential. In the direct breakup, a proton is excited to the continuum above the breakup threshold, 0.6 MeV. The breakup was assumed to occur at the distance of closest approach. This is the inverse reaction of the direct capture of a proton by 16O in 16O(p, γ)17F. The measured capture excitation function and the extracted S factor [11] were folded into the simulation to generate events. The other process, sequential breakup, takes place by populating excited states above the threshold which subsequently decay by proton emission. In all simulations, the breakup particles are assumed to be emitted isotropically in the rest frame of the 17F.

The breakup fragments can experience Coulomb repulsion of the target nucleus if the breakup process occurs near the target [16]. This final state interaction causes the relative energy of the fragments to differ from that at the time of breakup [17]. The observed energy distributions of the fragments will therefore be shifted. In the case of direct breakup, it may be necessary to consider this post-breakup proximity effect, since the measurement is performed near the grazing angle and it is assumed that the breakup takes place essentially instantaneously at the distance of closest approach. In the case of sequential breakup, the lifetimes of the low-lying excited states of 17F are very short, ≤ 5 × 10^{−22} s, so the post-breakup acceleration could also be important. Simulations were made with and without the postacceleration.

As mentioned above, the energy of the 16O cannot be determined experimentally because of the gain saturation of the amplifiers, so only the energy distributions of the protons can be compared with the Monte Carlo simulations. The measured proton energy distributions and the angular correlations of the fragments, shown in Fig. 1 and Fig. 2, respectively, are described reasonably well by the direct-breakup simulation with a proton excited between the 0.6 MeV breakup threshold and 1.4 MeV above the threshold. Simulations for exciting the proton to 2.4 and 4.4 MeV above the threshold are shown by the dashed and dotted histograms, respectively, for comparison. The angular correlations are presented in terms of the number of pixels on the DSSD with each pixel subtending approximately 2°. The counting statistics are too poor to draw any firm conclusions on whether postacceleration is significant. Simulations for the sequential breakup process in which states above the breakup threshold are populated fail to describe the data, as shown in panel (c) of Fig. 1 and Fig. 2.

The breakup cross section was obtained by normalizing to elastic scattering detected in the 10° monitor. The elastic cross section was taken to be the same as the Rutherford cross section. The coincidence efficiency of the detector system was calculated from the Monte Carlo simulations to be 0.054 and 0.091 with and without postacceleration, respectively. Sim-
Fig. 1. Energy distributions of the breakup protons. Panel (a) is for simulations without postacceleration and panel (b) is for simulations with postacceleration. The experimental data are shown by closed circles and the results of Monte Carlo simulations are shown by histograms for the cutoff in excitation energy of 2 (solid), 3 (dashed) and 5 MeV (dotted). Panel (c) shows the results for simulations of sequential breakup from the 3.1 MeV state with (dotted histogram) and without (solid histogram) postacceleration. The data are shown by the filled and open circles with and without consideration of postacceleration, respectively.

Calculations using angular distributions taken from theoretical calculations shown in Fig. 3 are compared with simulations with a uniform distribution to estimate uncertainties. The efficiency varies at most by 2% for different angular distributions. The major differences in efficiency come from the postacceleration effect. The resulting measured breakup cross section is $6.6 \pm 0.7 \text{ mb/sr}$ with postacceleration and $3.9 \pm 0.4 \text{ mb/sr}$ without postacceleration.

Theoretical calculations are compared with the measured data in Fig. 3. The angular distributions of the breakup cross section were calculated by a first-order perturbation theory (solid curve), dynamical calculation with E1 and E2 Coulomb fields (dashed curve) and dynamical calculation with Coulomb and nuclear fields (dotted curve). A description of the theoretical treatment used in these calculations can be found in Ref. [19]. The measured data are shown by the filled and open circles for considering breakup with and without postacceleration, respectively. The calculations overpredict the breakup by a factor of 4 or more. Unlike the Monte Carlo simulations, there was no cutoff in excitation energy in the calculations. When the 2 MeV cutoff is introduced in the perturbation calculation, the measured cross section is repro-
Fig. 3. Theoretical calculations of the $^{17}$F breakup cross section as a function of the center-of-mass angle. The solid curve is for calculations by a perturbation theory, the dashed curve is for dynamical calculations with E1 and E2 fields, and the dotted curve is for dynamical calculations with Coulomb and nuclear fields. The dash-dotted curve is for the perturbation calculation with a cutoff of 2 MeV in excitation energy. The data are shown by the filled and open circles with and without consideration of postacceleration, respectively.

The first excited state of $^{17}$F lies below the breakup threshold and can be excited from the ground state by an E2 transition with a large $B(E2)$ value [20]. The calculated probability for E2 excitation to the $1/2^+$ state is comparable to that of breakup. It is conceivable that a significant fraction of the $^{17}$F could have been excited to the first excited state. Due to the small binding energy, 0.1 MeV, and large rms radius of the $1/2^+$ state, the $^{17}$F may be polarized so that the valence proton moves away from the Pb nucleus and is shielded by the $^{16}$O core. Consequently, the breakup probability is smaller than it would be in the absence of polarization. Coupled-channels calculations are required to explore whether the breakup cross section is enhanced or suppressed by this excitation channel.

To compare breakup with fusion, the breakup probability was calculated to be 0.016 at the distance of closest approach in a head-on collision and at an energy close to the Coulomb barrier, cf. Fig. 2 of Ref. [10]. This result was obtained in a dynamical calculation that included the E1 and E2 fields to all orders. The same model predicts the dashed curve in Fig. 3, which is about a factor of 4 larger than our measured breakup cross section. If we adopt this discrepancy as a correction factor, the above mentioned breakup probability near the Coulomb barrier would be reduced to about 0.004, making it insignificant compared to the fusion probabilities extracted in Ref. [10]. This estimate suggests that the breakup of $^{17}$F is too small to have an observable effect on the $^{17}$F + $^{208}$Pb fusion.

Results have been reported from similar experiments in which the fusion cross sections for $^9$Be + $^{208}$Pb are found to be smaller than for neutron-rich $^{10}$Be- and $^{11}$Be-induced reactions at energies above the barrier [21] whereas the cross sections below the barrier are similar for the three reactions. The fusion excitation function of $^9$Be + $^{208}$Pb was remeasured to high precision by another group who observed a suppression of fusion at energies above the barrier [22]. From the analysis of the evaporation residues, the authors of Ref. [22] attribute the fusion suppression to the breakup of $^9$Be into charged fragments. They suggest that although the neutron binding energy of $^{11}$Be is small and the neutron dissociation probability is large, the influence of neutron breakup on fusion is not as important as breakup into charged fragments. If so,
weakly bound proton-rich nuclei such as $^{17}\text{F}$ will be more suitable for studying the influence of breakup on fusion. In the present experiment, the breakup probability of $^{17}\text{F}$ into two charged fragments was found to be small. Whether this can have noticeable effects on fusion at energies above the barrier has yet to be determined. Further studies of $^{17}\text{F}$ breakup and fusion with lighter targets, where the nuclear field plays a more dominant role, may provide better understanding of fusion involving weakly bound nuclei.

In summary, the breakup of $^{17}\text{F}$ into $^{16}\text{O}$ and $p$ was measured in coincidence near the grazing angle at an incident energy of 170 MeV on a $^{208}\text{Pb}$ target. Based on the angular correlations of the fragments and energy distributions of the protons, the primary breakup mechanism appears to be a direct process taking place at low excitation energies. The probability for exciting a proton to the $1/2^+$ state is calculated to be comparable to that of breakup. Calculations are required for studying how the breakup cross section is influenced by this coupled-channels effect. Further experimental and theoretical work are necessary for investigating the cutoff of the photon excitation energy introduced in the Monte Carlo simulations and understanding the breakup process. The breakup cross section measured in this work suggests that the breakup channel seems too small to influence fusion significantly near the barrier.

Acknowledgements

We wish to thank the HRIBF ion source development team and accelerator staff for providing excellent radioactive beams and support. We also thank A.C. Shotter and G.F. Bertsch for fruitful discussions. This research was supported in part by an appointment to the Oak Ridge National Laboratory Postdoctoral Research Associates Program administrated jointly by the Oak Ridge National Laboratory and the Oak Ridge Institute for Science and Education (ORISE). The ORISE is supported by the US Department of Energy under contract number DE-AC05-760R00033. Research at the Oak Ridge National Laboratory is supported by the US Department of Energy under contract DE-AC05-00OR22725 with UT-Battelle, LLC. One of us (H.E.) was supported by the US Department of Energy, Nuclear Physics Division, under Contract No. W-31-109-ENG-38.

References

Production of $\eta'$ mesons in the $pp \to pp\eta'$ reaction at 3.67 GeV/c

DISTO Collaboration

F. Balestra$^d$, Y. Bedfer$^c$, R. Bertini$^{c,d,*}$, L.C. Bland$^b$, A. Brenschede$^{b,1}$, F. Brochard$^{c,2}$, M.P. Bussa$^d$, Seonho Choi$^{b,3}$, M. Debowski$^{f,4}$, M. Dzemidzic$^{b,5}$, J.-Cl. Faivre$^c$, I.V. Falomkin$^{a,6}$, L. Fava$^e$, L. Ferrero$^d$, J. Foryciarz$^{e,f,7}$, I. Fröhlich$^b$, V. Frolov$^a$, R. Garfagnini$^d$, A. Grasso$^d$, S. Heinz$^c$, V.V. Ivanov$^a$, W.W. Jacobs$^b$, W. Kühn$^b$, A. Maggiora$^d$, M. Maggiora$^d$, A. Manara$^{c,d}$, D. Panzieri$^e$, H.-W. Pfaft$^h$, G. Piragino$^d$, G.B. Pontecorvo$^a$, A. Popov$^a$, J. Ritman$^b$, P. Salabura$^f$, V. Tchalyshev$^a$, F. Tosello$^d$, S.E. Vigdor$^b$, G. Zosi$^d$

$^a$ JINR, Dubna, Russia
$^b$ Indiana University Cyclotron Facility, Bloomington, IN, USA
$^c$ Laboratoire National Saturne, CEA Saclay, France
$^d$ Dipartimento di Fisica “A. Avogadro” and INFN, Torino, Italy
$^e$ Università ‘del Piemonte Orientale and INFN, Torino, Italy
$^f$ M. Smoluchowski Institute of Physics, Jagellonian University, Kraków, Poland
$^g$ H. Niewodniczanski Institute of Nuclear Physics, Kraków, Poland
$^h$ Il. Physikalisches Institut, University of Gießen, Gießen, Germany

Received 31 May 2000; received in revised form 16 August 2000; accepted 30 August 2000

Abstract

The ratio of the total exclusive production cross sections for $\eta'$ and $\eta$ mesons has been measured in the $pp$ reaction at $p_{beam} = 3.67$ GeV/c. The observed $\eta'/\eta$ ratio is $(0.83 \pm 0.11^{+0.23}_{-0.18}) \times 10^{-2}$ from which the exclusive $\eta'$ meson production

* Corresponding author.
E-mail address: bertini@to.infn.it (R. Bertini).
1 Current address: Brokat Infosystems AG, Stuttgart.
3 Current address: Temple University, Philadelphia.
4 Current address: FZ-Rossendorf.
5 Current address: IU School of Medicine, Indianapolis.
6 Deceased.
7 Current address: Motorola Polska Software Center, Kraków.

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.
PII: S0370-2693(00)01015-7
cross section is determined to be \((1.12 \pm 0.15^{+0.42}_{-0.31}) \mu b\). Differential cross section distributions have been measured. Their shape is consistent with isotropic \(\eta'\) meson production. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 14.40.Cs; 13.75.Cs; 25.40.Ve

Keywords: \(\eta'\) meson; Proton–proton final state interaction

The study of the \(\eta'\) meson production is of particular interest because of its large mass compared to the other members of the ground state pseudoscalar meson nonet. The spontaneous breaking of chiral symmetry causes the existence of massless Goldstone bosons, which acquire mass due to explicit chiral symmetry breaking, and are associated with the pseudoscalar meson nonet. In addition quantization effects in QCD lead to the so-called \(U_A(1)\) anomaly, which allows the \(\eta'\) meson to gain mass by a different mechanism than the Goldstone bosons \([2 – 5]\). Nevertheless, the origin of the \(\eta'\) mass and its structure in terms of quark and gluon degrees of freedom remain controversial.

Recent measurements of the \(\eta'\) meson by the CLEO collaboration show an anomalously large branching ratio of \(B\)-mesons to \(\eta'X\) and \(\eta'K\) \([6]\), which might indicate a strong coupling of the \(\eta'\) meson to gluons \([7]\). Furthermore, the quark component of the nucleon’s axial-vector matrix element measured in the EMC experiment \([8]\) suggests that the \(\eta'\) meson couples very weakly to the nucleon \([9,10]\).

First measurements \([11 – 13]\) of the reaction \(pp \rightarrow \eta'NN\) near the production threshold provide the possibility to determine the coupling constant \(g_{\eta'NN}\). However, a quantitative evaluation of this coupling constant requires answers to several open questions concerning the production mechanism, such as the roles of (i) meson-exchange currents, (ii) baryon resonances in the production mechanism (comparable with the role of the \(N^*(1535)\) for \(\eta\) meson production \([14]\)); and (iii) final-state interactions (FSI).

The existing data close to threshold are consistent with different one-boson-exchange models (OBE) including FSI \([15,16]\) given the ambiguities in the treatment of heavy-meson-exchange currents. Evaluation of the different models requires a consistent description over a wide energy range, but data have been lacking at higher energies, where identification of the \(\eta'\) production can no longer be made solely by detecting two protons in a small forward cone \([11 – 13]\).

In this letter, we report on a measurement of the \(\eta'/\eta\) production cross section ratio at an energy where proton–proton (\(pp\) FSI) have a much smaller relative influence on the production mechanism \([16,17]\) compared to the near to threshold data \([11 – 13]\).

In addition, we show the differential cross sections of the \(\eta'\) meson as a function of the polar angle in the CM (center of mass) reference frame and the momentum distributions of the final state particles. These distributions are related to the partial wave contribution in the exit channel (\(pp\eta'\)).

We studied the \(pp\) reaction at the SATURNE II accelerator facility at Saclay. The proton beam of momentum \(3.67\ \text{GeV}/c\) was incident on a liquid hydrogen target and charged products were detected using the DISTO spectrometer, which is described in detail elsewhere \([18]\). This spectrometer consisted of a large dipole magnet (40 cm gap size, operating at 1.0 Tm) which covered the target area and two sets of scintillating fiber hodoscopes. Outside the magnetic field, two sets of multi-wire proportional chambers (MWPC) were mounted, along with segmented plastic scintillator hodoscopes and water Čerenkov detectors. The scintillator hodoscopes and the Čerenkov detectors allow particle identification by combining either the energy loss, time of flight or Čerenkov light output with the particle momentum.

The large acceptance of all detectors (\(\simeq 2^\circ\) to \(\simeq 48^\circ\) horizontally and \(\simeq \pm 15^\circ\) vertically), on both sides of the beam, facilitated the coincident measurement of four charged particles, which was crucial for the reconstruction of many exclusive channels like \(ppK^+K^-\) \([19,20]\), \(pp\pi^+\pi^-\pi^0\), \(pK\Lambda\) and \(pK\Sigma\) \([21]\).

The multi-particle trigger \([22]\), which was based on the multiplicity of the hodoscope elements and the scintillating fibers, selected events with at least three charged tracks in the final state. The results presented in this work are based on \(4.2 \times 10^7\) reconstructed
events with four charged particles (mainly $pp\pi^+\pi^-$) detected.

The exclusive $\eta$ meson production ($pp \rightarrow pp\eta$) was identified via its dominant decay channel involving charged particles ($\pi^+\pi^-\pi^0$, branching ratio 23.1%) and the reaction $pp \rightarrow pp\eta'$ was reconstructed via the decay of the $\eta'$ meson into $\pi^+\pi^-\eta$ (branching ratio 43.8%). The selection of the exclusive $\eta'$ and $\eta$ meson production is based on two kinematical observables, the 4-particle ($pp\pi^+\pi^-$) missing mass ($M^4_{\text{miss}}$) and the proton–proton missing mass ($M^{pp}_{\text{miss}}$).

Furthermore the light output from the Čerenkov detectors together with the particle momentum were used to discriminate between $\pi^+$ mesons and protons in the final state.

Since neutral pions were not detected ($\pi^0 \rightarrow \gamma\gamma'$, branching ratio 98.8%), $pp\eta$ events were selected in which $M^4_{\text{miss}}$ was approximately consistent with a missing $\pi^0$ meson ($0.005 \text{ GeV}^2/c^4 < (M^4_{\text{miss}})^2 < 0.035 \text{ GeV}^2/c^4$). After imposing this constraint the proton–proton missing mass distribution is shown in Fig. 1 (upper frame). A broad signal from the $\eta$ meson is visible near $M^{pp}_{\text{miss}} \approx 0.3 \text{ GeV}^2/c^4$.

The assumption of a missing $\pi^0$ meson allowed a constraint ($M^{pp}_{\text{miss}} \approx M^{\pi^0}_{\text{miss}}$) to be imposed in order to improve the $M^{pp}_{\text{miss}}$ mass resolution by a kinematical refit procedure. In this procedure all particle momenta were simultaneously re-determined under the assumption of a missing $\pi^0$ meson. After applying the refit procedure, the resolution of the $\eta$ meson signal in the $M^{pp}_{\text{miss}}$ distribution is improved by about a factor 2 (see Fig. 1, lower frame).

In both frames the solid curve shows the sum of the signal (dotted curve) and the background (dashed curve). The signal line shape was taken from detailed Monte Carlo simulations of the detector performance and the yield was determined by scaling the line shape to match the data using a $\chi^2$ minimization procedure. The small deviations at $(M^{pp}_{\text{miss}})^2 \approx 0.325 \text{ GeV}^2/c^4$ result from imperfections of the modeling of the detector response and are included in the estimation of the systematic errors.

The non-resonant reaction $pp \rightarrow pp\pi^+\pi^-\pi^0$ accounts for most of the background under the $\eta$ meson signal. Since the exact shape of the background is not quantitatively known, the background has been parameterized with a 3rd order polynomial, that provides the best $\chi^2$ to the fit.

The reconstruction of the decay channel $pp \rightarrow pp\eta' \rightarrow pp\pi^+\pi^-\eta$ is analogous to the $\eta$ meson reconstruction described above, since the $\eta$ meson decays mostly by neutral modes (branching ratio 71.5%).

For this decay channel $M^{pp}_{\text{miss}}$ and $M^{\eta'}_{\text{miss}}$ must correspond to a missing $\eta'$ and a missing $\eta$, respectively. The $(M^{pp}_{\text{miss}})^2$ distribution for events with a 4-particle missing mass consistent with a missing $\pi^0$ meson (i.e. $|M^{pp}_{\text{miss}}|^2 - M^2_{\pi^0} < 0.03 \text{ GeV}^2/c^4$) is shown in Fig. 2 before (hatched area) and after (data points) the kinematical refit procedure with the constraint $M^{pp}_{\text{miss}} \approx M_{\eta'}$. The spectrum after the refit shows a clear sig-
nal from the \( \eta' \) meson near \( M_{\eta'}^2 \simeq 0.92 \text{ GeV}^2/c^4 \). In comparison to the reconstruction of the \( \eta \) meson the kinematical refit procedure only improves the resolution of the \( \eta' \) signal by about 25\%, due to the lower laboratory momenta of the outgoing protons.

The dashed curve represents the background contribution and the solid curve shows the sum of the background and of the signal (dotted curve). In addition the hatched spectrum indicates the spectrum before the refit procedure was applied.

The five degrees of freedom related to the three body decay of the \( \eta' \) and the \( \eta \) mesons were integrated in the simulations using an isotropic orientation of the decay plane and the measured matrix elements [23, 24].

After accounting for azimuthal symmetry in the production reaction, we divided the kinematically allowed phase space into four-dimensional bins and evaluated the efficiency for each bin separately. This was realized by storing the number of generated and the number of reconstructed events from the simulations for each phase space bin. The bin-by-bin ratio provides the efficiency correction, which was stored in the 4-dimensional acceptance correction matrix.

The acceptance values were typically larger than 1\% in each phase space bin including branching ratios, particle decay in flight, tracking efficiency and particle identification efficiency. The approximately 6\% efficiency loss due to the refit procedure was also accounted for. The data were corrected using the appropriate entry from this 4-dimensional acceptance correction matrix, on an event by event basis. For a detailed discussion of the relative acceptance correction method see [25].

The simulations indicated a very low acceptance of the apparatus for \( \eta \) mesons produced in the backward hemisphere in the CM (center of mass) rest frame. However, since the entrance consists of two identical particles a reflection symmetry about \( \theta_{\text{CM}} = 90^\circ \) exists, thus the backward data are redundant for determining total cross sections. Therefore, we only analyzed the acceptance-corrected production yield in the forward hemisphere, where the acceptance was non-zero in each phase space bin.

It should be noted that the acceptance correction is essentially independent of the event generator used in the simulations due to the complete phase space coverage of the DISTO spectrometer. The generator used for the correction assumed uniform phase space density for both reactions. This assumption was verified by using the same correction for different phase space populations in the simulations. The small deviations observed are included in the estimation of the systematic errors.

Furthermore, the simulations included all important decay modes for the \( \eta \) meson [26] for the acceptance correction of the \( pp \rightarrow pp\eta' \rightarrow pp\pi^+\pi^-\eta \) and the
$pp \rightarrow pp\eta$ reactions. Hence, background processes such as $\eta' \rightarrow \pi^+\pi^-\eta \rightarrow \pi^+\pi^-\pi^0(\pi^0\pi^-\pi^0)$, where one or both of the observed pions are from the $\eta$ or events with more than four charged particles in the acceptance of the detector are properly accounted for.

After correcting the $\eta'$ meson and the $\pi^0$ meson production yields for the respective acceptances and branching ratios [26] as described above, the measured total cross section ratio $R = \sigma_{pp \rightarrow pp\eta} / \sigma_{pp \rightarrow pp\bar{\eta}}$ is determined to be $(0.83 \pm 0.11^{+0.23}_{-0.18}) \times 10^{-2}$. Where the first error is statistical and the second error range is due to systematic uncertainties. Because both meson channels have been reconstructed in events with the same four-charged-particle final state and measured simultaneously within the same experiment, many systematic uncertainties cancel when considering the production ratio. As a result, the systematic error is dominated by the background subtraction and the relative acceptance correction.

The total cross section for the $pp \rightarrow pp\eta$ reaction is known over a wide energy range [27] above and below the beam momentum of this measurement. Interpolation of the existing data leads to a cross section of $\sigma_{pp \rightarrow pp\eta} = 135 \pm 35 \, \mu b$ at $p_{beam} = 3.67 \, \text{GeV}/c$ in good agreement of the value $\sigma_{model_{pp \rightarrow pp\eta}} \approx 120 \, \mu b$ calculated by Vetter et al. [14] using a meson exchange model. By multiplying $R$ by $\sigma_{exp_{pp \rightarrow pp\eta}}$ we obtain the total cross section $\sigma_{pp \rightarrow pp\eta} = 1.12 \pm 0.15^{+0.42}_{-0.31} \, \mu b$. The systematic error in $\sigma_{pp \rightarrow pp\eta}$ is geometrically added with the systematic error in the $pp \rightarrow pp\eta / pp \rightarrow pp\bar{\eta}$ ratio. This result is shown in Fig. 3 (filled circle) together with other data closer to the production threshold [11 – 13] and model calculations (solid and dashed curves) [16].

The calculation from Sibirtsev et al. [16] represents a one-pion-exchange model including the $pp$ FSI. The solid line denotes the full calculation and the dashed line corresponds to the same calculation excluding the FSI. The full calculation reproduces the near threshold data well, but predicts a cross section of about $2.3 \, \mu b$ at our energy, which is significantly above our measurement.

The distribution of the differential cross section as a function of the CM polar angle of the $\eta'$ meson ($\cos(\Theta_{CM})$) is shown in Fig. 4. The distribution displays no significant deviations from isotropy, indicating that the $\eta'$ meson is predominantly in a s-wave state relative to the two protons. The differential cross section distributions were determined by producing the corresponding ($M_{miss}^{pp}$)² spectra for each kinematical bin. Each spectrum was fitted analogously as described above to determine the yield of the signal for each bin. The statistical uncertainty of the yield for each bin are determined by the fitting procedure and are shown as vertical error bars in the differential distributions (see Fig. 4 and Fig. 5). The signal line shape was calculated for each spectrum individually from the Monte Carlo simulations and the background was also allowed to vary.

The relative partial wave contributions in the three particle final state $pp\eta'$ can also be determined from...
the momentum distributions \( q \) and \( p \), where \( q \) is the CM momentum of the \( \eta' \) meson and \( p \) is the momentum of a proton in the proton–proton rest frame.

Then, the total cross section is given as the sum of the individual partial wave contributions [28]:

\[
\sigma \sim \sum_{l_1,l_2} \int |M_{l_1,l_2}|^2 \, dp_{l_1l_2} \tag{1}
\]

here the sum extends over the angular momenta \( l_1,l_2 \) and the transition amplitude for the given exit channel \( (M_{l_1,l_2}) \), where \( l_1 \) is the orbital angular momentum of the two protons relative to each other and \( l_2 \) is the orbital angular momentum of the \( \eta' \) meson relative to the proton–proton system.

The corresponding phase space element \( dp_{l_1l_2} \) is determined by

\[
dp_{l_1l_2} \sim p^{2l_1+1} q^{2l_2+2} dq. \tag{2}
\]

If we assume the transition amplitude \( M_{l_1,l_2} \) to be almost constant over the available phase space then the differential cross section as a function of the particle momenta are given by the variation of the phase space volume.

The measured differential cross sections are plotted in Fig. 5 as a function of \( q \) (upper frame) and \( p \) (lower frame). The corresponding curves, that reproduce the data quite well, have been obtained from Eq. (2), assuming \( l_1 = l_2 = 0 \). The normalization was obtained by a simultaneous \( \chi^2 \) minimization to both differential cross section distributions. The introduction of higher values of \( l \) does not improve the \( \chi^2 \) fit to the data. The result is consistent with a dominant Ss-wave production of the \( \eta' \) meson, where S denotes \( l_1 = 0 \) and \( s \), \( l_2 = 0 \), respectively.

In conclusion, the production of the pseudoscalar \( \eta' \) meson has been studied in the \( pp \) reaction at \( p_{\text{beam}} = 3.67 \) GeV/c. The \( \eta' \) meson has been reconstructed by measuring its charged decay products. The \( \eta'/\eta \) cross section ratio has been determined and the extracted \( \eta' \) cross section has been compared to data very close to threshold and an one-pion exchange model including the \( pp \) FSI. While the model describes the data close to threshold very well it overestimates our data point by about 100%.

Furthermore, the differential cross sections indicates that the \( \eta' \) meson is predominantly produced in a s-wave state for the two protons relative to each other and the \( \eta' \) meson relative to the proton–proton system.

Calculations up to 100 MeV above the \( \eta' \) production threshold, in the framework of a relativistic meson-exchange model [29] should be extended to higher energies. Comparison with our results should help to learn more about the different \( \eta' \) meson production mechanisms, the potential influence of a \( \eta'/p \) FSI [30] and the magnitude of the coupling constant \( g_{\eta'pp} \).

In this context a comparison with a recent publication on the production of \( \pi^0 \), \( \eta \) and \( \eta' \) mesons in proton–proton collisions [31] should be very helpful. The novel approach therein factored out the \( pp \) FSI and the initial-state proton–proton interaction (ISI) from the total cross section to derive the phase space dependence of the total production amplitude for \( \pi^0 \) (\( |A_{\pi^0}^0| \)), \( \eta \) (\( |A_{\eta}^0| \)) and \( \eta' \) mesons (\( |A_{\eta'}^0| \)).

Our measurement will assist to evaluate different models used for the parameterization of the \( pp \) FSI which is essential for the determination of the absolute strength of \( |A_{\eta'}^0| \) and hence of \( g_{\eta'pp} \).

Acknowledgements

We acknowledge the work provided by the SAT-URNE II accelerator staff and technical support groups
in delivering an excellent proton beam and assisting this experimental program.

This work has been supported in part by: CNRS-IN2P3, CEA-DSM, NSF, INFN, KBN (2 P03B 117 10 and 2 P03B 115 15) and GSI.

References

   L. Bodini et al., Nuovo Cimento 58A (1968) 475; 
   C. Caso et al., Nuovo Cimento 55A (1968) 66; 
[28] R.G. Newton, Scattering Theory of Waves and Particles, 
   Springer Verlag, New York; 
   H.O. Meyer, Particles and Fields Series 41, AIP Conference 
   Proceedings, New York.
Branching processes and Koenigs function

O.G. Tchikilev*

Institute for High Energy Physics, Protvino, Russia

Received 28 July 2000; received in revised form 5 September 2000; accepted 6 September 2000

Editor: L. Montanet

Abstract

An explicit solution of non-critical time-homogeneous branching processes is described. © 2000 Published by Elsevier Science B.V.

1. Introduction

Branching processes are widely used in high energy physics [1]. For example, the well known Furry–Yule and negative binomial distributions occur in simple branching processes with allowed transition 1 → 2. The use of processes with higher order transitions 1 → n with n > 2 is rare due to the absence of explicit solutions in terms of elementary functions. In this paper we describe the solution for processes with higher order transitions using a recently found recursive procedure for the pure birth branching process [2]. The solution is based on the use of the Koenigs function [3] and the functional Schröder equation [4], sometimes called the Schröder–Koenigs equation (for a detailed description and extensive bibliography see [5,6]). In Section 2 we describe the solution [2] for the pure birth branching process. In Sections 3 and 4, respectively, the procedures for non-critical branching processes and branching processes with immigration are outlined. The results are summarized in the last section.

2. Solution for the general pure birth branching process

A branching process with continuous evolution parameter t is determined by the rates $\alpha_n$ for the transition (“splitting”) of one particle into n particles with all particles subsequently evolving independently. For a pure birth branching process $\alpha_0 = 0$. The probability distribution $p_n(t)$ for the process having one particle at $t = 0$ with $p_n(0) = \delta_{1n}$ is a solution of the forward Kolmogorov equation [7,8]

$$\frac{\partial m}{\partial t} = f(x) \frac{\partial m}{\partial x}$$

(1)

for the probability generating function

$$m(x, t) = \sum_{n=0}^{\infty} p_n(t) x^n$$

(2)

with

$$f(x) = \sum_{n=2}^{\infty} \alpha_n x^n - \alpha x$$

(3)

and with $\alpha = \sum \alpha_n$. The Taylor expansion of Eq. (1) leads to the following system of equations for the probabilities $p_n$.
\[
\frac{dp_1}{dt} = -\alpha p_1, \tag{4}
\]
\[
\frac{dp_2}{dt} = \alpha_2 p_1 - 2\alpha p_2, \tag{5}
\]
\[
\frac{dp_3}{dt} = \alpha_3 p_1 + 2\alpha_2 p_2 - 3\alpha p_3 \tag{6}
\]
and for arbitrary \( n \)
\[
\frac{dp_n}{dt} = \sum_{j=1}^{n-1} j\alpha_{n-j+1} p_j - n\alpha p_n. \tag{7}
\]

A simple interpretation of Eq. (7) is as follows: let us consider the state with \( n \) particles at the moment \( t \). The change in this state is due to the arrival from states with multiplicity lower than \( n \) and to the departure to states with higher multiplicity. The arrival rate from the state with \( j \) particles is proportional to the number of particles \( j \), to the transition rate (for one particle) to produce \( n - j \) new particles, i.e., \( \alpha_{n-j+1} \), and to the population density in the state \( j \), the sum in Eq. (7) goes over all states below \( n \). The departure rate is proportional to the total transition rate (for one particle) \( \alpha \), to the number of particles \( n \) in this state and to the population density, this explains second term in Eq. (7). Formally this system of equations is valid for any initial condition.

Let us recall the recursive solution of Eqs. (4)–(7) given in [2]: the probability \( p_1(t) \) is \( p_1 = \exp(-\alpha t) \) and
\[
p_n = \sum_{j=1}^{n} \pi_{jn} p_1 \tag{8}
\]
with the following recursion for the coefficients \( \pi_{jn} \)
\[
(n - j)\pi_{jn} = \sum_{l=1}^{n-j} (n-l)b_l \pi_{j(n-l)}. \tag{9}
\]
Here \( b_l = \alpha_{l+1}/\alpha \) is the relative probability to produce \( l \) new particles. The recursion starts from \( \pi_{11} = 1 \) and the coefficient \( \pi_{nn} \) can be found from the initial condition
\[
\pi_{nn} = -\sum_{j=1}^{n-1} \pi_{jn}. \tag{10}
\]
For the case with \( N \) initial particles with \( p_n^{(N)}(0) = \delta_{nN} \), the first \( N - 1 \) equations are automatically valid since
\[
p_1^{(N)} = p_2^{(N)} = \cdots = p_{N-1}^{(N)} = 0 \tag{11}
\]
and the solution in this case has the following form:
\[
p_n^{(N)} = \exp(-N\alpha t) = p_1^{(N)} \text{ and}
\]
\[
p_n^{(N)} = \sum_{j=N}^{n} \pi_{jn}^{(N)} p_1^{(N)} \tag{12}
\]
with the same recursion as (9) for the coefficients \( \pi_{jn}^{(N)} \)
\[
(n - j)\pi_{jn}^{(N)} = \sum_{l=1}^{n-j} (n-l)b_l \pi_{jn(n-l)}^{(N)}. \tag{13}
\]
This recursion starts from \( \pi_{1N}^{(N)} = 1 \) and the coefficient \( \pi_{nn}^{(N)} \) can be found from the relation
\[
\pi_{nn}^{(N)} = -\sum_{j=N}^{n-1} \pi_{jn}^{(N)}. \tag{14}
\]
One can calculate the coefficients \( \pi_{jn}^{(N)} \) using the concept of the Koenigs function [2,5,6,9]. For the branching process starting from one particle at \( t = 0 \) this function is defined as the limit
\[
K(x) = \lim_{n \to \infty} \frac{m(x, nt)}{p_1^{(N)}}. \tag{15}
\]
\( K(x) \) has the following Taylor expansion:
\[
K(x) = \sum_{j=1}^{\infty} \kappa_j x^j = \sum_{j=1}^{\infty} \pi_{1j} x^j. \tag{16}
\]
For the branching process starting from \( N \) particles the Koenigs function is defined analogously:
\[
K^{(N)}(x) = K^{(N)}(x) = \lim_{n \to \infty} \frac{m^{(N)}(x, nt)}{(p_1^{(N)})^n}
= \sum_{j=N}^{\infty} \kappa_j^{(N)} x^j = \sum_{j=N}^{\infty} \pi_{Nj}^{(N)} x^j. \tag{17}
\]
The recursion (13) leads to the following recurrence for the coefficients \( \kappa_j^{(N)}, N = 1, 2, \ldots; j = N + 1, N + 2, \ldots \)
\[
(j - N)\kappa_j^{(N)} = \sum_{l=1}^{j-N} (j-l)b_l \kappa_{j-l}^{(N)}. \tag{18}
\]
Let us denote \( \kappa_{k+n}^{(N)} = t_n(x) \), then \( t_0(x) = 1 \) and \( t_n(x) \) is given by the following recursion
\[ n t_n(x) = \sum_{l=1}^{n} (x - n - l) b_l t_{n-l}(x). \]  
\((19)\)

It is evident from Eq. (19) that the \( t_n(x) \) is a polynomial of order \( n \) in \( x \). The \( q_j^{(N)} \) in terms of \( t_n(x) \) is equal to \( t_{j-N}(N) \).

The remarkable property of the Koenigs function is that it satisfies the functional Schröder equation:

\[ K(m) = p_1 K(x). \]  
\((20)\)

It is convenient to introduce the function \( Q(x) \) the inverse of the Koenigs function. Then Eq. (20) gives

\[ m(x, t) = Q(p_1 K(x)). \]  
\((21)\)

The solution for the supercritical branching processes is obtained in the following way. Let us transform \( m \) and \( x \) to \( m' \) and \( x' \) by the linear transform

\[ x' = \frac{x - \beta}{1 - \beta}, \quad m' = \frac{m - \beta}{1 - \beta}. \]  
\((22)\)

The absorption transforms to the form for the pure birth branching process. Therefore one can introduce the Koenigs function with

\[ K\left(\frac{m - \beta}{1 - \beta}\right) = q_1 K\left(\frac{x - \beta}{1 - \beta}\right). \]  
\((23)\)

The solution for the supercritical branching processes

\[ m(x, t) = \beta + (1 - \beta) Q\left(q_1 K\left(\frac{x - \beta}{1 - \beta}\right)\right) \]  
\((24)\)

and to the infinite series in \( q_1 \) for the probabilities \( p_n \)

\[ p_n = \delta_{0n} + (\beta - 1) \sum_{l=0}^{\infty} \frac{(-\beta)^{-n} l!}{(1 - \beta)^l n!(n-1)!} \times \sum_{j=n}^{l} Q j_k Q l^{-j} k_j. \]  
\((25)\)

The solution for the subcritical branching processes \((\beta > 1)\) can be obtained in a similar way. In this case the linear transform should move \( 1 \) to zero and \( \beta \) to \( 1 \), i.e., \( x' = (x - 1)/(\beta - 1) \). This leads to the similar expressions

\[ m(x, t) = 1 + (\beta - 1) Q\left(q_1 K\left(\frac{x - 1}{\beta - 1}\right)\right) \]  
\((26)\)

and

\[ p_n = \delta_{0n} + (\beta - 1) \sum_{l=0}^{\infty} \frac{(-1)^{-n} l!}{(\beta - 1)^l n!(n-1)!} \times \sum_{j=n}^{l} Q j_k Q l^{-j} k_j. \]  
\((27)\)

The procedures described above are not suitable for critical branching processes with \( \beta = 1 \). In simplest case without higher order transitions the probability generating function \( m(x, t) \) is a linear fraction with coefficients having linear dependence on \( t \) (instead of \( \text{exp}(-\alpha t) \) dependence for non-critical processes). In general case with higher order transitions the interplay between \( t \) and \( \text{exp}(-\alpha t) \) dependencies leads to more complex equations.
4. Solution for branching processes with immigration

For the branching processes with immigration there is an additional external source of particles appearing in clusters of $j$ particles with the differential rates $\beta_j$ ($\sum \beta_j = b$). The generating function for the process starting with zero particles at $t = 0$ can be written [11, 12] as

$$M(x, t) = \exp \left( \int_0^t g(m(x, \tau)) \, d\tau \right)$$

with

$$g(x) = \sum_{j=1}^{\infty} \beta_j x^j - b,$$  \hspace{1cm} (31)

where $m(x, \tau)$ is the solution for the underlying branching process without immigration. For the underlying pure birth branching process, Eq. (30) leads to the following expression

$$M(x, t) = \exp (-bt) \exp \left( \sum_{n=1}^{\infty} v_n(t) x^n \right)$$

with

$$v_n(t) = \sum_{j=1}^{n} \sum_{j=1}^{n} a_{j,n} \frac{1 - p_j(t)}{j\alpha}.$$  \hspace{1cm} (33)

Let us denote

$$\exp \left( \sum_{n=1}^{\infty} v_n x^n \right) = 1 + \sum_{n=1}^{\infty} V_n x^n,$$  \hspace{1cm} (34)

then the final probability $P_n(t) = \exp (-bt) V_n(t)$. The coefficients $V_n(t)$ can be calculated using the recursive relation:

$$nV_n = \sum_{j=1}^{n} j v_j V_{n-j}.$$  \hspace{1cm} (35)

This relation is known in combinatorics and is used, for example, in the study of combiants [13–16].

5. Summary

In this letter we have derived explicit expressions for the probability distributions in various branching processes. Although we have not derived closed expressions for the polynomials $l_n(x)$, the given recursions can serve as a calculational tool both in theoretical and experimental studies.

References

Data on $\bar{p}p \rightarrow \eta'\pi^0\pi^0$ for masses 1960 to 2410 MeV/$c^2$

A.V. Anisovich $^c$, C.A. Baker $^b$, C.J. Batty $^b$, D.V. Bugg $^{a,*}$, C. Hodd $^a$, V.A. Nikonov $^c$, A.V. Sarantsev $^c$, V.V. Sarantsev $^c$, B.S. Zou $^{a,1}$

$^a$ Queen Mary and Westfield College, London E1 4NS, UK
$^b$ Rutherford Appleton Laboratory, Chilton, Didcot, OX11 OQX, UK
$^c$ St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg district 188350, Russia

Received 28 July 2000; accepted 29 August 2000

Editor: L. Montanet

Abstract

Data on $\bar{p}p \rightarrow \eta' (958)\pi^0\pi^0$ are presented at nine $\bar{p}$ momenta from 600 to 1940 MeV/$c$. Strong S-wave production of $f_2(1270)\eta'$ is observed, requiring a $J^{PC} = 2^{-+}$ resonance with mass $M = 2248 \pm 20$ MeV, $\Gamma = 280 \pm 20$ MeV. © 2000 Published by Elsevier Science B.V.

The first data are presented on $\bar{p}p \rightarrow \eta'\pi^0\pi^0$ in flight. These data were taken with the Crystal Barrel detector at LEAR. They are part of an extensive study of the $I = 0$, $C = +1$ system in several channels. Data have been reported earlier on $\pi^0\pi^0$ [1], $\eta\eta$ and $\eta\eta'$ [2], and $\eta\pi^0\pi^0$ [3]. A comparison will be made here specifically with the $\eta\pi^0\pi^0$ data, and with a combined amplitude analysis of all the earlier data [4].

The present channel is studied in 10$^5$ events, where $\eta'\rightarrow \eta\pi^0\pi^0, \eta \rightarrow \gamma\gamma$. Photons are detected with high efficiency down to 20 MeV in a barrel of 1380 CsI crystals covering 98% of the solid angle; the geometry is such that crystals point towards the target. The crystals have a length of 16 radiation lengths and provide an angular resolution of ±20 mrad in azimuth and polar angle. The energy resolution is given by $\Delta E/E = 2.5\%/E$ (GeV)$^{1/4}$.

The general procedures for event reconstruction and selection have been described in several earlier publications, of which the most detailed concern the study of $\pi^0\pi^0$, $\eta\eta$ and $\eta\eta'$ final states [1,2]. A Monte Carlo simulation of the detector is used to assess the efficiency for reconstruction of the $\pi^0\pi^0\eta'$ final state and the levels of background from competing channels.

$^*$ Corresponding author.
E-mail address: bugg@v2.rl.ac.uk (D.V. Bugg).
$^1$ Now at the Institute for High Energy Physics, Beijing 100039, China.

0370-2693/00/$ – see front matter © 2000 Published by Elsevier Science B.V.
PII: S0370-2693(00)01019-4
Fig. 1. The distribution of $M^2(\pi\pi\pi)$ at four beam momenta indicated by numerical values in MeV/c. The shaded areas show selected signal events in the $\eta'$ peak and those used for sideband subtraction.

Events are first submitted to a kinematic fit to $\bar{p}p \rightarrow 10\gamma$, requiring a confidence level > 5%. The best kinematic fit to $\bar{p}p \rightarrow 4\pi^0$ is then selected, again with confidence level > 5%. At this step, the main background to $4\pi^0$ comes from $5\pi^0$ events. This background is suppressed strongly by rejecting any event passing a kinematic fit to $5\pi^0$ with confidence level > 1% (or 0.1% at 600 MeV/c, where the background is more severe). Finally, those few events are rejected which fit $\eta\pi^0$ with confidence level better than $4\pi^0$.

Fig. 1 illustrates at four beam momenta the $\eta\pi\pi$ mass distribution of surviving events in the mass range around the $\eta'$. There is a clear $\eta'$ signal, agreeing in mass within 6 MeV of the standard value at all momenta. It is superposed on a smooth background, whose magnitude is largest at low beam momenta. The Monte Carlo simulation estimates that the background comes approximately equally from 3 sources: (i) $5\pi^0$ events, (ii) $\omega 4\pi^0$, ($\omega \rightarrow \pi^0\gamma\gamma$) after losing one photon, and (iii) $\eta 4\pi^0$ without an $\eta'$. The predicted background agrees with that observed (within 10% of the prediction). Tighter cuts do not improve the signal/background ratio significantly, but simply cause loss of events.

Signal events are selected from the peak region of the $\eta'$ by adjusting a mass cut around the peak at every individual momentum so as to optimise the signal/background ratio. Very rarely, two events fall within the window; in this case the one closer to the $\eta'$ is accepted. Statistics of the data selection are shown in Table 1. In the maximum likelihood fit used for amplitude analysis, sidebands events shown shaded in Fig. 1 are used to subtract the background. The areas of sidebands are chosen so that each covers twice the range of mass squared which is used to select $\eta'$ events; in this way, statistical errors on the background are small. A technicality is that the width of the mass cut is varied according to the accuracy with which the $\eta'$ mass is reconstructed. This is the reason that sidebands have diffuse edges: the width of the sidebin likewise varies with the width of the $\eta'$ mass cut. Technically, the way the subtraction is made is to include sidebin events into the fit with a weight $-0.25$ times that of events selected in the signal region. Amplitudes are constructed with tensor expressions using the measured mass of each $\eta'$.

Fig. 2 shows the Dalitz plots at all momenta for events from the signal region. There is an obvious contribution due to $f_{2}(1270)\eta'$, appearing at momenta

<table>
<thead>
<tr>
<th>Momentum (MeV/c)</th>
<th>Data</th>
<th>BG</th>
<th>Signal</th>
<th>$\epsilon$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>180</td>
<td>61</td>
<td>119</td>
<td>2.90</td>
</tr>
<tr>
<td>900</td>
<td>1017</td>
<td>399</td>
<td>618</td>
<td>4.61</td>
</tr>
<tr>
<td>1050</td>
<td>831</td>
<td>257</td>
<td>574</td>
<td>5.76</td>
</tr>
<tr>
<td>1200</td>
<td>2770</td>
<td>852</td>
<td>1918</td>
<td>6.33</td>
</tr>
<tr>
<td>1350</td>
<td>2296</td>
<td>595</td>
<td>1701</td>
<td>5.92</td>
</tr>
<tr>
<td>1525</td>
<td>1416</td>
<td>381</td>
<td>1035</td>
<td>5.06</td>
</tr>
<tr>
<td>1642</td>
<td>1530</td>
<td>330</td>
<td>1200</td>
<td>4.72</td>
</tr>
<tr>
<td>1800</td>
<td>1503</td>
<td>325</td>
<td>1178</td>
<td>4.57</td>
</tr>
<tr>
<td>1940</td>
<td>1063</td>
<td>240</td>
<td>823</td>
<td>4.34</td>
</tr>
</tbody>
</table>
\( \geq 1200 \text{ MeV}/c \) at the lower left edge of the plot. Fig. 3 shows the Dalitz plots for sidebin events. The distribution of background is not uniform, but peaks in the corners of the Dalitz plots. This peaking accounts for corresponding peaks observed in the corners of the Dalitz plots of Fig. 2. When the subtraction is made, the surviving signal outside the \( f_2(1270) \) peak is nearly uniform within the available statistics. At 1940 MeV/c, there is also some weak \( f_2(1270) \) in the background; we have checked that this is not due to \( \eta'\pi^0\pi^0 \) signal spilling into the mass ranges used for the sidebins.

There is no indication for the presence of \( a_2(1320) \to \eta'\pi \). The expected contribution may be predicted from fits which have been made to \( a_2(1320)\pi \) in \( \eta\pi^0\pi^0 \) data [3]. The predicted contribution is only \( \sim 3\% \) of \( \eta'\pi^0\pi^0 \), because of the small (0.53\%) branching fraction of \( a_2(1320) \) to \( \eta'\pi \). This contribution is included in the amplitude analysis using amplitudes fitted to the \( \eta\pi\pi \) data, but is so small as to have negligible effect on conclusions. Fig. 4 shows projections at two beam momenta on to masses of \( \pi\pi \) and \( \pi\eta' \); the latter is featureless. The histograms show results of the maximum likelihood fit described below.

We now turn to physics results. Data points on Fig. 5(a) show the integrated \( \eta'\pi^0\pi^0 \) cross section after background subtraction and after scaling to allow for all other unobserved decay modes of \( \eta' \), \( \eta \) and \( \pi^0 \). There is a peak around 2230 MeV, which is the nominal threshold for \( f_2(1270)\eta'(958) \). The absolute normalisation is obtained using beam counts, target length and density, and correcting the observed number of signal events for the reconstruction efficiency shown in Table 1. A correction is applied for observed dependence of the cross section on beam rate, as described in detail in Ref. [1].

The amplitude analysis is made using (a) S- and P-waves for \( \sigma\eta' \), where \( \sigma \) stands for the \( \pi\pi \) S-wave amplitude, for which we use the parametrisation of Zou and Bugg [6], (b) S- and P-waves for \( f_2(1270)\eta' \), and (c) a small, almost negligible contribution from \( ^3P_1 \to f_0(975)\eta' \), which helps marginally in fitting.
the $\pi\pi$ mass distribution at the lowest three beam momenta. It is to be expected that higher partial waves for $f_2\eta'\eta$ will be suppressed strongly by the centrifugal barrier in the final states. Contributions from $f_2\eta'\eta$ D-waves have been tried in the fit, but are not required; indeed, the P-wave contribution is quite small. Likewise, $\sigma\eta'$ contributions with $L \geq 2$ are negligible.

We shall present amplitudes for $f_2(1270)\eta'$ in partial waves $^3D_2$ ($J^{PC} = 2^{+-}$), $^3P_2$ and $^3F_2$ ($2^{++}$), $^3P_1$ ($1^{++}$) and $^3F_3$ ($3^{++}$); they will be compared with $f_2(1270)\eta$ observed in $\eta\pi\pi$ data [3,4]. These two channels are related by the composition of the $\eta'$ and $\eta$ in terms of strange and non-strange quarks:

$$|\eta| \simeq 0.8 \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} - 0.6 s\bar{s},$$

$$|\eta'| \simeq 0.6 \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} + 0.8 s\bar{s}.$$  

The coefficients 0.8 and 0.6 are derived from the well known pseudo-scalar mixing angle [7]. Our earlier analysis of $\bar{p}p \to \pi^-\pi^+, \pi^0\pi^0$, $\eta\eta$ and $\eta\eta'$ [8] finds that almost all $s$-channel resonances produced in $\bar{p}p$ interactions are consistent with small mixing angles $\leq 15^\circ$ between $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$. The naive prediction is therefore that amplitudes $a$ for $\bar{p}p \to f_2(1270)\eta'$ and $\bar{p}p \to f_2(1270)\eta$ will be related by

$$a(f_2\eta') \simeq 0.75 a(f_2\eta).$$

The peak in the full curve of Fig. 5(a) requires a resonance in $f_2\eta'$ close to the mass of the peak. However, the mass spectrum from a simple resonance will be pushed upwards by the rapidly increasing phase space for the final state $f_2\eta'$. This effect is visible in the dotted curve of Fig. 5(c), which shows the resonance contribution to $f_2\eta'$ fitted to $\eta_2(2248)$; this curve peaks above 2300 MeV because of the increasing phase space. In order to reproduce the integrated

---

**Fig. 4.** Projections on to $M(\pi\pi)$ and $M(\eta'\pi)$ at beam momenta of 1050 and 1800 MeV/c; in all cases, a background subtraction is made using sidebins. Histograms show the fit compared with data.

**Fig. 5.** (a) Points with errors show the integrated cross sections for the final state $\eta'\pi^0\pi^0$, after correction for backgrounds and for all decay modes of $\eta'$, $\eta$ and $\pi^0$; the full curve shows the fit from the amplitude analysis; the dashed curve shows the $\eta\pi\pi$ cross section from Ref. [3], multiplied by the SU(3) factor $(0.75)^2$; the dotted curve shows the $\sigma\eta'$ contribution; (b) the full curve shows the cross section for $f_2\eta$ fitted to $\eta\pi\pi$ data; the dotted curve shows the contribution to $\eta\pi\pi$ from $\eta_2(2248)$ alone and the dashed curve that from $\eta_2(1860) + \eta_2(2030)$; the chain curve shows the intensity fitted to $f_2(1270)\eta$ with $L = 2$ in Ref. [4]; (c) as (b) for $f_2\eta'$; (d) Argand diagram for the $f_2\eta'$ S-wave amplitude; crosses mark beam momenta; (e) intensities of contributions to $f_2\eta'$ from $^3P_2$ (full curve), $^3P_1$ (chain curve), $^3F_2$ (dashed) and $^3F_3$ (dotted); (f) intensities of contributions to $\sigma\eta'$ from 0 (full curve), 1 (dashed) and 1$^+$ (dotted).
cross section of Fig. 5(a), the amplitude analysis requires a strong interfering background peaking below threshold. The interference is constructive at low masses, and is required to give a large \( f_{2}\eta' \) cross section there, despite the limited phase space. Above the peak at 2230 MeV, the interference becomes destructive, and cuts off the \( f_{2}\eta' \) cross section on the upper side of the resonance.

The motivation for including this background contribution at low \( f_{2}\eta' \) masses arises from the new combined analysis [4] of \( \eta\pi\pi \) data, together with those on \( \bar{p}p \rightarrow \pi^{-}\pi^{+}, \pi^{0}\pi^{0}, \eta\eta \) and \( f_{2}\eta' \). Results for \( \eta\pi\pi \) from that analysis are shown in Fig. 5(b). That analysis requires a \( 2^{-+} \) resonance at 2267 \( \pm 14 \) MeV. It appears there most clearly in \( f_{2}(1270)\eta \) with \( L = 2 \) in the final state, shown by the chain curve in Fig. 5(b). However, for the dominant \( f_{2}\eta \) \( L = 0 \) channel, what one observes is a strong peak near 2 GeV, shown by the full curve. This comes mostly from \( \eta_{2}(1860) \), but partly from \( \eta_{2}(2030) \) reported in an analysis of data on \( \bar{p}p \rightarrow \eta\pi^{0}\pi^{0}\pi^{0} \) [9]. The intensities of contributions to the \( f_{2}\eta \) channel are shown in Fig. 5(b) from (i) all \( \eta_{2}(2248) \) contributions (dotted curve) and (ii) the coherent sum of \( \eta_{2}(1860) \) and \( \eta_{2}(2030) \) (dashed curve); the latter two resonances are not well resolved by the \( \eta\pi\pi \) data, because they lie close together near the \( \bar{p}p \) threshold. The contribution from \( \eta_{2}(2248) \) interferes destructively with \( \eta_{2}(1860) \) and \( \eta_{2}(2030) \), so as to cut off the full curve at high masses.

In present data, the width of the \( \eta_{2}(2248) \) is well determined by the width of the peak in Fig. 5(a): \( \Gamma = 280 \pm 20 \) MeV. This determination is superior to that in \( \eta\pi\pi \) data: 290 \( \pm 50 \) MeV. The mass is somewhat less well determined, since the interference with the tails of the lower resonances may shift the peak by an amount which is sensitive to their widths. Using the best estimates for the widths from Ref. [8], the mass from the present data is \( M = 2248 \pm 20 \) MeV, in reasonable agreement with the value derived from \( \eta\pi\pi \) data: 2267 \( \pm 14 \) MeV. The Argand diagram for the \( f_{2}\eta' \) S-wave amplitude is shown in Fig. 5(d).

A striking feature of the \( f_{2}\eta' \) signal is its large magnitude. The dashed curve on Fig. 5(a) shows the complete integrated \( \eta\pi^{0}\pi^{0} \) cross section, multiplied by \( (0.75)^{2} \) to allow for the expected inhibition of \( \eta' \) with respect to \( \eta \). It is surprising that the \( f_{2}\eta' \) signal is nearly as strong as the dashed curve, bearing in mind the difference in available phase space for \( f_{2}\eta' \) and \( f_{2}\eta \). The peak in the \( \eta'\pi\pi \) cross section (full curve) is much larger than the small peak observed at the same mass in the \( \eta\pi\pi \) cross section. Likewise, the S-wave peak due to \( \eta_{2}(2248) \rightarrow f_{2}\eta' \), shown by the dotted curve in Fig. 5(c), is considerably stronger than that in \( f_{2}\eta \) in Fig. 5(b). If one takes into account the available phase space for \( f_{2}\eta' \) and \( f_{2}\eta \), the coupling constant for \( \eta_{2}(2248) \rightarrow f_{2}\eta' \) relative to that in \( f_{2}\eta \) is stronger than predicted by Eq. (3) by a factor 5.2 in amplitude.

Vandermeulen has remarked that \( \bar{p}p \) annihilation usually favours high mass final states [10]. This may be understood as a form factor effect, arising from the sizes of the participating states. In present data, the final state \( f_{2}\eta' \) has very low momentum. However, in the process \( \eta_{2}(2248) \rightarrow f_{2}\eta \), the momentum \( q \) in the final state is \( \sim 635 \) MeV/c. The factor 5.2 would require a form factor \( \exp(-4.1q^{2}) \) in amplitude, with \( q \) in GeV/c; if this arises from a source having a Gaussian distribution in \( r \), the form factor takes the well known form \( \exp(-q^{2}R^{2}/6) \), and requires a radius of interaction \( R = 0.98 \) fm. Such a form factor is surprisingly strong. For comparison, the Vandermeulen form factor approximates to \( \exp(-1.5q^{2}) \).

A possibility is that \( \eta_{2}(2248) \) is an \( s\bar{s} \) state. However, strong production from \( \bar{p}p \) is unlikely and in disagreement with results for \( \pi\pi, \eta \) and \( \eta\eta' \) [4].

The strong sub-threshold contribution to the \( f_{2}\eta' \) S-wave is intriguing. A variety of explanations are possible, of which we mention one. In Ref. [9], evidence has been presented for three \( \eta_{2} \) resonances in a mass range where only two are likely to be \( q\bar{q} \). Of these, \( \eta_{2}(1860) \) is a candidate for a hybrid, because of its strong decay to \( f_{2}\eta \), despite limited phase space. If that conjecture is correct, it should be accompanied by an \( s\bar{s}g \) partner at about 2100 MeV. Such an \( s\bar{s}g \) hybrid is expected to decay strongly to \( f_{2}(1525)\eta' \) and \( f_{2}(1270)\eta' \). If it mixes into neighbouring \( q\bar{q} \) states, it could help to explain the anomalously strong \( f_{2}\eta' \) signal observed here.

We now consider other partial waves. The present data require a small but significant P-wave \( f_{2}\eta' \) contribution. This could arise from initial \( \bar{p}p \) states \( ^{3}P_{1}, ^{3}P_{2}, ^{3}F_{2} \) or \( ^{3}F_{3} \). The amplitude analysis of Ref. [4] requires all of these contributions in \( \eta\pi\pi \) data with a \( 3^{+} \) resonance at 2303 MeV, a \( 1^{+} \) resonance at 2310 \( \pm 60 \) MeV and \( 2^{+} \) resonances at 2240 and 2293 MeV. A good fit to present data may be obtained by fixing the relative magnitudes and phases of these partial waves.
from the fit to $\eta^0\pi^0\pi^0$ data. The absolute magnitude of the P-wave contribution is sensitive to the radius chosen for the Blatt–Weisskopf centrifugal barrier. This radius is therefore adjusted to give the best fit to the data, with the reasonable result 0.8 fm.

The magnitudes of the contributions are then 3.5% for $^3F_3$, 3.2% for $^3P_1$, 3.2% for the $^2\pi^+$ resonance at 2240 MeV and 1.0% for the $^2\pi^+$ resonance at 2293 MeV; in the latter two, the ratios of amplitudes for $^3P_2$ and $^3F_2$ are taken from Ref. [4]. Without these amplitudes, log likelihood of the fit to $\eta^0\pi^0\pi^0$ is worse by 142 for only one parameter fitted to the overall magnitude; so the P-wave contribution is highly significant. [Our definition of log likelihood is such that it a change of 0.5 corresponds to one standard deviation change in one variable.] If instead the magnitudes and phases of these amplitudes are fitted freely, the fit changes very little. It is not possible from the present data to separate $^3F_2$ and $^3P_2$, which need to be constrained in relative magnitude as determined in Ref. [4]. With this constraint, the freely fitted intensities are 3.9% for $^3F_3$, 4.7% for $^3P_1$ and 3.9% for $^2\pi^+$, close to the constrained fit.

Figs. 6 (a) and (b) show angular distributions for production of $\eta'/f_2(1270)$ in the mass range > 1.1 GeV in terms of the centre of mass angle $\theta$ of the $\eta'$. The distributions are uncorrected for acceptance, which is included in the maximum likelihood fit shown by the histograms. At high beam momenta, the acceptance for $\eta'$ falls in the forward direction, where the separation of its decay products becomes less efficient. A check on the reconstruction procedure is that angular distributions are symmetric forward-backward in the centre of mass system within errors, after correction for acceptance; this symmetry is required by charge conjugation invariance.

We now turn to the contributions from the broad $\sigma\eta'$ channel. From present data, the only firm conclusion which may be drawn is that contributions from both $^1S_0$ and $^3P_1$ initial states are required. At all momenta from 900 MeV/c upwards, the data require angular distributions of the form $A + B\cos^2 \theta_{\eta'}$, as shown in Figs. 6 (c) and (d). The $\eta\pi\pi$ data have been fitted including $0^-$ and $1^+$ resonances. Present data are fitted well by the same resonances. However, statistics are not sufficient to provide clear evidence of these resonances in present data. Fig. 6 shows that the fit to data is adequate.

In summary, the main feature of the $\eta^0\pi^0\pi^0$ data is a peak at 2230 MeV, requiring a dominant contribution from the $f_2(1270)\eta'$ S-wave. The data require a $2^+\pi^+$ resonance with mass 2288 ± 20 MeV and width $\Gamma = 280 \pm 20$ MeV; this result is closely consistent with an $\eta_2(2267)$ resonance observed in $\eta\pi\pi$ data. The $f_2\eta'$ S-wave amplitude is surprisingly strong compared with that for $f_2\eta$, even allowing for a form factor in the latter. Contributions from $f_2(1270)\eta'$ P-states are consistent with the amplitude analysis of the $\eta\pi\pi$ data.

Acknowledgement

We thank the Crystal Barrel Collaboration for allowing use of the data. We wish to thank the technical staff of the LEAR machine group and of all participating institutes for their invaluable contributions to the successful running of the experiment. We acknowledge financial support from the British Particle Physics and Astronomy Research Council (PPARC).
The St. Petersburg group wishes to acknowledge financial support from PPARC and INTAS grant RFBR 95-0267.

References

Abstract

A combined fit is presented to data on $\bar{p}p$ annihilation in flight to final states $\eta \pi^0, \pi^0 \pi^0, \eta \eta, \eta \eta'$ and $\pi^- \pi^+$. The emphasis lies in improving an earlier study of $\pi^0$ by fitting data at nine $N_p$ momenta simultaneously and with parameters consistent with the two-body channels. There is evidence for all of the $I^D_0$, $C_{DC}$ states expected in this mass range. New resonances are reported with masses and widths ($M; \Gamma$) as follows: $J^{PC} = 4^{+}C_{12285 \pm 20; 325 \pm 30}$ MeV, and $0^{+}C_{1971 \pm 15; 240 \pm 45}$ MeV. Errors on the masses and widths of other resonances are also reduced substantially. All states lie close to parallel straight line trajectories of excitation number vs. mass squared.

Data from $\bar{p}p$ interactions in flight have the great merit of allowing a direct study of $s$-channel meson resonances in formation reactions of the type $\bar{p}p \rightarrow R \rightarrow A + B$. Many decay channels $A, B$ may be studied. Earlier, we have presented data on $\bar{p}p \rightarrow \eta \pi^0 \pi^0$ [1], in which evidence was found for a number of $I = 0, C = +1 \bar{q}q$ states expected in this mass range. New resonances are reported with masses and widths ($M, \Gamma$) as follows: $J^{PC} = 4^{+}C_{12285 \pm 20; 325 \pm 30}$ MeV, and $0^{+}C_{1971 \pm 15; 240 \pm 45}$ MeV. Errors on the masses and widths of other resonances are also reduced substantially. All states lie close to parallel straight line trajectories of excitation number vs. mass squared.

The essential results of the new analysis are summarised by the masses and widths given in Table 2. These results supersede earlier determinations in Refs. [1] and [3]. The present

* Corresponding author.
E-mail address: bugg@v2.ac.uk (D.V. Bugg).
In analysing earlier data [1,3], we have found that resonances mostly cluster into two groups (a) from 1920 to 2050 MeV around $f_4(2050)$, (b) from 2220 to 2320 MeV, around $f_4(2300)$.

The background terms may parametrise the tails of resonances below the $\bar{p}p$ threshold or the effects of $t$-channel exchanges. Singularities due to those exchanges are distant ($s \lesssim 1 \text{ GeV}^2$), so it is to be expected that backgrounds will contribute mostly to low partial waves. This is what we find. The strong high partial waves with $J_{PC} = 4^{++}$ and $3^{++}$ are consistent with no background. In low partial waves ($J \leq 2$), the background is parametrised as a broad resonance (in most cases below the $\bar{p}p$ threshold) or as a constant. Parametrising with resonances or constants guarantees that partial wave amplitudes obey the important constraint of analyticity, since a Breit–Wigner amplitude is analytic. We see no evidence that strong threshold effects are present to perturb such a parametrisation.

Each partial wave amplitude then takes the form:

$$f = \sum_i g_i \exp(i\phi_i) B_i(\bar{p}p) B_i(AB) \frac{M_i^2 - s - i M_i \Gamma_i}{},$$ (1)

The factors $B(\bar{p}p)$ and $B(AB)$ are standard Blatt–Weisskopf centrifugal barrier factors which guarantee the correct threshold behaviour for each channel; expressions are given in Ref. [5]. A common radius for the centrifugal barrier is fitted to all partial waves. There is a strong optimum at 0.829 ± 0.021 fm. The full widths $\Gamma_i$ of all resonances are taken to be constant because of the large number of open channels. Each resonance is fitted with a real coupling constant $g_i$ and a phase $\phi_i$.

We now discuss the channels fitted to the $\eta \pi \pi$ data. Fig. 1 shows mass projections on to $\pi \pi$ and $\eta \pi$ at two representative momenta; further figures are to be found in Ref. [1]. In Fig. 1, data are uncorrected for (small) variations of acceptance, which are included in the maximum likelihood fit. Histograms show the results of the present fit. In Ref. [1], final states fitted to $\eta \pi \pi \pi^0$ data were dominantly $a_2(1320)\pi$, $f_2(1270)\eta$ and $\eta\sigma$, where $\sigma$ stands for the $\pi \pi$ S-wave amplitude. There were small additional contributions from $a_0(980)\pi$, $f_0(980)\eta$ and $f_0(1500)\eta$. In the $\eta \pi$ mass projection at the higher beam momenta there is a distinct shoulder in the mass range around 1450 MeV. A fit to this part of the mass spectrum and higher $\eta \pi$ masses requires further small contributions from $a_0(1450)\pi$ and $a_2(1660)\pi$. The evidence for $a_2(1660) \rightarrow \eta \pi$ from the Crystal Barrel experiment is given in Ref. [6]. Parameters for $a_0(1450)$ are fixed to values of the PDG. However, evidence presented here for $s$-channel resonances does not depend significantly on these small $a_2(1660)\pi$ or $a_0(1450)$ contributions. We have tried including $f_0(1370)$ in the fit, but find negligible effect. It is known to decay weakly to...
Fig. 1. Mass projections on to $M(\pi\pi)$ and $M(\pi\eta)$ for data at (a) and (b) 900 MeV/c, (c) and (d) 1525 MeV/c, (e) and (f) 1940 MeV/c; histograms show the result of the fit. Data are uncorrected for (small) variations of acceptance; these are included in the maximum likelihood fit.

$\pi\pi$ [7] compared with $f_0(1500)$, and the latter is already a small effect in the present data.

The 2-body channels $\pi\pi$, $\eta\eta$ and $\eta\eta'$ are related by SU(3), since $\pi$, $\eta$ and $\eta'$ are members of a single nonet. Formulae for these constraints are given in Ref. [3]. In outline, SU(3) allows $\eta\eta$ and $\eta\eta'$ amplitudes to be predicted from $\pi\pi$ using the well-known composition of $\eta$ and $\eta'$ in terms of $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$. Here we use the assumption that $\rho\rho$ does not couple directly to $s\bar{s}$; this assumption has been tested in Ref. [3] and is well obeyed with one striking exception. The $f_0(2105)$ decays more strongly to $\eta\eta$ than to $\pi\pi$ by a factor 1.88; this compares with the SU(3) prediction of 0.84 = 0.42. That is, the decay to $\eta\eta$ is a factor 4.5 stronger than predicted by the SU(3) relation. Since the $f_0(2105)$ is produced strongly, this feature suggests it has exotic character. Its coupling constants to $\pi\pi$, $\eta\eta$ and $\eta\eta'$ are therefore fitted freely.

For $\eta\pi\pi$, amplitudes for $a_2(1320)\pi$ and $f_2(1270)\eta$ are again in principle related by SU(2) constraints if one assumes ideal mixing for the $2^+$ nonet. These have been tried in the fit, but do not work well. This is probably because (a) there is a large mass difference between these channels and (b) the $f_2(1270)\eta$ threshold is close to the region we are analysing. Consequently, momenta in these channels are substantially different. This is likely to have large effects on matrix elements. We find it necessary to fit magnitudes and phases of $a_2(1320)\pi$ and $f_2(1270)\eta$ amplitudes freely.

Mass differences between $\pi$, $\eta$ and $\eta'$ also give rise to differences in momenta $q$ in $\pi\pi$, $\eta\eta$ and $\eta\eta'$ channels, hence significantly different form factors and centrifugal barrier effects. We fit a form factor
exp(−αq^2) to all decays. The value of α optimises at 0.92 ± 0.10 GeV^{-2}.

We find that two minor improvements may be made to the fit. The phases φ_i of each resonance arise from multiple scattering in initial and final states, as explained in Ref. [8]. Firstly, to allow for overlap of resonances (hence departure from strict Breit–Wigner forms), phases φ_i are allowed to vary by up to ±20° for ηη and ηη' with respect to the ππ value. Secondly, we find significant improvements by allowing coupling constants g_i to vary for ηη and ηη' from their SU(3) values by factors constrained to the range 0.7 to 1.3. This probably reflects differences in form factors for different matrix elements, which may be affected by masses, hence momenta, in the two-body final state. These refinements have only minor effects; without them, conclusions on fitted resonance masses and widths change by only a few MeV.

For J^P = 2^+ and 4^-, the phase factors, φ_{2,4} may couple with orbital angular momentum L = J ± 1 (e.g., 2^P_2 and 4^F_2). An ideal resonance should have the same phase for both L values (via multiple scattering through the resonance to each channel). We therefore take the ratio of coupling constants r_{J^P} = g_{J+1}/g_{J-1} to be real. In fitting ηππ data, where a final state such as f_2(1270)η may also have two or more L values, the ratio of coupling constant is likewise constrained to be real in initial fits. Again, we find small but significant improvements to the fit if these ratios for f_2η and a_2πη decays are allowed to depart from real values by phase angles in the range up to ±20°. But there is little effect on fitted masses and widths, merely a small improvement in the quality of the fit to data.

In Ref. [3], data on ππ, ηη, and ηη', required four J^{PC} = 2^{++} resonances. In addition, data on η'π on ηη provide evidence for a broad 2^+ ηπ contribution with M = 1980 ± 50 MeV [9]. Central production of 4π provides evidence for a small mass and width [10]. We find that this broad component improves the present combined fit to ηππ and 2-body channels strongly. Here we find a clear optimum for the mass at M = 2010 ± 40 MeV and for the width at 495 ± 50 MeV. These values are consistent with Refs. [9] and [10]. However, those earlier determinations have the advantage that the broad 2^+ component is clearly visible by eye, hence providing the incentive for trying it here. Without this component, large interferences develop between the four 2^+ states, so as to simulate this broad contribution.

We have searched for ambiguous solutions by (i) removing each resonance (or background) one by one and reoptimising the rest, (ii) changing signs of amplitude ratios r one by one and (iii) moving resonances in steps of 20 MeV in mass and 40 MeV in width over large ranges (9 steps each). Each iteration of the overall fit takes 10 minutes of computing and one solution converges in typically 20–50 iterations. So ~ 500 alternatives have been explored. All variants collapse back to the same solution.

Intensities of all amplitudes fitted to ηπ^0π^0 are displayed in Fig. 2. They have been corrected for all η and π^0 decays, i.e., for the 39.25% branching fraction of η → γγ and the 98.798% branching fraction of π^0 → γγ. Typical errors are ±5–10% for the larger partial waves, increasing to ±35% for the smallest. Amplitudes which contribute less than 0.3% of the integrated cross section are found to change log likelihood by < 40 and are omitted.

Table 2 shows masses and widths of fitted resonances. Statistical errors are negligible, so quoted errors cover the range of solutions observed when components of the fit are varied. It is apparent, for example, that there are small systematic discrepancies between data on final states π^+π^-π^+ and π^0π^0, and some masses and widths change by a few MeV according to the way one weights different data sets. Errors include systematic changes when the radius of the centrifugal barrier is varied within its error. The last column of Table 2 shows changes in log likelihood when each component is removed from the fit to ηπ^0π^0 data and others are reoptimised. Our definition of log likelihood is such that it changes by 0.5 when one parameter changes by one standard deviation. For 2^+ and 4^+ components, there are also large changes in χ^2 arising in the fit to two-body data; those changes are close to values quoted in Ref. [3]. Consequently, all of the components in the fit are highly significant. The last 6 lines of Table 1 show, for completeness, parameters of I = 1 resonance fitted to two-body data; there are small changes from values of Ref. [3].

The fit to ππ, ηη, and ηη' has changed little from that of Ref. [3], so attention will be concentrated here on fits to ηππ. For those data there are some significant changes from the fit described in Ref. [1]. The main origin of these changes is that ratios r of
amplitudes with $L = J \pm 1$ are defined well by the two-body data, specifically by the polarisation data on $\bar{p}p \rightarrow \pi^-\pi^+$. Using these ratios in the combined fit introduces significant changes to $2^+$ amplitudes in $\eta\pi\pi$. A knock-on effect is that there is also a large improvement in the fitted $3^+$ amplitude towards the top of the mass range.

Argand diagrams are displayed in Fig. 3. Crosses mark individual beam momenta; Table 1 lists these beam momenta and the corresponding centre of mass energies. In many cases, one discerns in Fig. 3 that the amplitude varies rapidly in the low mass region and again in the high mass region. This indicates the presence of two resonances. However, only in Figs. 3
Table 2
Resonances fitted to the data. Errors cover systematic variations observed in a variety of fits with different ingredients. Values in parentheses are fixed from other data [18]. The sixth column shows changes in $S = \log$ likelihood when each component is omitted in turn from the fit to $\eta\pi^0\pi^0$ data and remaining components are reoptimised. The final column highlights those resonances which are new.

<table>
<thead>
<tr>
<th>$I$</th>
<th>$j^{PC}$</th>
<th>Mass $M$ (MeV)</th>
<th>Width $r$ (MeV)</th>
<th>$r$</th>
<th>$\Delta S(\eta\pi\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$6^{++}$</td>
<td>2485±40</td>
<td>410±90</td>
<td>0</td>
<td>245</td>
</tr>
<tr>
<td>4</td>
<td>$4^{++}$</td>
<td>2283±17</td>
<td>310±25</td>
<td>2.7±0.5</td>
<td>2185</td>
</tr>
<tr>
<td>4</td>
<td>$4^{++}$</td>
<td>2018±6</td>
<td>182±7</td>
<td>0.0±0.04</td>
<td>1607</td>
</tr>
<tr>
<td>4</td>
<td>$4^{--}$</td>
<td>2328±38</td>
<td>240±90</td>
<td>558</td>
<td>New</td>
</tr>
<tr>
<td>3</td>
<td>$3^{++}$</td>
<td>2303±15</td>
<td>214±29</td>
<td>1173</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$3^{++}$</td>
<td>2048±8</td>
<td>213±34</td>
<td>1345</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$2^{++}$</td>
<td>2293±13</td>
<td>216±37</td>
<td>−2.2±0.6</td>
<td>1557</td>
</tr>
<tr>
<td>2</td>
<td>$2^{++}$</td>
<td>2240±15</td>
<td>241±30</td>
<td>0.46±0.09</td>
<td>468</td>
</tr>
<tr>
<td>2</td>
<td>$2^{++}$</td>
<td>2001±10</td>
<td>312±32</td>
<td>5.0±0.5</td>
<td>168</td>
</tr>
<tr>
<td>2</td>
<td>$2^{++}$</td>
<td>1934±20</td>
<td>271±25</td>
<td>0.0±0.08</td>
<td>1462</td>
</tr>
<tr>
<td>2</td>
<td>$2^{++}$</td>
<td>2010±25</td>
<td>495±35</td>
<td>1.51±0.09</td>
<td>694</td>
</tr>
<tr>
<td>2</td>
<td>$2^{--}$</td>
<td>2267±14</td>
<td>290±50</td>
<td>1349</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$2^{--}$</td>
<td>(2030)</td>
<td>(205)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$1^{++}$</td>
<td>2310±60</td>
<td>255±70</td>
<td>882</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$1^{++}$</td>
<td>1971±15</td>
<td>240±45</td>
<td>1451</td>
<td>New</td>
</tr>
<tr>
<td>0</td>
<td>$0^{++}$</td>
<td>2337±14</td>
<td>217±33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$0^{++}$</td>
<td>2102±13</td>
<td>211±29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$0^{++}$</td>
<td>2040±38</td>
<td>405±40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$0^{--}$</td>
<td>2285±20</td>
<td>325±30</td>
<td>1342</td>
<td>New</td>
</tr>
<tr>
<td>0</td>
<td>$0^{--}$</td>
<td>2010$^{+35}_{-60}$</td>
<td>270±60</td>
<td>1189</td>
<td>New</td>
</tr>
</tbody>
</table>

(h) and (e) does one see conspicuously separate loops due to different resonances.

The data require a larger and more linear phase variation than can be produced by a single resonance. A sequence of resonances conspires to produce approximately a linear phase variation with mass squared $s$. In electronics, it is well known that a linear phase variation with frequency may be obtained with
Fig. 3. Argand diagrams for prominent partial waves; for (a) and (b), dashed curves show the effect of removing the lower $0^-$ resonance. Crosses mark beam momenta of Table 1; in all cases, the amplitude moves anti-clockwise with increasing beam momentum.

A Bessel filter [11], which has a sequence of poles almost linearly spaced in frequency. Meson resonances appear to behave in an analogous way as a function of $s = M^2$. Physically, such a system generates a time delay which is independent of frequency, since group velocity is proportional to the gradient of phase vs. frequency.

We shall now comment on each $J^P$ in turn, beginning with the high partial waves which are most conspicuous. For $4^+$, the different peaks in Figs. 2 (g) and (h) for $f_2(1270)\eta$ (full curves) and $a_2(1320)\pi$ (dashed) obviously demand two resonances. The lower one optimises at a mass of $2018\pm6$ MeV, considerably lower than the PDG average of $2044\pm11$ MeV. The effect of the centrifugal barrier is large and the intensity of the $3F_4$ partial wave peaks at 2080 MeV in Fig. 2(g). Differences in earlier determinations of the mass may well depend on varying treatments of
the centrifugal barrier. Our mass is determined essentially by the maximum in the "speed plot", i.e., in the movement of the amplitude in the Argand diagram; it is found to be quite insensitive to the radius chosen for the centrifugal barrier. As the radius increases, all masses go up; however, with our error for the radius of the barrier, the contribution to the uncertainty in mass is only 0.7 MeV. The width of the resonance does, however, correlate much more strongly with the radius, and this contributes an uncertainty to the width of 3.3 MeV. The \( L = 3 \) centrifugal barrier in the \( \bar{p}p \) channel suppresses strongly the coupling of the lower 4\(^+\) resonance to \( f_2(1270)\eta \). The lower resonance couples purely to \( \bar{p}p\,^3F_4, \) while the upper one requires a large \(^3H_4\) component.

For \( J^P = 3^+ \), the different peaks in Fig. 2(d) for \( f_2\eta \) (full curves) and \( a_2\pi \) (dashed) again clearly demand two resonances. The Argand diagram of Fig. 3(h) also clearly demands two resonances. The upper resonance is considerably stronger in the present fit than in Ref. [1].

For \( J^P = 2^+ \), the \( \eta\pi\pi \) data alone do not resolve \( f_2(1934) \) and \( f_2(2001) \) clearly, because both lie at the bottom end of the available mass range. One sees from Table 2 that removing the \( f_2(2001) \) produces a change in log likelihood in \( \eta\pi\pi \) data alone of only 168. This is because it may be interchanged to some extent with contributions from \( f_2(1934) \). For \( f_2(1934) \) and \( f_2(2001) \), the two-body data play a very important role, for two reasons. Firstly there are extensive data on the \( \pi^-\pi^+ \) channel down to 360 MeV/c (a mass of 1910 MeV), which determine quite well the mass and width of the \( f_2(1934) \). This state is consistent in mass with the the \( f_2(1920) \) of GAMS [12] and VES [13], though they find a narrower width. Secondly, the polarisation data provide a sensitive determination of the ratios of amplitudes between \(^3F_2\) and \(^3P_2\). The values of \( r_2 \) given in Table 2 are for ratios of coupling constants \(^3F_2/\sqrt{2},\) i.e., after factoring out the effect of the centrifugal barriers. Both \( f_2(1934) \) and \( f_2(2001) \) are definitely required by the two-body data. If either of them is removed from the fit, \( \chi^2 \) for 2-body data increases by > 2000, as reported in Ref. [3].

The earlier analysis of \( \eta\pi\pi \) data [1] showed the requirement for three \( 2^+ \) states at 2020 ± 50, 2240 ± 40 and 2370 ± 50 MeV. These masses now adjust naturally by small amounts to those of Table 2. The lowest \( 2^+ \) state at 1934 MeV contributes strongly to \( \bar{p}p \to \pi\pi \) and is dominantly \(^3P_2\); that at 2001 MeV is largely \(^3F_2\) and is close in mass to \(^3F_3\) and \(^3F_4\) resonances. The \( f_2(2240) \) and \( f_2(2293) \) are dominantly \(^3P_2\) and \(^3F_2\), respectively. Each of the Argand diagrams of Figs. 3 (k) to (o) show at most a requirement for two \( 2^+ \) states. However, the two-body data require four \( 2^+ \) states, as reported in Ref. [3]. The \( f_2(1934) \) lies below the available mass range for \( \eta\pi\pi \) data.

We have speculated earlier on the possibility that the broad \( f_2(1980) \) has a large component due to the 2\(^+\) glueball [14,15]. In this context, its relative coupling to \( \pi\pi \) and \( \eta\eta \) is important. We fit these freely, with the result \( g_{\eta\eta}/g_{\pi\pi}^2 = 0.72 ± 0.06. \) This compares with the prediction 0.41 for \( (\eta\eta + d\bar{d})/\sqrt{2} \) and 1 for a glueball. The result lies midway between the two. In Ref. [3], the amplitude was expressed in terms of flavour mixing to a linear combination of states \( \cos\Phi|u\bar{u} + d\bar{d}|/\sqrt{2} + \sin\Phi|s\bar{s}| \). With this parametrisation, the flavour mixing angle \( \Phi \) optimises at 23.6 ± 3.5\(^\circ\) compared with the value 35.6\(^\circ\) for a glueball and 0\(^\circ\) for \( (u\bar{u} + d\bar{d}) \). Some mixing between a glueball and neighbouring \( q\bar{q} \) states is likely.

For \( J^P = 0^+ \), there are only minor changes from Ref. [3]. The \( f_0(2105) \) makes a large and very well defined contribution to \( \eta\eta \), and a small contribution to \( \pi\pi \). It requires a mixing angle \( \Phi = (58 ± 5)\(^\circ\). \) Its strong production and large flavour mixing angle suggests exotic character. It is clearly an "extra", non-\( q\bar{q} \) state. Its mass optimises at 2102 ± 13 MeV and its width at 211 ± 29 MeV. These values agree closely with those found in the E760 experiment [16]. Since their determinations of the width is somewhat more precise, namely 203 ± 10 MeV, we use this value in the final fit. The data also require the presence of another nearby broad \( f_0(2040) \), in good agreement with WA102 parameters [17] and a further \( f_0(2337) \) at higher mass.

For \( J^{PC} = 2^-+ \), earlier data on \( \bar{p}p \to \eta\pi^0\pi^0 \) demonstrate the presence of two states at 1860 and 2030 MeV [18]. The first of these decays largely to \( [f_2\eta]_{L=0} \) and the second decays weakly to this channel. The present \( \eta\pi\pi \) data extend only down to 1960 MeV and therefore do not resolve these two states. We therefore fix their masses, widths and branching ratios between \( f_2\eta, a_2\pi \) and \( a_0\pi \) channels to the values of Ref. [18]. The \( [f_2\eta]_{L=0} \) channel makes a dominant contribution to the \( 2^-+ \) \( \eta\pi\pi \) partial waves at low mass, see Fig. 2(c), full curve. The
presence of a further \( \eta_2 \) at high masses is seen most clearly in \( ^1D_2 \rightarrow [f_2 \eta]_{L=2} \). Fig. 2(c) chain curve; it is also visible in the smaller components \([\alpha_0(980)\pi]_{L=2} \). Fig. 2(k) dashed curve and in \( ^1D_2 \rightarrow [f_0(1500)\eta]_{L=2} \). Fig. 2(k) chain curve. Its mass, \( 2267 \pm 14 \) MeV, is somewhat lower than the earlier determination [1], \( 2300 \pm 40 \) MeV. However, it agrees closely with a conspicuous peak observed [19] in the integrated cross section. However, there is quite a large error on the width.\n
We now observe small but well determined contributions of a similar mass. We now observe small but well determined contributions from a \( 4^{-+} \) state at \( 2328 \pm 38 \) MeV in \( ^3G_2 \rightarrow [a_2 \pi]_{L=2} \), \([\alpha_2 \pi]_{L=4} \) and \([\alpha_0 \pi]_{L=4} \). Fig. 2(m). This resonance, although it contributes the smallest change in log likelihood in Table 2, is very stable throughout all fits. There is no doubt of its presence, despite the fact that it contributes only 2% of the integrated cross section. However, there is quite a large error on the width.\n
Next we consider \( 1^{++} \). The earlier work of Ref. [1] found a strong \([a_2 \pi]_{L=1} \) intensity near the \( \bar{p}p \) threshold. In the new combined fit, the \( 1^{++} \) amplitude is large in several channels around 1975 MeV, falling rapidly at higher masses, see Fig. 2(j). Despite the fact that it lies at the bottom end of the available mass range, parameters of the resonance in this range are very stable: \( M = 1971 \pm 15 \) MeV, \( \Gamma = 241 \pm 45 \) MeV. Argand diagrams of Figs. 3 (c) and (d) for \([f_2 \eta]_{L=1} \) and \([\sigma \eta]_{L=1} \) require rapidly varying phases at low mass when one remembers that the amplitude goes to zero at the \( \bar{p}p \) threshold; this rapid phase variation is a clear signature of the resonance.\n
At high masses, the phase variation observed in Ref. [1] suggested a \( 1^{++} \) resonance at \( 2340 \pm 40 \) MeV with \( \Gamma = 340 \pm 40 \) MeV. We now find that this second \( 1^{++} \) resonance is definitely required. Without it, log likelihood is worse by 882; statistically this is a 25 standard deviation effect when one allows for the number of fitted parameters. Nonetheless, the mass of this state is the least well determined of all the resonances: \( M = 2310 \pm 60 \) MeV. The reason is that it makes small contributions to the distinctive channels \( a_2(1320)\pi \) and \( f_2(1270)\eta \), and there are large interferences with the \( 0^- \) amplitude. As the resonance mass is changed, these interferences are able to absorb the variation with small changes in log likelihood. This resonance is most clearly visible in \([f_2 \eta]_{L=1} \). Fig. 2(b), full curve.\n
Lastly we consider \( 0^{++} \). There is a large change, compared with Ref. [1], in the fit to the 

The \( \sigma \eta \) and \( f_0(1500)\eta \) channels, Fig. 2(i) (full and dotted curves) now definitely require the presence of a high mass resonance with \( M = 2285 \pm 20 \) MeV. In addition, there is a very strong contribution close to the \( \bar{p}p \) threshold in several channels of Figs. 2 (a) and (i). A resonance is definitely required in this mass region. Without it, log likelihood gets worse by a very large amount, 1189. The effect of dropping it is illustrated by the dashed curves of Figs. 3 (a) and (b). For this partial wave, amplitudes do not go to zero at the \( \bar{p}p \) threshold, but instead go to values described by scattering lengths. It is not possible to exclude the possibility that the \( \bar{p}p \) threshold plays a strong role (a cusp at the \( \bar{p}p \) threshold). This makes the identification of the mass of the lower resonance difficult. The optimum is at \( 2010^{+35}_{-60} \) MeV.\n
There is just one significant change in the fit to two-body data, compared with Ref. [3]. There is a strong \( 3^- \) resonance in the low mass range. It interferes strongly with \( 2^{++} \) states in data on \( \bar{p}p \rightarrow \pi^- \pi^+ \). Small changes in the masses and widths fitted to the \( 2^{++} \) states have had the effect of increasing the mass of this \( \rho_3 \) from \( 1960 \pm 15 \) MeV [3] to \( 1981 \pm 14 \) MeV; the fitted width has also increased slightly.\n
Fig. 4 illustrates the quality of the fit at two momenta. It shows the angular distributions for production of mass regions centred on (a) and (d) \( f_2(1270) \), (b) and (e) \( a_2(1320) \), (c) and (f) \( a_0(980) \). In all cases, there are background contributions underneath these resonances, particularly in the case of the small \( a_0(980) \) signal; these backgrounds are included in

Fig. 4. Centre of mass angular distributions for production of (a) $f_2(1270)/\eta$, (b) $a_2(1320)/\pi$, (c) $a_0(980)/\pi$ at 1050 MeV/c; (d)–(f) show corresponding angular distributions at 1525 MeV/c. Histograms show the fit. Data and fit are uncorrected for angular acceptance of the detector; this acceptance is included in the maximum likelihood fit.

Fig. 4. The angular distributions are well reproduced by the fit in all cases.

We now turn to the interpretation of the results. Fig. 5 shows plots of mass squared vs. excitation for all $J^P$. In making this plot, we use the K-matrix mass of 1598 MeV for $f_2(1565)$ [20], and 1400 MeV for $f_0(1370)$ [7]. The reason for this choice is that the well known linear mass relation between $\Sigma(1230)$, $\Sigma(1385)$, $\Sigma(1520)$ and $\Omega^-(1672)$ works well when one uses for the $\Delta$ the mass at which the phase shift goes through 90°, i.e., the K-matrix mass. It does not work nearly so well using the T-matrix pole position of 1210 MeV. For Breit–Wigner resonances of constant width (used here), the K-matrix mass and the T-matrix pole position are the same.

A remarkably simple pattern is apparent in Fig. 5. All states lie close to parallel straight-line trajectories. These trajectories extrapolate well to known states in the mass range 1200–1700 MeV. [We do not, however, attempt to place the $\eta$ on the 0° trajectory, since its mass is affected strongly by the instanton interaction.] The simplicity of Fig. 5 suggests that observed states are $q\bar{q}$ rather than hybrids. For $^3P_1$ and $^1S_0$, states expected around 1670 MeV are presently missing. For quantum numbers $^3D_3$, $I = 1$ resonances observed in the $\pi^-\pi^+$ channel are shown; it is worthwhile to show these results, since the lowest two states are very well defined.

Using PDG masses for resonances below 1.9 GeV, slopes for different quantum numbers are shown in Table 3. They are consistent with the same slope within statistical errors; the mean slope is $1.143 \pm 0.013$ GeV$^2$ per excitation. The most accurate determination of the slope comes from the $^3P_2$ trajectory, which begins with the well known $f_2(1270)$. However, one must expect some deviations from straight lines
due to local perturbations, for example (a) from mixing with nearby glueballs or hybrids, (b) from level repulsion between $2^+$ states, (c) from nearby thresholds, particularly in the low mass region, and (d) from variation with $s$ of the effects of tensor and spin–orbit splitting. There is an “extra” $2^+$ state at 1860 MeV, which will perturb the $2^+$ trajectory; as discussed in Ref. [18], it is a candidate for a hybrid expected in that mass region.

The $0^+$ trajectory is drawn with $n = 1$ for $f_0(980)$. The line does, however, go through $f_0(1370)$, and it is possible that this state is the $n = 1$ ground state rather than a molecule. The error on the K-matrix mass of $f_0(1370)$ is quite large, so the $0^+$ trajectory is defined mostly by $f_0(1770)$ and $f_0(2337)$. Because of strong mixing between scalar states, the interpretation of the mass range around $f_0(980)$ and $f_0(1370)$ may be complicated; it has previously been considered in terms of the K-matrix approach by Anisovich et al. [21].

In the absence of tensor and spin-orbit splitting, $3^{F_3}$ states are degenerate in mass. Tensor and spin-orbit splitting may be assessed from the relations [22]

$$\Delta M_{LS} = \left[ -20M(F_2) - 7M(F_3) + 27M(F_3) \right]/54,$$

$$\Delta M_T = \left[ -4M(F_2) + 7M(F_3) - 3M(F_3) \right]/14.$$  

For both multiplets of Table 2, $3^{F_3}$ states lie highest in mass, requiring significant tensor splitting. This split-
ting is in the sense predicted by one-gluon exchange. Using a linear confining potential plus one-gluon exchange, Godfrey and Isgur [23] predict \( \delta M_T = 9 \) MeV for the lower multiplet and \( \Delta M_{LS} = -20 \) MeV. From Table 2, one finds \( \delta M_T = 20 \pm 4.5 \) MeV for the lower multiplet and \( 7.1 \pm 7.3 \) MeV for the upper one; errors take into account observed correlations in fitted masses. Because we use the same centrifugal barrier for \( 3F_4, 3F_3 \) and \( 3F_2 \) states, there is almost no correlation of \( \Delta M_T \) or \( \Delta M_{LS} \) with the radius of the barrier. The observed spin–orbit splittings are \( \Delta M_{LS} = 2.4 \pm 4.4 \) MeV for the lower multiplet and \( -6.3 \pm 5.1 \) MeV for the upper one; these average approximately to zero.

In summary, the expected \( q\bar{q} \) states in this mass range are observed with well determined masses and widths, except for the lower 0+ and the upper 1+ state, where errors are sizeable. They follow a simple pattern requiring approximately linear trajectories of mass squared against excitation number, with a slope of \( 1.143 \pm 0.013 \) GeV\(^2\) per excitation. In addition, there is evidence for a broad 2+ component with \( M = 2010 \pm 25 \) MeV, \( I^G = 495 \pm 35 \) MeV. Tensor splitting for \( 3F \) states is, within sizeable errors, consistent with that predicted from one-gluon exchange and a linear confining potential. Spin–orbit splitting of \( 3F \) states is consistent with zero.

### Acknowledgements

We thank the Crystal Barrel Collaboration for allowing use of the data. We acknowledge financial support from the British Particle Physics and Astronomy Research Council (PPARC). We wish to thank Prof. V.V. Anisovich for helpful discussions. The St. Petersburg group wishes to acknowledge financial support from PPARC and INTAS grant RFBR 95-0267.

### References

[7] A.V. Anisovich et al., Resonances in \( \bar{p}p \to \eta \pi^+ \pi^- \pi^+ \pi^- \), submitted to Nucl. Phys. A.
[15] D.V. Bugg, Resonances around 2 GeV: \( q\bar{q} \) and glueballs, Hadron99 Proceedings (to be published).
[19] A.V. Anisovich et al., Data on \( \bar{p}p \to \eta \pi^0 \pi^0 \) for masses 1960 to 2410 MeV, submitted to Phys. Lett. B.

### Table 3

<table>
<thead>
<tr>
<th>( J^P )</th>
<th>Slope (GeV(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4++</td>
<td>1.139 ± 0.037</td>
</tr>
<tr>
<td>3++</td>
<td>1.107 ± 0.078</td>
</tr>
<tr>
<td>3( F_2 )</td>
<td>1.253 ± 0.066</td>
</tr>
<tr>
<td>3( P_2 )</td>
<td>1.131 ± 0.024</td>
</tr>
<tr>
<td>2++</td>
<td>1.217 ± 0.055</td>
</tr>
<tr>
<td>1++</td>
<td>1.120 ± 0.027</td>
</tr>
<tr>
<td>0++</td>
<td>1.164 ± 0.041</td>
</tr>
<tr>
<td>0––</td>
<td>1.183 ± 0.028</td>
</tr>
<tr>
<td>3( ^–) ( I = 1 )</td>
<td>1.099 ± 0.029</td>
</tr>
</tbody>
</table>
Production of $\phi$-mesons in p + p, p + Pb and central Pb + Pb collisions at $E_{\text{beam}} = 158$ A GeV

NA49 Collaboration

S.V. Afanasiev\textsuperscript{1}, T. Anticic\textsuperscript{u}, J. Bächler\textsuperscript{f,h}, D. Barma\textsuperscript{c}, L.S. Barnby\textsuperscript{c}, J. Bartke\textsuperscript{g}, R.A. Barton\textsuperscript{c}, L. Betev\textsuperscript{n}, H. Bialkowska\textsuperscript{a}, A. Billmeier\textsuperscript{b}, C. Blume\textsuperscript{h}, C.O. Blyth\textsuperscript{c}, B. Boimska\textsuperscript{a}, J. Bracinik\textsuperscript{d}, F.P. Brady\textsuperscript{i}, R. Brun\textsuperscript{f}, P. Bunčić\textsuperscript{f,k}, L. Carr\textsuperscript{s}, D. Cebra\textsuperscript{i}, G.E. Cooper\textsuperscript{b}, J.G. Cramer\textsuperscript{a}, P. Csató\textsuperscript{e}, V. Eckardt\textsuperscript{p}, F. Eckhardt\textsuperscript{r}, I. Ferenc\textsuperscript{i}, H.G. Fischer\textsuperscript{f}, Z. Fodor\textsuperscript{e}, P. Foka\textsuperscript{b}, P. Freund\textsuperscript{p}, V. Friese\textsuperscript{o}, J. Ftácnik\textsuperscript{d}, J. Gál\textsuperscript{e}, R. Ganz\textsuperscript{p}, M. Gaździcki\textsuperscript{k}, E. Gladysz\textsuperscript{e}, J. Grebieszek\textsuperscript{w}, J.W. Harris\textsuperscript{i}, S. Hegyi\textsuperscript{e}, V. Hlinka\textsuperscript{d}, C. Höhne\textsuperscript{g}, I. Gigo\textsuperscript{n}, M. Ivanov\textsuperscript{d}, P. Jacobs\textsuperscript{b}, R. Janik\textsuperscript{d}, P.G. Jones\textsuperscript{c}, K. Kadija\textsuperscript{u,p}, V.I. Kolesnikov\textsuperscript{j}, M. Kowalski\textsuperscript{g}, B. Lasiuk\textsuperscript{i}, P. Lévai\textsuperscript{e}, A.I. Malakhov\textsuperscript{j}, S. Margetis\textsuperscript{m}, C. Markert\textsuperscript{h}, B.W. Mayes\textsuperscript{l}, G.L. Melkumov\textsuperscript{j}, A. Mischke\textsuperscript{h}, J. Molnár\textsuperscript{e}, J.M. Nelson\textsuperscript{c}, G. Odyniec\textsuperscript{b}, M.D. Oldenburg\textsuperscript{k}, G. Pálla\textsuperscript{e}, A.D. Panagiotou\textsuperscript{a}, A. Petridis\textsuperscript{a}, M. Pikna\textsuperscript{d}, L. Pinsky\textsuperscript{l}, A.M. Poskanzer\textsuperscript{b}, D.J. Prindle\textsuperscript{e}, F. Pühlhofer\textsuperscript{a,s}, J.G. Reid\textsuperscript{a}, R. Renfordt\textsuperscript{k}, W. Retyk\textsuperscript{f}, H.G. Ritter\textsuperscript{b}, D. Röhrich\textsuperscript{k,l}, C. Roland\textsuperscript{h}, G. Roland\textsuperscript{k}, A. Rybicki\textsuperscript{g}, T. Sammer\textsuperscript{p}, A. Sandoval\textsuperscript{h}, H. Sann\textsuperscript{h}, A.Yu. Semenov\textsuperscript{j}, E. Schäfer\textsuperscript{p}, N. Schmitz\textsuperscript{p}, P. Seyboth\textsuperscript{f}, P. Siklér\textsuperscript{a,l}, B. Sitarczyk\textsuperscript{r}, E. Skrzypczak\textsuperscript{f}, R. Snellings\textsuperscript{b}, G.T.A. Squier\textsuperscript{e}, R. Stock\textsuperscript{b}, P. Strmen\textsuperscript{d}, H. Ströbele\textsuperscript{k}, T. Susa\textsuperscript{a}, I. Szarka\textsuperscript{d}, I. Szentpétery\textsuperscript{e}, J. Sziklai\textsuperscript{e}, M. Toy\textsuperscript{h,n}, T.A. Trainor\textsuperscript{s}, S. Trentalange\textsuperscript{n}, T. Ullrich\textsuperscript{l}, D. Varga\textsuperscript{c}, M. Vassiliou\textsuperscript{a}, G.I. Veres\textsuperscript{e}, G. Vesztergombi\textsuperscript{e}, S. Voloshin\textsuperscript{b}, D. Vranic\textsuperscript{f}, F. Wang\textsuperscript{b}, D.D. Weerasundara\textsuperscript{a}, S. Wenig\textsuperscript{f}, C. Whitten\textsuperscript{n}, N. Xu\textsuperscript{b}, T.A. Yates\textsuperscript{c}, I.K. Yoo\textsuperscript{o}, J. Zimányi\textsuperscript{e}

\textsuperscript{a} Department of Physics, University of Athens, Athens, Greece
\textsuperscript{b} Lawrence Berkeley National Laboratory, University of California, Berkeley, CA, USA
\textsuperscript{c} Birmingham University, Birmingham, England, UK
\textsuperscript{d} Institute of Physics, Bratislava, Slovakia
\textsuperscript{e} KFKI Research Institute for Particle and Nuclear Physics, Budapest, Hungary
\textsuperscript{f} CERN, Geneva, Switzerland
\textsuperscript{g} Institute of Nuclear Physics, Cracow, Poland
\textsuperscript{h} Gesellschaft für Schwerionenforschung (GSI), Darmstadt, Germany
\textsuperscript{i} University of California at Davis, Davis, CA, USA
\textsuperscript{j} Joint Institute for Nuclear Research, Dubna, Russia
\textsuperscript{k} Fachbereich Physik der Universität, Frankfurt, Germany
\textsuperscript{l} University of Houston, Houston, TX, USA
\textsuperscript{m} Kent State University, Kent, OH, USA
\textsuperscript{n} University of California at Los Angeles, Los Angeles, CA, USA
Abstract

Yields and phase space distributions of $\phi$-mesons emitted from $p+p$ (minimum bias trigger), $p+Pb$ (at various centralities) and central $Pb+Pb$ collisions are reported ($E_{\text{beam}} = 158$ A GeV). The decay $\phi \rightarrow K^+K^-$ was used for identification. The $\phi/\pi$ ratio is found to increase by a factor of $3.0 \pm 0.7$ from inelastic $p+p$ to central $Pb+Pb$. Significant enhancement in this ratio is also observed in subclasses of $p+p$ events (characterized by high charged-particle multiplicity) as well as in the forward hemisphere of central $p+Pb$ collisions. In $Pb+Pb$ no shift or significant broadening of the $\phi$-peak is seen. © 2000 Published by Elsevier Science B.V.

1. Introduction

Strange-particle production is one of the observables expected to deliver detailed information on the reaction dynamics of ultrarelativistic nucleus–nucleus collisions [1]. In experiments at the CERN SPS accelerator it was found that the ratio of the number of produced kaons to that of pions is higher by a factor of about two compared to that in proton–proton reactions at the same energy [2–5]. In the past, several possible reasons for this strangeness enhancement have been discussed. Secondary interactions between the many hadrons created — about 2400 in a central $Pb+Pb$ collision at 158 A GeV beam energy — may lead to an increase of the strange-particle fraction above that in $p+p$ reactions, although the rate for this process is estimated to be relatively small. Secondly, and more interestingly, if nucleus–nucleus reactions proceed through a deconfined stage — in the limiting case with formation of a quark–gluon plasma (QGP) — then strange-quark production should be abundant [1]. Clearly, even if the colliding nucleons did not dissolve into a partonic phase, they would on average undergo several collisions. The stronger excitation might in turn lead to a strangeness enhancement [6]. This suggests to study particle production rates also as a function of the inelasticity or impact parameter in $p+p$ and $p+nucleus$ reactions.

In this context, $\phi$-mesons play a particular role due to the $s\bar{s}$ composition of these mesons. Their yield should depend more sensitively than that of kaons on a strangeness enhancement stemming from the early partonic phase. Significantly enhanced production was proposed [7] as a QGP signature in nucleus–nucleus collisions. As hadrons they are in total strangeness-neutral; the strangeness is “hidden” and therefore without influence on a hadro-chemical equilibrium.

The present paper reports yields of $\phi$-mesons measured via their $K^+K^-$ decay channel by the NA49 experiment at the CERN SPS. The systems studied are:

- proton + proton (with minimum-bias trigger and with multiplicity selection),
- proton + Pb (at various centralities),
- Pb + Pb (with triggering on central collisions),
each at 158 A GeV beam energy and with fixed target. The event selections represent an attempt to investigate the evolution of strangeness enhancement when going from more elementary to increasingly complex reactions.
2. Experiment

NA49 is a fixed-target experiment at CERN using external proton and, in particular, heavy-ion beams from the SPS. It is based on a hadron spectrometer that covers a large fraction of the solid angle and of the relevant momentum range. Four time-projection chambers (TPCs) provide charged-particle tracking as well as particle identification by dE/dx measurement. In a limited momentum range — which, however, is very important for mid-rapidity kaons — time-of-flight detectors (TOF) supplement the particle identification capabilities of the system. The apparatus is described in detail in [8]. In the following, we focus on the features that are important for the present work.

For the investigation of the Pb + Pb reaction a 158 GeV lead beam was used (typically 10^5 particles per 4.8 s spill/19.2 s cycle time). Beam contaminations were negligible. The solid Pb target had a thickness of 224 mg/cm², equivalent to 1% interaction probability. The 4% most central interactions, corresponding to an impact parameter range b < 3.5 fm, were selected by applying an appropriate upper limit on the energy transmitted in beam direction and measured by the zero-degree calorimeter. 10 to 20 events/spill were recorded. 380 000 events are used for this study.

For the measurement with protons the beam line was set to select secondary protons of 158 GeV (produced by the 450 GeV SPS beam). Typical intensities were 5 \times 10^4 to 10^5 protons per 2.4 s extraction within the 14.4 s SPS cycle. The protons were identified by Cerenkov counters. The contamination by pions and kaons was below 10^-3.

In the p + p case a 14 cm long liquid-hydrogen target (1.95% interaction probability) was inserted. Only interactions in the central 11 cm were accepted in order to minimize contributions from interactions in the mylar windows. By means of “empty-target runs” they were found to be of the level of 1% over the whole multiplicity range of interest. The losses introduced by this fiducial cut depend on the accuracy, with which the position of the interaction vertex can be determined by back-extrapolating the tracks from the first TPC, and which therefore varies with the charged-particle multiplicity of the event. They decrease from 16% at n_{ch} = 4 to 5% at n_{ch} = 10 (the mean value of the multiplicity in p + p is n_{ch} = 7.2). In this context, it is of importance that the position of the individual beam particles was measured by multi-wire chambers in the beam line with sub-millimeter accuracy. The “minimum-bias” event trigger used (28.9 mb) corresponds to 91% of the known inelastic p + p cross section (31.7 mb). 30 events/spill were recorded. The number of analyzed events is 400 000.

An essential additional feature in the p + Pb case was a centrality detector [8] counting the number n_{CD} of particles emitted from the Pb target nucleus in backward direction (mostly protons). n_{CD} is a measure of the mean number of collisions ν of the projectile inside the target (approximately ν ∝ √n_{CD} [9]). 180 000 events were analyzed.

The NA49 spectrometer accepts about 80% of all emitted particles. The requirements of kaon identification restricted the analysis of the φ-yields to the forward hemisphere, i.e., to above midrapidity y_{cm} = 2.9.

For the symmetric reactions p + p and Pb + Pb the total particle multiplicities can be obtained by doubling the measured forward yields. In the Pb + Pb case the two TPCs outside the magnetic field (MTPCs) were used for tracking. For the proton-induced reactions with their considerably lower charged-particle multiplicity (7.2 in p + p compared to about 1500 in central Pb + Pb) a “global” tracking through all TPCs was employed. As discussed, this is important for localizing the reaction vertex in the extended target volume in the p + p case.

For particle identification the momentum range 3.5–25 GeV/c (Pb + Pb) and 3–35 GeV/c (p + p, p + Pb) was selected. Here, the specific energy loss dE/dx of charged particles in the TPC gas lies in the region of the relativistic rise of the Bethe–Bloch function. The dE/dx resolution of the TPCs (σ/(dE/dx) = 5% (Pb + Pb case) and 4% (p + p case)) was sufficient to separate pions from the group of kaons and protons, but the latter two particle types could only be resolved on a statistical basis. In practice, particles in a window of ±1.5 σ around the mean kaon dE/dx (calculated using the Bethe–Bloch function with parameters adjusted to pions and TOF-identified protons and kaons) were selected. It contains 87% of all kaons. The pion and proton contamination eliminates itself to a large extent, because only kaon pairs from φ-decay can contribute to the φ-peak in the invariant-
mass spectrum. Misidentified particles, however, may distort the background. Near midrapidity, the combination of \( dE/dx \) and TOF information provides nearly perfect kaon separation. Here, the identification efficiency is about 85%, the contamination by pions and protons small (<12%).

In all cases of kaon identification, the \( \phi \)-signal was obtained by calculating the invariant mass distribution of the accepted \( K^+K^- \) pairs and subtracting the combinatorial background. The latter was reconstructed by the usual event mixing method. Both were normalized to the same number of particle pairs in the spectrum [10]. Fig. 1 shows examples. In some cases, one observes remaining background, which was subtracted. It can be shown to stem mostly from misidentified particles. The peak was then fitted with a relativistic Breit–Wigner distribution \( \Gamma = 4.43 \text{ MeV} \) folded with a Gaussian with adjustable width \( \sigma_m \) representing the spectrometer resolution. The parameters of the fit function were adjusted by a fit to the signal in the total acceptance.

The data were corrected for geometrical detector acceptance, tracking efficiency, kaon decay in flight, kaon identification efficiency, and, in the TOF case, for double hits and other losses in the scintillators. The geometrical acceptance, including losses by kaon decay, was obtained by GEANT [11] simulations using parametrized phase space distributions of isotropically decaying \( \phi \)-mesons as input. In order to give some typical values: In the \( \phi \)-rapidity range \( 3 < y < 3.8 \) used for the analysis of the \( m_T \)-distribution in the \( p + p \) case it varies from 62% at \( p_T < 0.3 \text{ GeV}/c \) to 36% at \( p_T = 2 \text{ GeV}/c \); for the \( p + p \) and \( p + \text{Pb} \) reactions, where all TPCs were used, the corresponding values are 80% at \( p_T < 0.3 \text{ GeV}/c \) and 73% at \( p_T = 1 \text{ GeV}/c \). The tracking efficiency was determined by simulations (embedding of tracks into real events) to be close to 100% in the relevant momentum range.

Details of the analysis can be found in [12]. Preliminary reports on this work were given in [5,13,14].

3. Results

The position of the \( \phi \)-signal in the \( K^+K^- \) invariant-mass spectrum is found at \((1019.4 \pm 0.2) \text{ MeV} \) in \( p + p \), at \((1019.0 \pm 0.3) \text{ MeV} \) in \( p + \text{Pb} \), and at \((1018.7 \pm 0.5) \text{ MeV} \) in \( \text{Pb} + \text{Pb} \). The \( \sigma_m \) values are \((1.1 \pm 0.2) \text{ MeV} \) for \( p + p \) and \((1.6 \pm 0.3) \text{ MeV} \) for \( \text{Pb} + \text{Pb} \). As 1.0 MeV is the calculated contribution from multiple scattering in the TPC material and an additional amount depending on track density is expected from the tracking accuracy, this agrees approximately with the spectrometer resolution. Therefore, within our errors mass and width of the \( \phi \)-peak are consistent with the free-particle values \( m = (1019.41 \pm 0.01) \text{ MeV} \) and \( \Gamma_0 = (4.43 \pm 0.05) \text{ MeV} \) [15] even in the \( \text{Pb} + \text{Pb} \) case. However, for \( \text{Pb} + \text{Pb} \) we cannot exclude a parameter combination \( \Gamma_0 = 6 \text{ MeV} \), \( \sigma_m = 1.2 \text{ MeV} \), i.e., a slightly increased width.

Transverse distributions of the \( \phi \) are shown in Fig. 2(a). From a fit to the \( m_T \)-distribution using an exponential function \( dn/(dm_T dm) \propto \exp(-m_T/T) \) one
obtains an inverse slope or temperature parameter $T = (305 \pm 15)$ MeV for Pb + Pb, to be compared with $(169 \pm 17)$ MeV for $p + p$. The quoted uncertainties are statistical errors. The large $T$-parameter in the Pb + Pb case fits into the systematics obtained for the dependence on the mass of various emitted particles (e.g., [16]). This behavior is characteristic of a transverse velocity field. Its origin is in debate; it may stem from the hadronizing partonic stage or develop in the hadronic phase [17].

The longitudinal distributions were obtained by integration over $m_t$ in individual rapidity intervals using the slope parameter determined before (Fig. 2(b)). In the Pb + Pb case the distribution is clearly broader than in $p + p$ and similar to that measured for other produced particles, e.g., pions and kaons. From a fit with a Gaussian ($dn/dy \propto \exp\left(-\frac{(y-y_{cm})^2}{2\sigma_y^2}\right)$) one obtains for the widths $\sigma_y = 0.89 \pm 0.06$ ($p + p$) and $1.22 \pm 0.16$ (Pb + Pb). This difference is remarkable in view of the fact that the shape of the distributions of charged pions and kaons is very similar in both reactions (e.g., $\sigma_y = 1.5$ [4]).

By integrating the fit functions discussed before over the whole kinematical range one obtains for the total average $\phi$-multiplicities

- for $p + p$ (inelastic): $\langle \phi \rangle = 0.012 \pm 0.0015$,
- for Pb + Pb (central): $\langle \phi \rangle = 7.6 \pm 1.1$.

The error estimates include contributions from statistics, background and extrapolation to full phase space.

For the $p + p$ reaction a series of data exist for comparison from other experiments [18] over a wide range of energies (Fig. 3). Our data point is consistent with the other results. For the nucleus–nucleus system the data are scarce. $\phi$-production was measured at the much lower AGS energy of 13.6 $A$ GeV [19]. For 158 $A$ GeV preliminary data were reported by the CERN NA50 collaboration [20].

The topology dependence of the $\langle \phi \rangle / \langle \pi \rangle$ ratio in $p + p$ as measured in this experiment is shown in Fig. 4(a). $n_{ch}$ is the number of the emitted charged particles within the acceptance of the NA49 spectrometer (typically 80%). For $p + Pb$ interactions the yields were obtained in four different centrality intervals (Fig. 4(b)). For the extrapolation of the mea-
Fig. 4. (a) Multiplicity dependence of the $\phi/\pi$ ratio in p + p. The cross-section weighted average is indicated by the horizontal dashed line. (b) Centrality dependence of the $\phi/\pi$ ratio in the forward hemisphere in p + Pb normalized to the average p + p value. The minimum-bias value is indicated by the horizontal dashed line. Vertical dashed lines indicate bin sizes in the abscissa.

sured to the total forward $\phi$-yield the parameters $\sigma_y$ and $T$ from p + p were used. This introduces a systematic error of possibly 5 to 10%, which has to be added to the statistical errors displayed in Fig. 4(b). It has to be emphasized that the p + Pb data hold for the forward hemisphere only. When comparing them with the symmetric reactions p + p and Pb + Pb one has to take into account two effects, (a) the feed-over of $\phi$-yield from the target hemisphere which may depend on centrality, and (b) the projectile energy loss which leads to a small ($\ll 0.2$ rapidity units) backward shift of the projectile fragmentation center of mass.

4. Discussion

When interpreting the total $\phi$-production cross section it is natural to use the emitted pions as reference. Approximately, this is equivalent to a normalization to the number of nucleons participating in the reaction as shown by data for minimum-bias p + p as well as for peripheral and central Pb + Pb collisions [4]. The comparison of the normalized $\phi$-yield between central Pb + Pb and inelastic p + p gives a “$\phi$-enhancement factor”

$$
\frac{\langle \phi \rangle}{\langle \pi \rangle} (\text{Pb + Pb central}) = 3.0 \pm 0.7, \\
\frac{\langle \phi \rangle}{\langle \pi \rangle} (\text{p + p inelastic})
$$

where $\langle \phi \rangle$ designates the average $\phi$-multiplicity, and $\langle \pi \rangle = (\langle \pi^+ \rangle + \langle \pi^- \rangle)/2$ is the corresponding quantity for pions (numerical values: 2.87 for p + p [21] and 611 for Pb + Pb (NA49 data)).

The enhancement factor obtained here is significantly larger than the one measured for kaons (approximately 2.0 [3,5]), but clearly smaller than in the case of $|S| = 2$ and $|S| = 3$ baryons [22,23]. It agrees approximately with the preliminary result of NA50 [20] for the $\phi/(\rho + \omega)$ ratio, that was found to rise by a factor of about 3 between deuteron–carbon and central Pb + Pb.

The magnitude of the experimental $\phi$-enhancement is moderate in comparison with estimates based solely on the flavor composition in an assumed QGP. According to [7] the $\phi/\omega$ ratio should then rise by more than an order of magnitude. As pointed out by various authors [1,24] this model neglects the influence of the hadronization process and a likely redistribution of strangeness between hadrons. Both effects are expected to reduce this enhancement considerably [24].

It has been shown [25] that for nucleus–nucleus collisions the strangeness yield is indeed consistent with that expected for QGP formation when taking into account the hadronization.

Hadro-chemical models have also been applied successfully to $\phi$-production data. It has been shown [26] that the particle composition in the final state of the collision corresponds approximately to a thermal equilibrium immediately after hadronization, except for a suppression of strange particles. It is tacitly assumed that there is no change afterwards. Apparently, the hadronization process fills the available hadronic phase space. The only information remaining from the
preceding partonic state is preserved in the value of the “strangeness saturation factor” \( \gamma_S \). In this picture, the s-quark production in the partonic state is insufficient to support full hadrochemical equilibrium at the high temperature of the initial hadronic state. Strange enhancement in nucleus–nucleus as compared to p + p collisions is traced back to a change of \( \gamma_S \) (0.45 for p + p and 0.6–0.7 for nucleus–nucleus collisions).

A thermodynamical model [27], which attempts to reproduce the particle composition without introduction of \( \gamma_S \) would favour a 1.6 times higher \( \phi \)-yield than measured here.

Uncertainties in the interpretation of strangeness enhancement in nucleus–nucleus collisions as well as experimental facts, in particular the enhancement of strange particles already in the forward hemisphere of the p + Pb reaction [5], motivate a more detailed investigation of p + p and p + Pb reactions. Whereas the previous discussion of p + p is concerned with production cross sections in average inelastic collisions, Fig. 4(a) shows that there is already a significant variation between subgroups of p + p events. In more violent collisions — defined by selecting a higher charged multiplicity \( n_{ch} \) — the \( \phi/\pi \) ratio is considerably higher than on average. This is qualitatively consistent with previous observations [28] for p + Be collisions. For p + Pb (where we are restricted to the forward hemisphere), Fig. 4(b) demonstrates an even stronger variation as a function of \( \sqrt{n_{cm}} \), which is a measure of the number of interactions of the projectile inside the target. On average, the \( \phi/\pi \) ratio increases by a factor 1.7 above that in average inelastic p + p collisions. Apparently, the consecutive interactions of the projectile nucleon with several target nucleons in p + Pb (3.7 on average) lead to an increased \( \phi \)-production in the forward hemisphere. Similar observations have been made for K\(^+\) production [5].

5. Summary

An enhancement of a factor of 3.0 ± 0.7 was found for the ratio \( \langle \phi \rangle/\langle \pi \rangle \) when comparing central Pb + Pb to minimum-bias p + p reactions. No change of the mass or the width of the \( \phi \) in its K\(^+\)K\(^-\) decay channel was observed. Centrality-selected p + p and p + Pb collisions show a significant rise in \( \langle \phi \rangle/\langle \pi \rangle \) over minimum-bias p + p, which, however, is smaller than that observed in central Pb + Pb collisions. A model based on the assumption of a transient partonic phase with subsequent statistical hadronization is consistent with the measured \( \langle \phi \rangle/\langle \pi \rangle \) enhancement in Pb + Pb.

Acknowledgements

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the US Department of Energy (DE-AC03-76SF00515 and DE-FG02-91ER40609), the US National Science Foundation, the Bundesministerium für Bildung und Forschung, Germany, and its International Office, the Alexander von Humboldt Foundation, the UK Engineering and Physical Sciences Research Council, the Polish State Committee for Scientific Research (2 P03B 02615, 01716, 02418 and 09916), the Hungarian Scientific Research Foundation (T14920 and T23790), the EC Marie Curie Foundation, and the Polish–German Foundation.

References

Determination of the $e^+e^-\rightarrow \gamma \gamma$ cross-section at centre-of-mass energies ranging from 189 GeV to 202 GeV

DELPHI Collaboration

Abstract

A test of the QED process $e^+e^- \rightarrow \gamma\gamma$ is reported. The data analysed were collected with the DELPHI detector in 1998 and 1999 at the highest energies achieved at LEP, reaching 202 GeV in the centre-of-mass. The total integrated luminosity amounts to 375.7 pb$^{-1}$. The differential and total cross-sections for the process $e^+e^- \rightarrow \gamma\gamma$ were measured, and found to be in
agreement with the QED prediction. 95% confidence level (C.L.) lower limits on the QED cut-off parameters of $\Lambda_+ > 330$ GeV and $\Lambda_- > 320$ GeV were derived. A 95% C.L. lower bound on the mass of an excited electron of 311 GeV/$c^2$ (for $\lambda_Y = 1$) was obtained. $s$-channel virtual graviton exchange was searched for, resulting in 95% C.L. lower limits on the string mass scale, $M_S$: $M_S > 713$ GeV/$c^2$ ($\lambda = 1$) and $M_S > 691$ GeV/$c^2$ ($\lambda = -1$). © 2000 Published by Elsevier Science B.V.

1. Introduction

An analysis of two-photon final states using the high energy data sets collected with the DELPHI detector in 1998 and 1999 is reported. The data analysed were collected at $e^+e^-$ collision energies ranging from 188.6 GeV up to 201.6 GeV, corresponding to a total integrated luminosity of 375.7 pb$^{-1}$.

Final states with two photons are mainly produced by the standard process $e^+e^- \rightarrow \gamma \gamma (\gamma)$. This reaction is an almost pure QED process: at orders above $\alpha^2$, it is mainly affected by QED corrections, such as soft and hard bremsstrahlung and virtual corrections, compared to which the weak corrections due to the exchange of virtual massive gauge bosons are very small [1 – 3]. Therefore, any significant deviation between the measured and the QED cross-section could unambiguously be interpreted as the result of non-standard physics.

The Born cross-section for $e^+e^- \rightarrow \gamma \gamma (\gamma)$ is given by

$$\sigma_{\text{QED}}^0 = K \frac{2\pi \alpha}{s},$$

(1)

$K$ depends on the angular acceptance for the final state photons, $\alpha$ is the electromagnetic coupling constant and $s$ is the centre-of-mass energy squared.

Since $\sigma_{\text{QED}}^0$ scales with $s^{-1}$, the combination of measurements taken at different centre-of-mass energy values is straightforward and data taken at neighbouring values of $\sqrt{s}$ can be combined by applying this scaling function.

Previous DELPHI results concerning the process $e^+e^- \rightarrow \gamma \gamma (\gamma)$, using LEPI and LEPII data, can be found in Refs. [4,5]. The most recently published results from the other LEP experiments can be found in Refs. [6–8].

2. Data sample and apparatus

The data analysed were taken at $e^+e^-$ collision energies of $188.63 \pm 0.04$ GeV, $191.6 \pm 0.04$ GeV, $195.5 \pm 0.04$ GeV, $199.5 \pm 0.04$ GeV and $201.6 \pm 0.04$ GeV [9], corresponding to integrated luminosities of $151.9 \pm 0.9$ pb$^{-1}$, $25.1 \pm 0.1$ pb$^{-1}$, $76.1 \pm 0.4$ pb$^{-1}$, $82.6 \pm 0.5$ pb$^{-1}$ and $40.1 \pm 0.2$ pb$^{-1}$, respectively. The luminosity was measured by counting the number of Bhabha events at small polar angles, recorded with DELPHI’s luminometer: the small angle tile calorimeter (STIC), made of two modules located at $|z| = 220$ cm from the interaction point and with polar angle coverage between 2$^\circ$ and 10$^\circ$ (170$^\circ$ and 178$^\circ$).

Photon detection and reconstruction relies on the trigger and energy measurement based on two electromagnetic calorimeters: the high density projection chamber (HPC) in the barrel region and the forward electromagnetic calorimeter (FEMC) in the endcaps. The HPC is a gas-sampling calorimeter, made of 144 modules, each one with 10 lead layers in $R\phi$ embedded in a gas mixture. It covers polar angles between 42$^\circ$ and 138$^\circ$. The FEMC is a lead glass calorimeter, covering the polar angle region [11$^\circ$, 35$^\circ$] and its complement with respect to 180$^\circ$. The barrel DELPHI electromagnetic trigger requires coincidence between scintillator signals and energy deposits in HPC while in the forward region the electromagnetic trigger is given by energy deposits in the FEMC lead-glass counters.

The tracking system allows the rejection of charged particles and the recovery of photons converting inside the detector. The DELPHI barrel tracking system relies on the vertex detector (VD), the inner detector (ID), the time projection chamber (TPC) and the outer detector (OD). In the endcaps, the tracking system relies also on the VD and the TPC (down to about 20$^\circ$ in polar angle), and on the forward chambers A and B (FCA, FCB). The VD plays an important role in the detection of charged particle tracks coming from the interaction point. A more detailed description of the
DELPHI detector, of the triggering conditions and of the readout chain can be found in [10].

3. Photon reconstruction and identification

The process $e^+e^- \rightarrow \gamma\gamma(\gamma)$ yields not only neutral final states but also final states characterized by the presence of charged particle tracks from photon conversions.

Photons converting inside the tracking system, but after the vertex detector, are characterized by charged particle tracks and will be referred to as converted photons. Photons reaching the electromagnetic calorimeters before converting, yielding no reconstructed charged particle tracks, will be referred to as unconverted photons. According to this classification, two different algorithms were applied in the photon reconstruction and identification.

The main contamination to $e^+e^- \rightarrow \gamma\gamma(\gamma)$ final states comes from radiative Bhabha ($e^+e^- \rightarrow e^+e^-\gamma(\gamma)$) events with one non-reconstructed electron and the other electron lost in the beam pipe, and from Compton ($e^\pm\gamma$) events. Compton events are produced by the scattering of beam electrons by a quasi-real photon radiated by another incoming electron, resulting mostly in final states with one photon and one electron in the detector, the remaining $e^\pm$ going undetected through the beam-pipe. Both the Bhabha and the Compton backgrounds can however be dramatically reduced if the vertex detector is used as a veto for charged particles coming from the interaction point. The event generator used to simulate $e^+e^- \rightarrow \gamma\gamma(\gamma)$ was that of Berends and Kleiss [1], while the Bhabha and Compton event generators are BHWIDE and TEEG, described in Refs. [11] and [12], respectively. The generated samples were processed through the full DELPHI simulation and reconstruction chains [10].

3.1. Unconverted photons

Unconverted photon candidates were reconstructed by applying an isolation algorithm to energy deposits in the calorimeters. The algorithm relied on a double cone centered on each energy deposit, with internal and external half angles of 5° and 15°, respectively, where the vertices of both cones correspond to the geometric centre of DELPHI. Showers were considered isolated if the total energy inside the double cone was less than 1 GeV. The energy of the isolated neutral particles was re-evaluated as the sum of the energy of all associated deposits inside the inner cone where no charged particles of more than 250 MeV/c were allowed. The direction of the isolated showers was the energy weighted mean of the directions of all associated energy deposits. Such particles, with a total energy above 3 GeV, were identified as photons if the following criteria were fulfilled:

- The polar angle of the energy deposit was inside [25°, 35°], [42°, 88°], [92°, 138°] or [145°, 155°], in order to reduce VD and calorimeter edge effects.
- No VD track element pointed to the direction of the energy deposit within 3° (10°) in azimuthal angle in the barrel (forward) region of DELPHI (a VD track element was defined as at least two hits in different VD layers aligned within an azimuthal angle interval of 0.5°).
- If more than 3 GeV of hadronic energy was associated to a deposit, then at least 90% of it had to be in the first layer of the hadronic calorimeter (HCAL).
- For an energy deposit in the HPC, there had to be at least three HPC layers with more than 5% of the total electromagnetic energy, unless the deposit was within 1 degree of the HPC azimuthal intermodular divisions.1

3.2. Converted photons

Converted photon candidates were reconstructed with the help of a jet clustering algorithm: all particles in the event, with the exception of isolated neutral particles, were forced to be clustered in jets (isolated charged particles were not treated as single particles but as low multiplicity jets). The DURHAM jet algorithm [13] was applied, using as resolution variable $y_{\text{cut}} = 0.003$. Low multiplicity jets with less than 6 charged particles were treated as converted photon candidates. These candidates were recovered if they were associated to energy deposits above 3 GeV fulfilling the photon identification criteria described in Section 3.1. The requirement that no correlated signals

1 The HPC modules are distributed in 6 rings of 24 modules located at \( \text{mod}(\phi, 15°) = 7.5° \).
were observed in the VD was a particularly important criterion for the rejection of electrons.

4. Two photon events: $e^+e^- \rightarrow \gamma\gamma$ (a)

The selected $\gamma\gamma$ sample consisted of events with at least two photons, where at most one was converted. The electromagnetic calorimeters (HPC and FEMC), the TPC and the VD were required to be nominally operational. The analysis was performed in the polar angle interval corresponding to $|\cos\theta^*| \in [0.035, 0.731] \cup [0.819, 0.906]$, where the variable $\theta^*$ stands for the polar angle of the photons relative to the direction of the incident electron in the centre-of-mass of the $e^+e^-$ collision after allowing for ISR. The two most energetic photons were required to have energies above 15% of the collision energy and isolation angle of 30° (the isolation angle is the minimum of the angles between the photon and the remaining reconstructed particles in the event). No other particles (with exception of isolated photons) with energy above 3 GeV were allowed in the event. The application of these criteria resulted in an almost pure $\gamma\gamma$ sample, where the contamination from Bhabha and Compton events is about 0.3% and 3%, respectively.

The radiation of a third hard photon constrains the two harder photons to be produced at effective $\sqrt{s}$ values which have been tested more accurately using lower energy data. Since the aim of this analysis is to test the QED $e^+e^- \rightarrow \gamma\gamma$ reaction at the highest available energies, such final states were not allowed in the selected sample: events with a third hard bremsstrahlung photon can be considered as a higher order contribution to $e^+e^- \rightarrow \gamma\gamma$ (like the soft bremsstrahlung and the virtual contributions), which can be deconvoluted from data by applying a radiative correction factor when evaluating the $e^+e^- \rightarrow \gamma\gamma$ Born cross-section. Moreover, the $e^+e^- \rightarrow \gamma\gamma\gamma$ contribution can be dramatically reduced if the spatial angle between the two most energetic photons is required to be large. Therefore, a final selection criterion, consisting in requiring that the acollinearity between the two most energetic photons was below 30°, was applied, eliminating most events with a third visible hard photon, and reducing the Compton background to 0.3%. The acollinearity distribution prior to the cut is shown in Fig. 1 (a) for the full data sample, and compared to the $e^+e^- \rightarrow \gamma\gamma$ simulation and to the remaining background expectations. After imposing all selection criteria, the contamination from Bhabha and Compton events to the selected $\gamma\gamma$ sample was estimated to be 0.6%, and taken into account in the systematic uncertainty.

4.1. $\gamma\gamma$ trigger and selection efficiencies

The trigger efficiency for neutral two-photon final states was computed with Bhabha events using the redundancy of the electromagnetic trigger with the track trigger. It was calculated for each centre-of-mass energy as a function of $|\cos\theta^*|$. The global values obtained for the barrel and endcaps are displayed in Table 1.

Final states with one converted photon are triggered by the single track coincidence trigger, whose effi-

---

2 The parameterization of the photon polar angle with $\theta^*$ enables the cross-section measurement to be insensitive to photons lost in the beam pipe.

3 The acollinearity between two directions is the complement to $\pi$ of the spatial angle between them.
efficiency is known to be near 100%. Two dedicated samples of Compton \((e^+\gamma)\) events, one with a triggered FEMC photon and another with a triggered HPC photon, were used to cross-check the track trigger efficiency in the barrel and endcaps. The global efficiency for triggering events with one converted photon was confirmed to be above 99% in both regions of the detector, for all data sets, and the resulting uncertainty was taken into account in the global systematic uncertainty, for the two data taking periods.

The selection efficiency for the two-photon event sample was evaluated as a function of \(|\cos \theta^*|\) using events from the \(e^+e^- \to \gamma\gamma(y)\) generator of Berends and Kleiss [1] passed through the full DELPHI simulation and reconstruction chains [10]. The effect of the calorimeter requirements on the selection efficiency obtained from simulation was cross-checked using a sample of \(e^+e^-\) events. These events were selected using information coming exclusively from the tracking detectors. The efficiency was defined as the ratio between the number of events in the subsample of \(e^+e^-\) final states fulfilling the calorimetric selection and the total number of selected \(e^+e^-\) events. This efficiency was computed as a function of \(|\cos \theta^*|\) for both real and simulated Bhabha events. The difference observed between the efficiency for the data and for the simulation was taken as a systematic uncertainty in the \(e^+e^- \to \gamma\gamma(y)\) selection efficiency determination.

The global values for the selection efficiency, both in the barrel and in the forward region of DELPHI, are displayed in Table 2 along with their statistical and systematic uncertainties. A change in the forward acceptance was divided into 8 bins: the barrel part of the detector, corresponding to \(|\cos \theta^*|\in [0.035, 0.731]\) with 7 bins, (each covering \(\Delta \cos \theta^* = 0.101\), except for the last bin, for which \(\Delta \cos \theta^* = 0.09\)) and the forward region with one bin, \(|\cos \theta^*|\in [0.819, 0.906]\). The number of events found in data for each centre-of-mass energy and the expected contribution from the QED process \(e^+e^- \to \gamma\gamma(y)\) (corrected for trigger efficiency) are displayed in Table 3 as a function of \(|\cos \theta^*|\).

| \(\sqrt{s}\) [GeV] | \(|\cos \theta^*| \in [0.035, 0.731]\) | \(|\cos \theta^*| \in [0.819, 0.906]\) |
|---------------------|---------------------------------|---------------------------------|
| 188.6               | 0.985 ± 0.002                   | 1.000 ± 0.003                   |
| 191.6               | 0.977 ± 0.007                   | 1.000 ± 0.002                   |
| 195.5               | 0.977 ± 0.004                   | 0.9995 ± 0.0005                 |
| 199.5               | 0.968 ± 0.005                   | 0.9995 ± 0.0005                 |
| 201.6               | 0.983 ± 0.005                   | 1.000 ± 0.001                   |

DELPHI particle reconstruction algorithms resulted in a better performance for \(\gamma\gamma\) final states for the 1999 data processing compared with that of 1998. However, there was an increase of the systematic uncertainty in the \(\gamma\gamma\) selection efficiency.

### 4.2. \(e^+e^- \to \gamma\gamma\) cross-section

The retained \(|\cos \theta^*|\) acceptance was divided into 8 bins: the barrel part of the detector, corresponding to \(|\cos \theta^*|\in [0.035, 0.731]\) with 7 bins, (each covering \(\Delta \cos \theta^* = 0.101\), except for the last bin, for which \(\Delta \cos \theta^* = 0.09\)) and the forward region with one bin, \(|\cos \theta^*|\in [0.819, 0.906]\). The number of events found in data for each centre-of-mass energy and the expected contribution from the QED process \(e^+e^- \to \gamma\gamma(y)\) (corrected for trigger efficiency) are displayed in Table 3 as a function of \(|\cos \theta^*|\).

The Born cross-section for the reaction \(e^+e^- \to \gamma\gamma(y)\) was evaluated through expression (2) for each centre-of-mass energy value,

\[
\sigma_{dat}^0 = \frac{N_{\gamma\gamma}}{\mathcal{L} \epsilon R} [\text{pb}].
\]

\(N_{\gamma\gamma}\) is the number of selected events after background subtraction, \(\mathcal{L}\) is the integrated luminosity, \(\epsilon\) is the product of the selection and trigger efficiencies and \(R\) is a radiative correction factor. The radiative correction factor was evaluated using the Monte Carlo generator of [1]. It was taken as the ratio between the \(e^+e^- \to \gamma\gamma(y)\) cross-section computed up to order \(\alpha^3\) to the Born cross-section \((\mathcal{O}(\alpha^2))\) and found to be of the order of 1.07 (1.04) for high (low) photon scattering angles.

A combined value of the Born cross-section at an average centre-of-mass energy of 193.8 GeV, corresponding to a total integrated luminosity of
samples by the corresponding weighting the integrated luminosities of the different average value of the centre-of-mass energy is obtained

375.7 pb\(^{-1}\), was obtained through expression (2). The average value of the centre-of-mass energy is obtained weighting the integrated luminosities of the different samples by the corresponding \(s^{-1}\) factor.

\(N^{\gamma\gamma}\) is taken as the total number of selected events in the five data samples. The average trigger and selection efficiencies were obtained by weighting the global trigger and selection efficiencies of each data

<table>
<thead>
<tr>
<th>(\sqrt{s}), GeV</th>
<th>([\cos\theta^*])</th>
<th>(N^{\gamma\gamma}_{\text{dat}})</th>
<th>((N_{\text{QED}}\pm\Delta N_{\text{dat}}))</th>
<th>(N^{\gamma\gamma}_{\text{dat}})</th>
<th>((N_{\text{QED}}))</th>
<th>(d\sigma^{0}_{\text{dat}}/d\Omega) [pb/str]</th>
</tr>
</thead>
<tbody>
<tr>
<td>188.6</td>
<td>0.035–0.136</td>
<td>46</td>
<td>(41.5 ± 1.4)</td>
<td>5</td>
<td>(6.2)</td>
<td>0.65 ± 0.10 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>0.136–0.237</td>
<td>48</td>
<td>(47.9 ± 1.5)</td>
<td>3</td>
<td>(3.8)</td>
<td>0.62 ± 0.09 ± 0.01</td>
</tr>
<tr>
<td></td>
<td>0.237–0.338</td>
<td>64</td>
<td>(52.6 ± 1.6)</td>
<td>5</td>
<td>(6.4)</td>
<td>0.84 ± 0.11 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>0.338–0.439</td>
<td>57</td>
<td>(54.8 ± 1.5)</td>
<td>5</td>
<td>(6.2)</td>
<td>0.81 ± 0.11 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>0.439–0.540</td>
<td>77</td>
<td>(71.1 ± 1.8)</td>
<td>11</td>
<td>(8.5)</td>
<td>0.97 ± 0.11 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>0.540–0.641</td>
<td>76</td>
<td>(90.0 ± 2.0)</td>
<td>19</td>
<td>(10.8)</td>
<td>1.01 ± 0.12 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>0.641–0.731</td>
<td>108</td>
<td>(111.7 ± 2.3)</td>
<td>11</td>
<td>(15.8)</td>
<td>1.59 ± 0.15 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>0.819–0.906</td>
<td>176</td>
<td>(170.3 ± 2.8)</td>
<td>47</td>
<td>(53.4)</td>
<td>4.27 ± 0.32 ± 0.06</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>652</td>
<td>(639.7 ± 5.4)</td>
<td>106</td>
<td>(111.1)</td>
<td></td>
</tr>
<tr>
<td>191.6</td>
<td>0.035–0.136</td>
<td>6</td>
<td>(6.4 ± 0.3)</td>
<td>2</td>
<td>(0.9)</td>
<td>0.53 ± 0.22 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>0.136–0.237</td>
<td>6</td>
<td>(7.2 ± 0.3)</td>
<td>0</td>
<td>(0.7)</td>
<td>0.52 ± 0.21 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>0.237–0.338</td>
<td>8</td>
<td>(8.5 ± 0.3)</td>
<td>1</td>
<td>(0.9)</td>
<td>0.62 ± 0.22 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>0.338–0.439</td>
<td>6</td>
<td>(9.9 ± 0.3)</td>
<td>1</td>
<td>(1.1)</td>
<td>0.48 ± 0.20 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>0.439–0.540</td>
<td>10</td>
<td>(12.4 ± 0.4)</td>
<td>1</td>
<td>(1.5)</td>
<td>0.79 ± 0.25 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>0.540–0.641</td>
<td>14</td>
<td>(14.7 ± 0.4)</td>
<td>5</td>
<td>(1.8)</td>
<td>1.09 ± 0.29 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>0.641–0.731</td>
<td>13</td>
<td>(17.8 ± 0.4)</td>
<td>1</td>
<td>(2.7)</td>
<td>1.17 ± 0.32 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>0.819–0.906</td>
<td>27</td>
<td>(31.3 ± 0.6)</td>
<td>8</td>
<td>(7.7)</td>
<td>3.42 ± 0.66 ± 0.09</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>90</td>
<td>(108.2 ± 1.1)</td>
<td>19</td>
<td>(17.3)</td>
<td></td>
</tr>
<tr>
<td>195.5</td>
<td>0.035–0.136</td>
<td>21</td>
<td>(19.3 ± 0.8)</td>
<td>4</td>
<td>(2.5)</td>
<td>0.61 ± 0.13 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>0.136–0.237</td>
<td>29</td>
<td>(21.5 ± 0.8)</td>
<td>5</td>
<td>(2.1)</td>
<td>0.80 ± 0.15 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>0.237–0.338</td>
<td>9</td>
<td>(24.6 ± 0.9)</td>
<td>0</td>
<td>(2.5)</td>
<td>0.23 ± 0.08 ± 0.01</td>
</tr>
<tr>
<td></td>
<td>0.338–0.439</td>
<td>23</td>
<td>(27.4 ± 0.9)</td>
<td>4</td>
<td>(3.2)</td>
<td>0.64 ± 0.13 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>0.439–0.540</td>
<td>48</td>
<td>(36.6 ± 1.1)</td>
<td>2</td>
<td>(4.4)</td>
<td>1.23 ± 0.18 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>0.540–0.641</td>
<td>47</td>
<td>(43.2 ± 1.2)</td>
<td>6</td>
<td>(5.3)</td>
<td>1.21 ± 0.18 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>0.641–0.731</td>
<td>58</td>
<td>(51.7 ± 1.7)</td>
<td>12</td>
<td>(7.9)</td>
<td>1.72 ± 0.23 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>0.819–0.906</td>
<td>102</td>
<td>(91.0 ± 1.7)</td>
<td>28</td>
<td>(22.6)</td>
<td>4.29 ± 0.42 ± 0.08</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>337</td>
<td>(315.3 ± 3.1)</td>
<td>61</td>
<td>(50.5)</td>
<td></td>
</tr>
</tbody>
</table>

(continued on next page)
set by the corresponding integrated luminosities. The measured Born cross-section for each of the five centre-of-mass energies and the combined result are compared to the QED predictions in Table 4 and in the upper right corner of Fig. 2. The \( \chi^2 \) of the measured values for the cross-section for the different centre-of-mass energies with respect to the QED prediction was 5.5 with 5 degrees of freedom.

The Born cross-section values for the five centre-of-mass energies measured in the region \( 0.035 < |\cos \theta^*| < 0.731 \), were corrected to the full barrel acceptance of DELPHI, \( 0.000 < |\cos \theta^*| < 0.742 \), and the obtained values are presented in Table 4. These are also displayed in Fig. 2 as a function of the centre-of-mass energy, along with the previously published results, which include LEP I data collected between 1990 and 1992 [4] and former LEP II data collected between 1995 and 1997 [5].

The total systematic errors were obtained by adding in quadrature the uncertainties on the selection efficiency, trigger efficiencies, residual background, luminosity determination and on the radiative corrections (amounting to \( \pm 0.5\% \)). The systematic uncertainty in the selection efficiency determination is the dominant contribution to the systematic error; with a typical value of \( \pm 2.5\% \). This uncertainty reflects residual differences between the real detector response and the simulated one. It is due to effects that cannot be fully described by the detector simulation such as detector instabilities and edge effects of calorimeters. The uncertainty in the luminosity determination was \( \pm 0.56\% \). It was obtained by adding in quadrature the \( \pm 0.5\% \) systematic uncertainty in the luminosity measurement and the \( \pm 0.25\% \) theoretical error in the Bhabha cross-section determination [14].

The \( e^+e^- \rightarrow \gamma\gamma \) differential Born cross-section was computed as:

| \( \sqrt{s}, \text{GeV} \) | \( |\cos \theta^*| \) | \( N_{\text{dat}}^{\gamma\gamma\gamma\gamma} \) | \( N_{\text{QED}} \pm \Delta N_{\text{stat}} \) | \( N_{\text{dat}}^{\gamma\gamma\gamma\gamma} \) | \( N_{\text{QED}} \) | \( d\sigma/d\Omega \) [pb/str] |
|-----------------|-----------------|-----------------|---------------------|-----------------|-----------------|-------------------|
| 199.5 | 0.035-0.136 | 19 | (21.2 ± 0.8) | 3 | (2.5) | 0.51 ± 0.12 ± 0.03 |
| 0.136-0.237 | 17 | (23.0 ± 0.9) | 0 | (2.5) | 0.42 ± 0.10 ± 0.03 |
| 0.237-0.338 | 28 | (23.7 ± 0.9) | 2 | (2.4) | 0.70 ± 0.13 ± 0.04 |
| 0.338-0.439 | 34 | (25.0 ± 0.9) | 3 | (3.7) | 0.93 ± 0.16 ± 0.05 |
| 0.439-0.540 | 39 | (35.2 ± 1.1) | 1 | (4.5) | 0.91 ± 0.15 ± 0.03 |
| 0.540-0.641 | 45 | (45.4 ± 1.2) | 4 | (6.3) | 1.11 ± 0.16 ± 0.05 |
| 0.641-0.731 | 40 | (54.9 ± 1.4) | 9 | (6.8) | 1.07 ± 0.17 ± 0.03 |
| 0.819-0.906 | 88 | (96.0 ± 1.8) | 29 | (24.4) | 3.37 ± 0.36 ± 0.09 |
| Total | 310 | (324.3 ± 3.3) | 51 | (53.1) | |
| 201.6 | 0.035-0.136 | 14 | (10.7 ± 0.4) | 2 | (1.2) | 0.72 ± 0.19 ± 0.06 |
| 0.136-0.237 | 8 | (10.9 ± 0.4) | 0 | (1.2) | 0.40 ± 0.14 ± 0.04 |
| 0.237-0.338 | 17 | (11.7 ± 0.4) | 4 | (1.2) | 0.84 ± 0.20 ± 0.08 |
| 0.338-0.439 | 12 | (13.3 ± 0.5) | 1 | (1.8) | 0.60 ± 0.17 ± 0.03 |
| 0.439-0.540 | 13 | (16.6 ± 0.5) | 0 | (2.1) | 0.63 ± 0.17 ± 0.03 |
| 0.540-0.641 | 21 | (21.7 ± 0.6) | 1 | (3.0) | 1.06 ± 0.23 ± 0.04 |
| 0.641-0.731 | 19 | (26.1 ± 0.6) | 4 | (3.2) | 1.05 ± 0.24 ± 0.03 |
| 0.819-0.906 | 43 | (45.6 ± 0.6) | 15 | (11.6) | 3.39 ± 0.52 ± 0.10 |
| Total | 147 | (156.6 ± 1.6) | 27 | (25.3) | |
Table 4
Measured Born cross-sections for $e^+e^- \rightarrow \gamma \gamma$ (with statistical and systematic uncertainties) at the different centre-of-mass energies, for the analysis $\cos \theta^*$ acceptance and for the barrel region ($42^< \theta^* < 138^>$), compared to the corresponding QED predictions. In the last line the combined results are displayed along with the QED cross-sections at a centre-of-mass energy of 193.8 GeV.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>Analysis acceptance</th>
<th>$\sigma_{\text{QED}}^0$ [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>188.6</td>
<td>$6.34 \pm 0.25 \pm 0.16$</td>
<td>6.27</td>
</tr>
<tr>
<td>191.6</td>
<td>$5.09 \pm 0.54 \pm 0.13$</td>
<td>6.08</td>
</tr>
<tr>
<td>195.5</td>
<td>$6.31 \pm 0.34 \pm 0.13$</td>
<td>5.83</td>
</tr>
<tr>
<td>199.5</td>
<td>$5.34 \pm 0.30 \pm 0.17$</td>
<td>5.60</td>
</tr>
<tr>
<td>201.6</td>
<td>$5.14 \pm 0.42 \pm 0.16$</td>
<td>5.49</td>
</tr>
<tr>
<td>193.8</td>
<td>$5.89 \pm 0.15 \pm 0.16$</td>
<td>5.94</td>
</tr>
</tbody>
</table>

The differential cross-section was computed for each centre-of-mass energy, taking into account the $|\cos \theta^*|$ dependence of trigger and selection efficiencies, radiative corrections and their respective uncertainties. Comparisons between the measured and predicted Born differential cross-sections for each centre-of-mass energy are shown in Fig. 3. The deficit of $\gamma \gamma$ events for $|\cos \theta^*|$ between 0.237 and 0.338 for $\sqrt{s} = 195.5$ GeV was concluded to be a statistical fluctuation: the trigger efficiency for this region was estimated to be about 98% and the counting of energy deposits associated to Bhabha electrons in the same $|\cos \theta^*|$ region showed a good agreement with the simulation expectations.

The differential cross-section extracted from the combined data sets (corresponding to $\sqrt{s_{\text{eff}}} = 193.8$ GeV), is compared to the QED prediction in Table 5 and in Fig. 4. The $\chi^2$ of the differential cross-section binned distribution at the mean centre-of-mass energy with respect to the QED prediction was 3.6 with 8 degrees of freedom.

4.3. Deviations from QED

Possible deviations from QED are described in the context of several models, which express the Born differential cross-section for $e^+e^- \rightarrow \gamma \gamma$ as the sum of the QED term and of a deviation term:

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_i^0}{2\pi \Delta \cos \theta^*} \text{[pb/str]},$$

where $\sigma_i^0$ stands for the measured Born cross-section in each $|\cos \theta^*|$ interval, $(i)$.

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s} \left(1 + \cos^2 \theta^*\right) + \left(\frac{d\sigma}{d\Omega}\right)_D.$$
Table 5
Measured and predicted Born differential cross-section (the measured cross-section uncertainties are statistical and systematic) for the QED process $e^+e^-\rightarrow \gamma\gamma$ at a mean centre-of-mass energy of 193.8 GeV obtained by combining the data sets corresponding to centre-of-mass energies of 189.6 GeV, 191.6 GeV, 195.5 GeV, 199.5 GeV, and 201.6 GeV.

<table>
<thead>
<tr>
<th>$\cos^2 \theta^*$</th>
<th>$\frac{d\sigma}{d\Omega}$ [pb/str]</th>
<th>$\frac{d\sigma^{\text{QED}}}{d\Omega}$ [pb/str]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035–0.136</td>
<td>0.61 ± 0.06 ± 0.04</td>
<td>0.56</td>
</tr>
<tr>
<td>0.136–0.237</td>
<td>0.58 ± 0.06 ± 0.03</td>
<td>0.59</td>
</tr>
<tr>
<td>0.237–0.338</td>
<td>0.67 ± 0.06 ± 0.03</td>
<td>0.65</td>
</tr>
<tr>
<td>0.338–0.439</td>
<td>0.75 ± 0.07 ± 0.03</td>
<td>0.75</td>
</tr>
<tr>
<td>0.439–0.540</td>
<td>0.96 ± 0.07 ± 0.03</td>
<td>0.90</td>
</tr>
<tr>
<td>0.540–0.641</td>
<td>1.08 ± 0.08 ± 0.04</td>
<td>1.14</td>
</tr>
<tr>
<td>0.641–0.731</td>
<td>1.41 ± 0.09 ± 0.03</td>
<td>1.53</td>
</tr>
<tr>
<td>0.819–0.906</td>
<td>3.90 ± 0.19 ± 0.08</td>
<td>3.76</td>
</tr>
</tbody>
</table>

Fig. 3. Differential Born cross-section distributions obtained for the five centre-of-mass energies compared to the corresponding QED theoretical predictions.

Among the models predicting deviations from QED are those described in Table 6. The most general parameterization consists of introducing a cut-off parameter in the electron propagators ($\Lambda$), reflecting the energy scale up to which the $e\gamma$ interaction can be described as point-like [15,16].

Deviations from QED could also follow from the t-channel exchange of an excited electron, which, in composite models [17], is parameterized as a function of $\lambda/\Lambda^2$ (the ratio between the coupling of the excited electron to the photon and to the electron and the excited electron mass) and of a kinematic factor, $H(\cos^2 \theta^*)$,

$$H(\cos^2 \theta^*) = \frac{2M_{e'}^2}{s} \left( \frac{2M_{e'}^2}{s} + \frac{1 - \cos^2 \theta^*}{1 + \cos^2 \theta^*} \right) \int \left[ \left( 1 + \frac{2M_{e'}^2}{s} \right)^2 - \cos^2 \theta^* \right].$$  \( (5) \)

Deviations from the QED $e^+e^-\rightarrow \gamma\gamma$ cross-section due to s-channel exchange of virtual gravitons were also probed. These can be parameterized as a function of $\lambda/M_s^2$, where $M_s$ is the string mass scale, which in some string models could be of the order of the electroweak scale [18,19]. $\lambda$ is a parameter enter-
The reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ was studied using the LEP 1998 and 1999 high energy data, collected with the DELPHI detector at centre-of-mass energies of 188.6 GeV, 191.6 GeV, 195.5 GeV, 199.5 GeV and 201.6 GeV, corresponding to integrated luminosities of 151.9 pb$^{-1}$, 25.1 pb$^{-1}$, 76.1 pb$^{-1}$, 82.6 pb$^{-1}$ and 40.1 pb$^{-1}$, respectively. The differential and total cross-sections for the process $e^+e^- \rightarrow \gamma\gamma$ were measured. Good agreement between the data and the QED prediction for this process was found. Lower limits on possible deviations from QED were derived by combining the present analysis result with a previously published one [5]. The 95% C.L. lower limits on the QED cut-off parameters of $\Lambda_+$ and $\Lambda_-$ were obtained. In the framework of composite models, a 95% C.L. lower limit for the mass of an excited electron, $M_{e^*} > 311$ GeV/$c^2$, was obtained considering an effective coupling value of 1 for $\lambda_{\gamma\gamma}$. The possible contribution of virtual gravitons to the process $e^+e^- \rightarrow \gamma\gamma$ was probed, resulting in 95% C.L. lower limits in the string mass scale of $M_S > 713$ GeV/$c^2$ and $M_S > 691$ GeV/$c^2$ for $\lambda = 1$ and $\lambda = -1$, respectively (where $\lambda$ is a $O(1)$ parameter of quantum gravity models).

5. Summary

The reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ was studied using the LEP 1998 and 1999 high energy data, collected with the DELPHI detector at centre-of-mass energies of 188.6 GeV, 191.6 GeV, 195.5 GeV, 199.5 GeV and 201.6 GeV, corresponding to integrated luminosities of 151.9 pb$^{-1}$, 25.1 pb$^{-1}$, 76.1 pb$^{-1}$, 82.6 pb$^{-1}$ and 40.1 pb$^{-1}$, respectively. The differential and total cross-sections for the process $e^+e^- \rightarrow \gamma\gamma$ were measured. Good agreement between the data and the QED prediction for this process was found. Lower limits on possible deviations from QED were derived by combining the present analysis result with a previously published one [5]. The 95% C.L. lower limits on the QED cut-off parameters of $\Lambda_+$ and $\Lambda_-$ were obtained. In the framework of composite models, a 95% C.L. lower limit for the mass of an excited electron, $M_{e^*} > 311$ GeV/$c^2$, was obtained considering an effective coupling value of 1 for $\lambda_{\gamma\gamma}$. The possible contribution of virtual gravitons to the process $e^+e^- \rightarrow \gamma\gamma$ was probed, resulting in 95% C.L. lower limits in the string mass scale of $M_S > 713$ GeV/$c^2$ and $M_S > 691$ GeV/$c^2$ for $\lambda = 1$ and $\lambda = -1$, respectively (where $\lambda$ is a $O(1)$ parameter of quantum gravity models).

The reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ was studied using the LEP 1998 and 1999 high energy data, collected with the DELPHI detector at centre-of-mass energies of 188.6 GeV, 191.6 GeV, 195.5 GeV, 199.5 GeV and 201.6 GeV, corresponding to integrated luminosities of 151.9 pb$^{-1}$, 25.1 pb$^{-1}$, 76.1 pb$^{-1}$, 82.6 pb$^{-1}$ and 40.1 pb$^{-1}$, respectively. The differential and total cross-sections for the process $e^+e^- \rightarrow \gamma\gamma$ were measured. Good agreement between the data and the QED prediction for this process was found. Lower limits on possible deviations from QED were derived by combining the present analysis result with a previously published one [5]. The 95% C.L. lower limits on the QED cut-off parameters of $\Lambda_+$ and $\Lambda_-$ were obtained. In the framework of composite models, a 95% C.L. lower limit for the mass of an excited electron, $M_{e^*} > 311$ GeV/$c^2$, was obtained considering an effective coupling value of 1 for $\lambda_{\gamma\gamma}$. The possible contribution of virtual gravitons to the process $e^+e^- \rightarrow \gamma\gamma$ was probed, resulting in 95% C.L. lower limits in the string mass scale of $M_S > 713$ GeV/$c^2$ and $M_S > 691$ GeV/$c^2$ for $\lambda = 1$ and $\lambda = -1$, respectively (where $\lambda$ is a $O(1)$ parameter of quantum gravity models).

The reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ was studied using the LEP 1998 and 1999 high energy data, collected with the DELPHI detector at centre-of-mass energies of 188.6 GeV, 191.6 GeV, 195.5 GeV, 199.5 GeV and 201.6 GeV, corresponding to integrated luminosities of 151.9 pb$^{-1}$, 25.1 pb$^{-1}$, 76.1 pb$^{-1}$, 82.6 pb$^{-1}$ and 40.1 pb$^{-1}$, respectively. The differential and total cross-sections for the process $e^+e^- \rightarrow \gamma\gamma$ were measured. Good agreement between the data and the QED prediction for this process was found. Lower limits on possible deviations from QED were derived by combining the present analysis result with a previously published one [5]. The 95% C.L. lower limits on the QED cut-off parameters of $\Lambda_+$ and $\Lambda_-$ were obtained. In the framework of composite models, a 95% C.L. lower limit for the mass of an excited electron, $M_{e^*} > 311$ GeV/$c^2$, was obtained considering an effective coupling value of 1 for $\lambda_{\gamma\gamma}$. The possible contribution of virtual gravitons to the process $e^+e^- \rightarrow \gamma\gamma$ was probed, resulting in 95% C.L. lower limits in the string mass scale of $M_S > 713$ GeV/$c^2$ and $M_S > 691$ GeV/$c^2$ for $\lambda = 1$ and $\lambda = -1$, respectively (where $\lambda$ is a $O(1)$ parameter of quantum gravity models).
Acknowledgements

We are greatly indebted to our technical collaborators, to the members of the CERN-SL Division for the excellent performance of the LEP collider, and to the funding agencies for their support in building and operating the DELPHI detector. We acknowledge in particular the support of Austrian Federal Ministry of Science and Traffics, GZ 616.364/2-III/2a/98; FNRS–FWO, Belgium; FINEP, CNPq, CAPES, FUJB and FAPERJ, Brazil; Czech Ministry of Industry and Trade, GA CR 202/96/0450 and GA AVCR A1010521; Danish Natural Research Council; Commission of the European Communities (DG XII); Direction des Sciences de la Matière, CEA, France; Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Germany; General Secretariat for Research and Technology, Greece; National Science Foundation (NWO) and Foundation for Research on Matter (FOM), The Netherlands; Norwegian Research Council; State Committee for Scientific Research, Poland, 2P03B06015, 2P03B03311 and SPUB/P03/178/98; JNICT — Junta Nacional de Investigação Científica e Tecnológica, Portugal; Vedecka grantova agentura MS SR, Slovakia, Nr. 95/5195/134; Ministry of Science and Technology of the Republic of Slovenia; CICYT, Spain, AEN96-1661 and AEN96-1681; The Swedish Natural Science Research Council; Particle Physics and Astronomy Research Council, UK; Department of Energy, USA, DE–FG02–94ER40817.

References

Observation of the $\phi \to \pi^+\pi^-\pi^+\pi^-$ decay

R.R. Akhmetshin$^a$, E.V. Anashkin$^a$, M. Arpagauss$^a$, V.M. Aulchenko$^{a,b}$, V.Sh. Banzarov$^a$, L.M. Barkov$^{a,b}$, N.S. Bashtovoy$^a$, A.E. Bondar$^{a,b}$, D.V. Bondarev$^a$, A.V. Bragin$^a$, D.V. Chernyak$^a$, S.I. Eidelman$^{a,b}$, G.V. Fedotovitch$^{a,b}$, N.I. Gabyshev$^a$, A.A. Grebeniuk$^a$, D.N. Grigoriev$^a$, V.W. Hughes$^c$, F.V. Ignatov$^{a,b}$, P.M. Ivanov$^a$, S.V. Karpov$^a$, V.F. Kazanin$^{a,b}$, I.A. Koop$^a$, M.S. Korostylev$^a$, P.P. Krokovny$^{a,b}$, L.M. Kurtxdy$^{a,b}$, A.S. Kuzmin$^{a,b}$, I.B. Logashenko$^a$, P.A. Lukin$^a$, K.Yu. Mikhailov$^{a,b}$, A.I. Milstein$^{a,b}$, I.N. Nesterenko$^a$, V.S. Okhapkin$^a$, A.V. Otboev$^a$, E.A. Perevedentsev$^{a,b}$, A.S. Popov$^{a,b}$, T.A. Purlatz$^{a,b}$, S.I. Redin$^a$, N.I. Root$^{a,b}$, A.A. Ruban$^a$, N.M. Ryskulov$^a$, A.G. Shamov$^a$, Yu.M. Shatunov$^a$, B.A. Shwartz$^{a,b}$, A.L. Sibidanov$^{a,b}$, V.A. Sidorov$^a$, A.N. Skrinsky$^a$, V.P. Smakhtin$^a$, I.G. Snopkov$^a$, E.P. Solodov$^{a,b}$, P.Yu. Stepanov$^a$, A.I. Sukhanov$^{a,*}$, J.A. Thompson$^d$, V.M. Titov$^a$, A.A. Valishev$^a$, Yu.V. Yudin$^a$, S.G. Zverev$^a$

$^a$ Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia
$^b$ Novosibirsk State University, Novosibirsk, 630090, Russia
$^c$ Yale University, New Haven, CT 06511, USA
$^d$ University of Pittsburgh, Pittsburgh, PA 15260, USA

Received 15 August 2000; accepted 22 August 2000

Editor: L. Montanet

Abstract

Using 11.6 pb$^{-1}$ of data collected in the energy range 0.984–1.06 GeV by CMD-2 at VEPP-2M, the cross section of the reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ has been studied. For the first time an interference pattern was observed in the energy dependence of the cross section near the $\phi$ meson. The branching ratio of the $\phi \to \pi^+\pi^-\pi^+\pi^-$ decay double suppressed by the G-parity and OZI rule is measured $Br(\phi \to \pi^+\pi^-\pi^+\pi^-) = (3.93 \pm 1.74 \pm 2.14) \times 10^{-6}$. The upper limits have been placed for the decays $\phi \to \pi^+\pi^-\pi^+\pi^-0$ and $\phi \to \eta\pi^+\pi^- Br(\phi \to \eta\pi^+\pi^-) < 4.6 \times 10^{-6}$ 90% CL, $Br(\phi \to \eta\pi^+\pi^-) < 1.8 \times 10^{-5}$ 90% CL. © 2000 Published by Elsevier Science B.V.

1. Introduction

Production of four pions in $e^+e^-$ annihilation is now well studied in the c.m. energy range 1.05 to 2.5 GeV (see [1] and references therein). Results on the measurements of the cross section of the reaction
$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ in the energy range from 0.60 to 0.97 GeV as well as the probability of the $\rho^0$ meson decay into the $\pi^+\pi^-\pi^+\pi^-$ final state were recently presented by the CMD-2 group [2]. However, the behavior of the cross section of the process $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ in the vicinity of the $\phi$ meson has not been as well studied. In the previous experiments performed in Orsay [3,4] and in Novosibirsk [5–7] the cross section was measured at single points at $E_{cm} \approx m_{\phi}$. Because of the small data samples in these experiments, no detailed studies of the cross section structure in the $\phi$ meson region could be made. Under the assumption that the visible cross section is due to the $\phi$ decay, an upper limit was set on the value of the decay probability $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-$ [4]. The intensity of this decay is of interest since it is twice suppressed, by G-parity and the OZI rule. Two other rare $\phi$ decays which violate the OZI rule are $\phi \rightarrow \eta\pi^+\pi^-$ (also forbidden by G-parity) and $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$. A search for the decay $\phi \rightarrow \eta\pi^+\pi^-$ based on part of the total data sample was performed by CMD-2 [8] using the $\eta \rightarrow \gamma\gamma$ decay mode. No events of this decay were observed and an upper limit was placed. Earlier the CMD group set an upper limit for the branching ratio of the decay $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-$ [7].

In 1992 the upgraded high luminosity collider VEPP-2M resumed its operation at the Budker Institute of Nuclear Physics in Novosibirsk [9]. Two modern detectors CMD-2 [10] and SND [11] started a series of experiments which include various high precision measurements in the c.m. energy range from the threshold of hadron production to 1.4 GeV. High data samples collected by both detectors in the $\phi$ meson energy range allowed the first observation of various rare decay modes among which are G-parity and OZI rule suppressed decays to $\pi^+\pi^-\pi^+\pi^-\pi^0$ [12,13] and $\omega\pi^0$ [14].

In this paper we extend the analysis of the process $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ started by CMD-2 in Refs. [1,2] to the $\phi$ meson c.m. energy range from 0.984 to 1.06 GeV. The high integrated luminosity allowed the observation of the clear interference pattern at $E_{cm} \approx m_{\phi}$ indicating the presence of the decay $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^-$. The same data sample was used to search for the decays $\phi \rightarrow \eta\pi^+\pi^-$ and $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-$. The collider energy was roughly set (\(\delta E/E \lesssim 10^{-3}\)) by the dipole magnet currents. In the energy range 1.010 to 1.028 GeV of the main scans the beam energy was more precisely (\(\delta E/E \lesssim 10^{-4}\)) determined by measuring the average momentum $p_{av}$ of $K^+K^-$ pairs in the DC: $E_{K^+K^-}^{K^+K^-} = \sqrt{p_{av}^2 + M_K^2} + \Delta$. Here $\Delta$ is a correction for the contributions of kaon ionization losses inside the detector and radiative losses of initial electrons. Its magnitude depends on $p_{av}$ and varies from 5 to 3 MeV for $p_{av}$ in the range 80 to 130 MeV/c (see [19] for more detail). Then the effective beam energy at each energy point was determined by averaging $E_{K^+K^-}^{K^+K^-}$ weighted by the integrated luminosity:

2. Experiment and data analysis

Three scans of the energy range from 0.984 to 1.06 GeV were performed in winter 1997–1998. The scan step was 1 MeV near the $\phi$ meson (1.016–1.023 GeV) and 6–10 MeV outside the resonance. Some luminosity has also been collected at 1.019 and 1.020 GeV before the main scans and at 1.017 and 1.020 GeV after them. For the final analysis data samples from the same energy points of different scans were combined.

The general purpose detector CMD-2 has been described in detail elsewhere [10]. It consists of a drift chamber (DC) [15] and a proportional Z-chamber [16], both used for the trigger, and both inside a thin (0.4X0) superconducting solenoid with a field of 1 T.

The barrel calorimeter [17] which is placed outside the solenoid, consists of 892 CsI crystals of 6 × 15 cm³ size and covers polar angles from 46° to 134°. The energy resolution for photons is about 9% in the energy range from 50 to 600 MeV. The angular resolution is about 0.02 radians.

The end-cap calorimeter [18] which is placed inside the solenoid, consists of 680 BGO crystals of 2.5 × 2.5 × 15 cm³ size and covers forward–backward polar angles from 16° to 49° and from 131° to 164°. The energy and angular resolution varies from 8% to 4% and from 0.03 to 0.02 radians, respectively, for the photons in the energy range from 100 to 700 MeV.

The luminosity was determined from the events of Bhabha scattering at large angles [19].

The collider energy was roughly set (\(\delta E/E \lesssim 10^{-3}\)) by the dipole magnet currents. In the energy range 1.010 to 1.028 GeV of the main scans the beam energy was more precisely (\(\delta E/E \lesssim 10^{-4}\)) determined by measuring the average momentum $p_{av}$ of $K^+K^-$ pairs in the DC: $E_{K^+K^-}^{K^+K^-} = \sqrt{p_{av}^2 + M_K^2} + \Delta$. Here $\Delta$ is a correction for the contributions of kaon ionization losses inside the detector and radiative losses of initial electrons. Its magnitude depends on $p_{av}$ and varies from 5 to 3 MeV for $p_{av}$ in the range 80 to 130 MeV/c (see [19] for more detail). Then the effective beam energy at each energy point was determined by averaging $E_{K^+K^-}^{K^+K^-}$ weighted by the integrated luminosity:
\[ E_i = \sum_{j=1}^{3} L_{ij} E_{ij}^{K^+K^-} / \sum_{j=1}^{3} L_{ij}, \]

where \( E_{ij}^{K^+K^-} \) and \( L_{ij} \) are the kaon energy and the integrated luminosity measured at the \( i \)th energy point of the \( j \)th scan.

The first two columns of Table 1 present the corresponding energy values and integrated luminosities. The total integrated luminosity was 11.63 pb\(^{-1}\).

The analysis of the reaction \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^- \) was performed similarly to our analysis described in [2]. However, because of the completely different background situation, other methods of background suppression were used. Events with four charged tracks coming from the interaction region were selected:

- the impact parameter of each track \( r_{\text{min}} \) is less than 1 cm;
- the vertex coordinate along the beam axis \( z_{\text{vert}} \) is within \( \pm 10 \) cm.

Table 1
Summary of the cross section calculations

<table>
<thead>
<tr>
<th>( E_{\text{cm}}, \text{GeV} )</th>
<th>( L, \text{nb}^{-1} )</th>
<th>( N_{\Delta \pi} )</th>
<th>( \sigma, \text{nb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.984</td>
<td>382.0</td>
<td>69</td>
<td>0.68 ± 0.08</td>
</tr>
<tr>
<td>1.004</td>
<td>485.9</td>
<td>112</td>
<td>0.90 ± 0.09</td>
</tr>
<tr>
<td>1.0103</td>
<td>503.4</td>
<td>125</td>
<td>0.98 ± 0.09</td>
</tr>
<tr>
<td>1.0157</td>
<td>442.3</td>
<td>103</td>
<td>0.92 ± 0.09</td>
</tr>
<tr>
<td>1.0168</td>
<td>1036.4</td>
<td>246</td>
<td>0.94 ± 0.06</td>
</tr>
<tr>
<td>1.0178</td>
<td>1562.6</td>
<td>393</td>
<td>1.01 ± 0.05</td>
</tr>
<tr>
<td>1.0187</td>
<td>1555.9</td>
<td>368</td>
<td>0.96 ± 0.05</td>
</tr>
<tr>
<td>1.0197</td>
<td>1361.6</td>
<td>416</td>
<td>1.26 ± 0.06</td>
</tr>
<tr>
<td>1.0206</td>
<td>923.7</td>
<td>280</td>
<td>1.27 ± 0.08</td>
</tr>
<tr>
<td>1.0215</td>
<td>476.7</td>
<td>134</td>
<td>1.17 ± 0.10</td>
</tr>
<tr>
<td>1.0227</td>
<td>584.6</td>
<td>177</td>
<td>1.25 ± 0.09</td>
</tr>
<tr>
<td>1.0278</td>
<td>573.9</td>
<td>181</td>
<td>1.29 ± 0.10</td>
</tr>
<tr>
<td>1.034</td>
<td>519.7</td>
<td>183</td>
<td>1.45 ± 0.11</td>
</tr>
<tr>
<td>1.040</td>
<td>491.5</td>
<td>182</td>
<td>1.53 ± 0.11</td>
</tr>
<tr>
<td>1.050</td>
<td>333.2</td>
<td>132</td>
<td>1.64 ± 0.14</td>
</tr>
<tr>
<td>1.060</td>
<td>399.0</td>
<td>184</td>
<td>1.92 ± 0.14</td>
</tr>
</tbody>
</table>

To have good reconstruction efficiency, tracks were also required to cross at least two superlayers of the drift chamber: \(| \cos \theta | < 0.8\).

For selected events a kinematic fit was performed, assuming that all tracks are pions and under the constraint that the sum of the 3-momenta \( \sum_{i=1}^{4} \vec{p}_i = 0 \). Then the requirement that the fit quality \( \chi^2_{\text{kin}} / \text{n.d.f.} < 100/3 \) was applied. This condition has high efficiency (about 95\%) for the process under study and rejects about 70\% of the background reactions:

\[ \phi \rightarrow K^0_SK^0_L \]

and

\[ \phi \rightarrow K^+K^- . \]

Further analysis was performed using the normalized "apparent energy":

\[ \epsilon_{\text{app}} = \frac{\sum_{i=1}^{4} \sqrt{\vec{p}_i^2 + m_\pi^2}}{2E_{\text{beam}}} . \]

Fig. 1 shows the distribution of \( \epsilon_{\text{app}} \) versus the minimum space angle between the tracks with the opposite charges \( \psi_{\text{min}} \). A band with \( \epsilon_{\text{app}} \approx 1 \) corresponding to \( \pi^+ \pi^- \pi^+ \pi^- \) events is clearly observed in the region "a". The lower part of the region "b" is populated by events from the process:

![Fig. 1. Distribution of the normalized apparent energy versus the minimum space angle between tracks with opposite charges.](image-url)


\[ e^+ e^- \rightarrow \pi^+ \pi^- \pi^0, \quad \pi^0 \rightarrow e^+ e^- \gamma, \quad (3) \]

while events from the processes:

\[ e^+ e^- \rightarrow e^+ e^- \gamma, \quad (4) \]
\[ e^+ e^- \rightarrow \pi^+ \pi^- \gamma \quad (5) \]

with the subsequent photon conversion into an \( e^+ e^- \)-pair at the beam pipe, fall into the upper part of the region “b”. Events of the \( \phi \) meson decay (1), where \( K_L^0 \) and \( K_S^0 \) decay to \( \pi^+ \pi^- \) and \( \pi^+ \pi^- \pi^0 \), respectively, contribute to the region “c”. Events of another \( \phi \) meson decay (2), where products of kaon nuclear interactions scatter back to the drift chamber and induce two “extra” tracks, fall in the region “e”. Events with the decay of one of the kaons \( K^\pm \rightarrow \pi^+ \pi^- \pi^\pm \) populate the region “d”. Thus, using the parameter \( \varepsilon_{\text{app}} \), we are able to separate events of the classes “c” and “d” from events of the process (2). Thus, selection of events below the solid line effectively rejects events from both classes “d” and “e”. To estimate the number of remaining background events in class “e”, events of the process \( \phi \rightarrow K^+ K^- \), \( K^\pm \rightarrow \pi^\pm \pi^+ \pi^- \) were selected using the conditions \( \varepsilon \leq 0.7 \) and \( \psi_{\text{min}} > 1 \). Then the efficiency \( \varepsilon_{K^+ K^-} \) of such a cut was determined for these events. Since in class “e” both charged kaons are detected in the drift chamber, the probability of kaon misidentification is \( \varepsilon_{\text{app}} \). Using this probability, the expected number of remaining background events was found to be \( N_{K^+ K^-} < 5 \).

Events of the reactions (3), (4) and (5) (region “b” in Fig. 1) with low momentum of \( e^\pm \) tracks give a small tail in the lower left corner of Fig. 2. Selection of events above the dashed line suppresses the background from the processes with the photon conversion into an \( e^\pm e^- \)-pair. Additional suppression of the process (4) is provided by the requirement \( \varepsilon < 0.9 \), where the parameter \( \varepsilon_{\text{app}} \) is the normalized total energy assuming that all particles are electrons.

After that, using the conditions \( \psi_{\text{min}} > 0.3 \) and \( |\varepsilon_{\text{app}} - 1| < 0.1 \), we selected our data sample of about 4200 events consisting mostly of the events of the process \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^- \). The number of remaining background events from the process (3) was estimated using the \( \psi_{\text{min}} \) distribution in the region \( \varepsilon_{\text{app}} > 1.1 \) (see Fig. 1). This area is populated

![Fig. 2. Ionization losses (arbitrary units) versus the track momentum: (a) positively charged tracks, (b) negatively charged tracks. The solid and dashed lines show the selection boundaries.](image-url)
Fig. 3. The invariant mass $M_{\pi^+\pi^-}$ of pion pairs versus the momentum of the same pair $P_{\pi^+\pi^-} = |p_{\pi^+} + p_{\pi^-}|$.

by events from the reactions (3), (4) and (5). Assuming that $v_{\text{min}}$ distributions have similar shape in the regions $\varepsilon_{\text{app}} > 1.1$ and $\varepsilon_{\text{app}} < 1.1$, we obtained $N_{\text{bg}} < 80$.

Fig. 3 shows the distribution of the invariant mass $M_{\pi^+\pi^-}$ for pairs of opposite charged pions versus the total momentum of the same pair $P_{\pi^+\pi^-}$ for the selected data sample (4 entries per event). The enhanced concentration of events in the region $M_{\pi^+\pi^-} \approx m_{K^0_S}$ and $P_{\pi^+\pi^-} \approx P_{K^0_S}^0$ is caused by the reaction (1), where $K^0_S$ decays to $\pi^+\pi^-$ and $K^0_L$ decays to one of the semileptonic modes $K^0_L \rightarrow \pi^+e^+\nu_e$ or $K^0_L \rightarrow \pi^+\mu^+\nu_\mu$. Here $m_{K^0_S} = 497.67$ MeV [20] and $P_{K^0_S}^0 = \sqrt{E^2_{\text{beam}} - m^2_{K^0_S}}$ are the $K^0_S$ meson mass and momentum. To reject this type of the background we excluded events in which at least one of the $\pi^+\pi^-$ pairs satisfies both of the following conditions:

$$|M_{\pi^+\pi^-} - m_{K^0_S}| < 30 \text{ MeV}/c^2,$$

$$|P_{\pi^+\pi^-} - P_{K^0_S}^0| < 30 \text{ MeV}/c.$$  \hspace{1cm} (6)

The expected number of remaining background events from this process was estimated using the complete Monte Carlo simulation (MC) of the CMD-2 detector [21] and was found to be $N_{\text{bg}}^{K^0_SK^0_L} < 30$.

3. Determination of cross section

At each energy the cross section of the process $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ was calculated using the formula:

$$\sigma_i = \frac{N_i}{L_i \varepsilon_i (1 + \delta_i)}, \hspace{1cm} (7)$$

where $N_i$ is the number of selected $\pi^+\pi^-\pi^+\pi^-$ events, $L_i$ is the integrated luminosity, $\varepsilon_i$ is the detection efficiency, and $\delta_i$ is the radiative correction at the $i$th energy point.

The detection efficiency was determined from MC assuming the $a_1(1260)\pi$ quasi-twobody production mechanism, which clearly dominates at higher energy [1]. Comparison of various experimental distributions with the simulation shows that the assumption of the $a_1(1260)\pi$ mechanism does not contradict the data. The detection efficiency decreases with energy, smoothly varying from 30% to 27%.

Radiative corrections were calculated according to [22]. Since radiative corrections themselves depend on the energy behavior of the cross section, the calculation was performed by the iteration method. “Visible” values of the cross section (with $\delta = 0$ in (7)) were used as the first approximation. Then the cross section was recalculated with the new values of the radiative corrections and the whole procedure was repeated until the convergence was reached. Fig. 4 demonstrates the energy dependence of radiative corrections.

Table 1 presents the summary of the cross section calculations. Fig. 5 shows the energy dependence of the cross section near the $\phi$ meson. The obtained values of the cross sections measured below and above the $\phi$ meson match our previous results well [1,2]. Only statistical errors are shown in Fig. 5. The systematic uncertainty comes from the following sources:

- selection criteria and background suppression — 11%;
- event reconstruction — 5%;
- detection efficiency dependence on the production mechanism — 3%;
- beam energy spread — 2%;
- radiative corrections — 1.6%;
- luminosity determination — 1.5%.

The overall systematic uncertainty was estimated to be $\approx 13%$. 
The energy dependence of the cross section shown in Fig. 5 demonstrates a clear interference pattern in the vicinity of the $\phi$ meson. To describe the interference behavior we parameterize the cross section according to the following formula:

$$\sigma_{e^+e^-\rightarrow\pi^+\pi^-\pi^+\pi^-}(E) = \sigma_0 f(E) \left| 1 - Z \frac{m_\phi \Gamma_\phi}{m_\phi^2 - E^2 - i E \Gamma_\phi} \right|^2,$$

where $E = 2E_{\text{beam}}$; $\sigma_0$ is the nonresonant cross section of the process $e^+e^-\rightarrow\pi^+\pi^-\pi^+\pi^-$ at $E = m_\phi$; $f(E)$ is a smooth function describing the nonresonant behavior of the cross section and normalized to 1 at $E = m_\phi$; $m_\phi$ and $\Gamma_\phi$ are the $\phi$ meson mass and width; and $Z = |Z| e^{i\theta}$ is a complex interference amplitude. The following values of the parameters were obtained from the fit with $f(E) = e^{A(E-m_\phi)}$ ($A$ is a slope parameter):

$$\sigma_0 = 1.114 \pm 0.035 \pm 0.056 \text{ nb},$$

$$\text{Re } Z = 0.122 \pm 0.027 \pm 0.033,$$

$$\text{Im } Z = -0.003 \pm 0.025 \pm 0.058. \quad (9)$$

The obtained value of $\chi^2$/n.d.f. characterizing the fit quality was 8.31/12 corresponding to a 75% confidence level. Since the observed value of the resonance amplitude is about the same order ($\sim 10\%$) as the statistical errors in any one of the measured values of the cross section, the interference pattern in the cross section behavior could appear due to statistical fluctuations rather than the decay $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-$. To check the consistency of the data with this assumption, we performed a fit with the amplitude $|Z|$ fixed to 0. The obtained $\chi^2$/n.d.f. value was 27.90/12 corresponding to the 0.5% confidence level. Thus, we can really claim evidence for the decay $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-$. The first error in each fit parameter in (9) is statistical, while the second one is systematic. Table 2 lists the main sources of systematic uncertainties contributing to the overall systematic error of the fit parameters. Let us discuss some of these uncertainties in more detail.

The influence of the event selection procedure on the fit parameters was estimated in the following way. Three data samples were selected with an additional requirement for background suppression and variations of the criteria described in Section 2:

1. Application of a stricter requirement on the impact parameter of each track $r_{\text{min}} < 0.3$ cm suppresses $\sim 80\%$ of the background events from the reaction (1), leaving about 150 $K_S^0 K_L^0$ events in the $\pi^+\pi^-\pi^+\pi^-$ data sample.

2. The probability $W_{\pi^+\pi^-}$ for two tracks with the smallest angle between them to be pions was calculated (see [2] for more detail). The requirement
Uncertainties in parameters and reconstruction efficiencies lead to systematic uncertainties in the luminosity determination, detection and reconstruction efficiencies, which are estimated to be about 3.8%.

Using real and imaginary parts $\text{Re} Z$ and $\text{Im} Z$ of the interference amplitude $Z$, the values of $|Z|$ and $\psi$ were calculated:

$$|Z| = 0.122 \pm 0.027 \pm 0.033,$$

$$\psi = (-1 \pm 12 \pm 27)^\circ.$$

The branching ratio of the decay $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-$ can be calculated using the following expression:

$$\text{Br}(\phi \rightarrow \pi^+\pi^-\pi^+\pi^-) = \frac{\sigma_0 |Z|^2}{\sigma_0},$$

where $\sigma_0 = 12\pi \text{Br}(\phi \rightarrow e^+e^-)/m_\phi^2 = 4224 \pm 113 \text{ nb}$ [20] is the cross section of the $\phi$ meson production. This is the first measurement of this quantity, and it supersedes the upper limit on the branching ratio of $8.7 \times 10^{-4}$ obtained in Orsay [4] as well as the upper limit of $1 \times 10^{-4}$ placed by CMD-2 and based on part of the whole data sample [23].

### 4. Search for decays $\phi \rightarrow \eta \pi^+\pi^-$ and $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$

The same data sample of preselected events with four charged tracks was used for the search of the decays $\phi \rightarrow \eta \pi^+\pi^-$, $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$. In this analysis at least two photons detected in the calorimeter were required. The kinematic fit was performed taking into account energy-momentum conservation. In the reconstruction procedure all charged particles were assumed to be pions.

One of the main problems is additional (“fake”) photons induced by the products of nuclear interactions of charged pions in the detector material. The following simple method was used to suppress such fake photons. In the kinematic fit the energy resolution of photons was loosened to the value $\sigma_{E_\gamma} = E_\gamma + 20 \text{ MeV}$. Thus, the photon energy was allowed to vary in a wide range during the fit. Only two photons were included in the fit. For events with more than two detected photons, the fit was repeated with all possible pairs of photons and the pair with the smallest $\chi^2$ characterizing the fit quality was selected. Events with the reconstructed photon energy below
30 MeV were rejected from the subsequent analysis. The following requirements were additionally applied: $\chi^2/\nu < 10/4$ and the invariant mass of the photon pair is near the $\pi^0$ mass; $|M_{\gamma\gamma} - m_{\pi^0}| < 30$ MeV.

The process (2) in which products of kaon nuclear interactions scatter back to the drift chamber and induce two extra tracks or one of the kaons decays via the $K^\pm \to \pi^\pm \pi^+ \pi^-$ channel, accompanied by fake photons, can contribute to the background for the decays under study. Another source of the background is the reaction

$$e^+e^- \to \omega \pi^0, \quad \omega \to \pi^+ \pi^- \pi^0$$

(10)

with the Dalitz decay of one of the neutral pions. The main background, however, comes from the process (1) followed by the $K^0_S \to \pi^+ \pi^- \pi^0$ and $K^0_L \to \pi^+ \pi^- \pi^0$ decays.

The same restrictions on the $dE/dx$ of the tracks as applied in the search for the decay $\phi \to \pi^+ \pi^- \pi^0 \pi^0$ (see Fig. 2), were used to suppress the background from the decay (2).

To reject the background from the reaction (10), we searched for a pair oppositely charged particles with the minimum space angle $\psi_{\min}$ between the tracks. Assuming this pair to be $e^+e^-$ and taking the photon with the smaller energy, the invariant mass $M_{\gamma e^-\gamma e^-}$ was calculated. The requirements $\psi_{\min} > 0.3$ and $M_{\gamma e^-\gamma e^-} > 170$ MeV reduced the background from the reaction $e^+e^- \to \omega \pi^0$ to a negligible level: $N_{\omega\pi^0} < 0.1$.

To reject the background from the decay (1) events in which at least one of the $\pi^+ \pi^-$ pairs satisfies the conditions (6) were excluded. Additional suppression of $K^0_S K^0_L$ events was achieved by restricting the impact parameter of each track: $r_{\min} < 0.3$ cm. Only $N_{\pi^+ \pi^-}^{\pi^+ \pi^-} = 2$ candidate events survive after applying these selection criteria.

Events for which the impact parameter of at least one track has the value $r_{\min} > 0.3$ cm are mostly coming from the decay (1). The observed number of such events satisfying all above criteria but $r_{\min} < 0.3$ cm is $N_{K^0_S K^0_L} = 6$. Applying the whole set of selection criteria and requiring that for at least one $\pi^+ \pi^-$ pair the conditions (6) are held, one can obtain a practically pure $K^0_S K^0_L$ sample. From the distribution of $r_{\min}$ for thus selected events of the process (1) the ratio $N_{K^0_S K^0_L}(r_{\min} < 0.3)/N_{K^0_S K^0_L}(r_{\min} > 0.3) = 0.20 \pm 0.03$ was obtained in good agreement with simulation. Using this ratio, the expected background in the region $r_{\min} < 0.3$ cm was estimated to be $N_{bg} = 1.2^{+0.7}_{-0.4}$. Thus, the upper limit can be set on the number of signal events: $N_{\pi^+ \pi^-} < 5.1$ at 90% CL [24]. The 90% CL upper limit can be correspondingly obtained for the decay probability:

$$Br(\phi \to \eta \pi^+ \pi^-) < \frac{N_{\eta \pi^+ \pi^-}}{N_\phi Br(\eta \to \pi^+ \pi^- \pi^0) \varepsilon_{\eta \pi^+ \pi^-}} = 1.8 \times 10^{-5},$$

(11)

where $N_\phi \approx 16 \times 10^6$ [19] is the total number of $\phi$ meson events recorded by CMD-2 in the experiment. $Br(\eta \to \pi^+ \pi^- \pi^0) = 0.231 \pm 0.005$ [20] is the branching ratio of the $\eta$ decay and $\varepsilon_{\eta \pi^+ \pi^-} = 0.09 \pm 0.01$ is the detection efficiency obtained from simulation. To take into account the uncertainties in the $Br(\eta \to \pi^+ \pi^- \pi^0)$ and $\varepsilon_{\eta \pi^+ \pi^-}$, their values were lowered by one standard deviation while calculating the upper limit in (11). This upper limit is approximately 15 times better than the previous one also set by the CMD-2 group using the $\eta \to \gamma \gamma$ decay mode and based on part of the total data sample [8].

Calculating the detection efficiency for the process $e^+e^- \to \pi^+ \pi^- \pi^+ \pi^- \pi^0$ under the assumption of the constant matrix element and skipping the probability of the decay $\eta \to \pi^+ \pi^- \pi^0$ in (11), one can obtain the upper limit on the branching ratio of the direct decay $\phi \to \pi^+ \pi^- \pi^+ \pi^- \pi^0$:

$$Br(\phi \to \pi^+ \pi^- \pi^+ \pi^- \pi^0) < 4.6 \times 10^{-6} \ 90\% \mathrm{CL.}$$

This limit is 25 times better than the previous one placed in Ref. [7].

5. Discussion

At the present time no theoretical calculations exist for the value $Br(\phi \to \pi^+ \pi^- \pi^+ \pi^-)$. A simple estimate can be performed [25] taking into account the $\phi \to \gamma$ transition:

$$Br(\phi \to \gamma \gamma \to \pi^+ \pi^- \pi^+ \pi^-) = 9 \cdot \frac{(Br(\phi \to e^+e^-))^2}{\alpha^2} \frac{\sigma_0}{\sigma_\phi} = 3.99 \times 10^{-6},$$

where the values $\sigma_0 = 1.114$ nb and $\sigma_\phi = 4224$ nb were used for the nonresonant cross section of the
process $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ and the total cross section of the $\phi$ meson production, respectively. The measured branching ratio is consistent with this estimate.

Note that this measurement is also of interest to clarify the problem of two conflicting results for the branching ratio of the related decay $\phi \rightarrow \pi^+\pi^-$ recently measured by CMD-2 [12] and SND groups [13]. While in the CMD-2 measurement the imaginary part of the interference amplitude is consistent with zero, SND claims a statistically significant non-zero imaginary part measured: by G-parity conservation and OZI rule has been measured:

$$Br(\phi \rightarrow \eta\pi^+\pi^-) = 0.35 \times 10^{-6}.$$  

This value is 50 times lower than the obtained upper limit.

6. Conclusion

The reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ has been studied in the energy range 0.984 to 1.06 GeV. About 3300 $\pi^+\pi^-\pi^+\pi^-$ events were detected. For the first time the interference behavior of the cross section has been observed in the vicinity of the $\phi$ meson. The branching ratio of the decay $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-$ suppressed by G-parity conservation and OZI rule has been measured:

$$Br(\phi \rightarrow \pi^+\pi^-\pi^+\pi^-) = (3.93 \pm 1.74 \pm 2.14) \times 10^{-6}.$$  

Upper limits have been set on the branching ratios of the decays $\phi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$ and $\phi \rightarrow \eta\pi^+\pi^- $:

$$Br(\phi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0) < 4.6 \times 10^{-6} \text{ 90\% CL},$$  

$$Br(\phi \rightarrow \eta\pi^+\pi^- ) < 1.8 \times 10^{-5} \text{ 90\% CL}.$$  

Acknowledgements

The authors are grateful to the staff of VEPP-2M for excellent performance of the collider, to all engineers and technicians who participated in the design, commissioning and operation of CMD-2. Special thanks are due to N.N. Achasov for useful discussions.

References

Isospin dependence of liquid–gas phase transition in hot asymmetric nuclear matter

Wei Liang Qian a,*, Ru-Keng Su a,b, Ping Wang a,c

a Department of Physics, Fudan University, Shanghai 200433, People's Republic of China
b China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing, People’s Republic of China
c Institute of High Energy Physics, P.O. Box 918, Beijing 100039, People’s Republic of China

Received 5 June 2000; accepted 14 August 2000
Editor: W. Haxton

Abstract

By using the Furnstahl, Serot and Tang’s model, the effect of density dependence of the effective nucleon–nucleon–ρ-meson (NNρ) coupling on the liquid–gas phase transition in hot asymmetric nuclear matter is investigated. A limit pressure $p_{\text{lim}}$ has been found. We found that the liquid–gas phase transition cannot take place if $p > p_{\text{lim}}$. The binodal surface for density dependent NNρ coupling situation is addressed.

Ó 2000 Published by Elsevier Science B.V.

PACS: 21.65.+f; 25.75.+r; 64.10.+h

It is generally recognized that the liquid–gas (LG) phase transition of one component system is of first order. The chemical potential continues at the phase transition point but its first order derivatives, namely, entropy and volume, are discontinuous. But for a multi-components or multi-conserved charges system, as was pointed out by Müller and Serot [1], because of the greater dimensionality of the binodal surface, the LG phase transition can be of second order, i.e., the entropy continues but the second order derivatives of chemical potential (for example, capacity) are discontinuous. An asymmetric nuclear matter has two components of proton and neutron, and two conserved charges of baryon number and the third component of isospin, will undergo a continuous second order phase transition.

Obviously, because of charge independence, the basic difference between proton and neutron be isospin. The isospin dependent interactions of nucleon–nucleon–isovector mesons play the key role to address the LG phase transition. As was pointed out by our previous papers [2–4], if one employed the isospin independent model, for example, Walecka model [5] or Zimanyi–Moszkowski model [6], to investigate asymmetric nuclear matter, a lot of difficulties, e.g., Coulomb instability and negative asymmetric parameter in the vapor phase, will emerge. To overcome these difficulties, the isospin vector ρ-meson must be introduced. It can be shown that the chemical potentials of proton and neutron may depend on the third component of isospin when NNρ interaction exists. A model without isospin vector ρ-meson, or even if it...
has $\rho$-meson, but the chemical potentials of the proton or neutron are still independent of the NN$\rho$ interaction because the third component $I_{3}$ of isospin be zero such as in a symmetric nuclear matter, the LG phase transition is still of first order.

In fact, the chemical potentials of proton and neutron not only depend on $I_{3}$ but also on the effective NN$\rho$ coupling $g_{\rho}$. Then the effective coupling $g_{\rho}$ is also essential for studying the LG phase transition because the chemical potentials determine the binodal surface directly. In our previous papers [7–9], we have shown that the effective couplings of $g_{\pi}$, $g_{\rho}$, $g_{\omega}$ and $g_{\rho}^{0}$ all decrease as the nucleon density increases. In an asymmetric nuclear matter, one can easily prove that the chemical potentials of nucleons have a term which is proportional to $g_{\rho}^{2}$ and $I_{3}$. This term has opposite signs for proton and neutron due to their different third component of isospin. Obviously, if $g_{\rho}$ depends on density, this term will change and then the chemical potentials of proton and neutron, as well as the binodal surface of LG phase transition will also be changed. The objective of the present Letter is to investigate the effect of the density dependence of $g_{\rho}$ on LG phase transition. We will prove that if $g_{\rho}$ is a decreasing function of density, a limit pressure $p_{\text{lim}}$ will occur, when $p > p_{\text{lim}}$, the coexisted equations have no solution and the LG phase transition cannot be existed. The chemical potential of neutron will become a monotonous function of asymmetry $\alpha$ in this case.

To illustrate our result, we employ a model suggested by Furnstahl, Serot and Tang (FST) [11–14] recently. This model is an extension of quantum hadrodynamics and has been proven to be successful to explain many experimental properties of both nuclear matter and the finite nuclei in mean field approximation. The Lagrangian density of FST model under mean field approximation is

\[
L_{\text{MFT}} = \bar{\Psi} \left[ i \gamma^{\mu} \partial_{\mu} - (M - g_{\pi} \phi_{0}) - g_{\pi} \gamma^{0} V_{0} - \frac{1}{2} g_{\rho}^{2} V_{0}^{2} \right] \Psi
\]

\[
+ \frac{1}{2} m_{\pi}^{2} V_{0}^{2} \left( 1 + \eta \frac{\phi_{0}}{S_{0}} \right) + \frac{1}{4!} \xi (g_{\pi} V_{0})^{4}
\]

\[
+ \frac{1}{2} m_{\rho}^{2} b_{0}^{2} - H_{Q} \left( 1 - \frac{\phi_{0}}{S_{0}} \right)^{4/d}
\]

\[
\times \left[ \frac{1}{d} \ln \left( 1 - \frac{\phi_{0}}{S_{0}} \right) - \frac{1}{4} \right].
\]

(1)

where $g_{\pi}$, $g_{\rho}$, $g_{\rho}^{0}$ are, respectively, the couplings of light scalar meson $\sigma$, vector meson $\omega$ and isovector meson $\rho$ fields to the nucleon, $\phi_{0}$, $V_{0}$, $b_{0}$ are the expectation values $\phi_{0} \equiv \langle \phi \rangle$, $V_{0} \equiv \langle V_{\mu} \rangle$, $b_{0} \equiv \langle b_{\mu} \rangle$. The scalar fluctuation field $\phi$ is related to $S$ by $S(x) = S_{0} - \phi(x)$ and $H_{Q}$ is given by $m_{\pi}^{2} = 4 H_{Q}/(d^{2} S_{0}^{4})$, $d$ the scalar dimension. By using the standard technique of statistical mechanics, we get the thermodynamic potential $\Omega$ as [15]

\[
\Omega = V \left[ H_{Q} \left( 1 - \frac{\phi_{0}}{S_{0}} \right)^{4/d}
\times \left( \frac{1}{d} \ln \left( 1 - \frac{\phi_{0}}{S_{0}} \right) - \frac{1}{4} \right) + \frac{1}{4} \right]
\]

\[
- \frac{1}{2} m_{\rho}^{2} b_{0}^{2} - \frac{1}{2} \left( 1 + \eta \frac{\phi_{0}}{S_{0}} \right) m_{\pi}^{2} V_{0}^{2}
\]

\[
- \frac{1}{4!} \xi (g_{\pi} V_{0})^{4}
\]

\[
- 2 k_{B} T \left[ \sum_{k, \tau} \ln \left( 1 + e^{-\beta (E^{k} - \nu_{k})} \right)
\right]
\]

\[
+ \sum_{k, \tau} \ln \left( 1 + e^{-\beta (E^{k} + \nu_{k})} \right)
\].

(2)

where $\beta = 1/k_{B} T$ and the quantity $\nu_{i}$ ($i = n, p$) is related to the usual chemical potential $\mu_{i}$ by the equations

\[
\nu_{n} = \mu_{n} - g_{\pi} V_{0} + \frac{g_{\rho}^{2} \rho_{3}}{4 m_{\rho}^{2}}
\]

(3)

\[
\nu_{p} = \mu_{p} - g_{\pi} V_{0} - \frac{g_{\rho}^{2} \rho_{3}}{4 m_{\rho}^{2}}
\]

(4)

where $\rho_{3} = \rho_{p} - \rho_{n}$ and the third component of isospin $I_{3} = (N_{p} - N_{n})/2 = V_{R} \rho_{3}/2$. The third term of the right hand side of Eq. (3) and Eq. (4) depends on $\rho_{3}$ and $g_{\rho}^{2}$. They have opposite signs and play the essential role to determine the LG phase transition.
Having obtained the thermodynamic potential, all other thermodynamic quantities, for example, pressure \( p = -\Omega / V \), can be calculated. The two-phase coexistence equations are

\[
\mu_l^1(T, \rho_l^1) = \mu_l^V(T, \rho_l^V), \quad \mu_l^1(T, \rho_l^1) = \mu_l^V(T, \rho_l^V),
\]

where subscripts of one phase L and V stand for liquid and vapor, respectively. The stability conditions are given by \([1]\)

\[
\frac{\partial \mu_p}{\partial \rho} \bigg|_{T, \alpha} \geq 0, \quad \frac{\partial \mu_n}{\partial \alpha} \bigg|_{T, \rho} < 0 \text{ or } \frac{\partial \mu_n}{\partial \alpha} \bigg|_{T, \rho} > 0,
\]

where \( \mathcal{F} \) is the density of free energy, \( \alpha = (\rho_n - \rho_p) / \rho \) the asymmetric parameter, and \( \rho = \rho_n + \rho_p \).

The numerical calculations have been done by adopting the parameters set T1 of FST model \([11–13]\). The parameters of set T1 are

\[
\begin{align*}
\delta_0 &= 99.3, & \delta_0 &= 154.5, & \delta_0 &= 70.2, \\
m_0 &= 509 \text{ MeV}, & \delta_0 &= 90.6 \text{ MeV}, \\
\zeta &= 0.0402, & \eta &= -0.496, & d &= 2.70.
\end{align*}
\]

Our results for \( g_p = (70.2)^{1/2} \) are shown in Fig. 1 and Fig. 2 by solid curves. The Gibbs conditions (5) and (6) for phase equilibrium demand equal pressures and chemical potentials for two phase with different concentrations. The collection of all such pairs \( \alpha_1(T, \rho) \) and \( \alpha_2(T, \rho) \) form the binodal surface. In Fig. 1, the chemical isobar vs. \( \alpha \) curves at fixed temperature \( T = 10 \text{ MeV} \) and \( p = 0.100 \text{ MeV fm}^{-3} \) are labeled by A and A' for neutron and proton, respectively. The two desired solutions form the edges of a rectangle and can be found by means of the geometrical construction shown in Fig. 1 \([1]\). The critical curves with \( T = 10 \text{ MeV} \) and \( p_{\text{crit}} = 0.165 \text{ MeV fm}^{-3} \) are shown in Fig. 1 by B and B' where the chemical potential curve arrive at a inflection point and the rectangle is degenerate to a line vertical to the \( \alpha \) axis. The behaviour of the nuclear matter under isothermal compression, or in other words, the section of binodal surface at finite temperature \( T = 10 \text{ MeV} \) are shown in Fig. 2. The physical behaviour of this processes has been discussed by Ref. \([1]\). Assume that the system is initially prepared with \( \alpha = 0.6 \) (gas), during the compression, the two-phase region is encountered at point A, and the liquid phase emerges at point B. The gas phase evolves from A to D, while the liquid phase evolves from B to C. The system leaves the region of instability at point C, while the original gas phase is
about to disappear. The critical point (CP), the point of equal concentration (EC) and the maximal asymmetry (MA) are indicated in Fig. 2.

Now we are in a position to extend our discussion to the case of $g$ density dependence. In fact, the effective masses of nucleons, effective masses or screening masses of mesons, and the effective couplings of NN-mesons are all dependent on density and temperature. We can sum the tadpole diagrams and the exchange diagrams for nucleon, the vacuum polarization diagrams for mesons and the three-lines vertex diagrams for effective couplings to get their density and temperature dependence [4,7–9,15–17]. But in order to illustrate our result more transparently, instead of the exact calculation of three-lines vertex, we introduce an ansatz

$$g'_{\rho} = g_{\rho}[1 - A\rho + B\rho^2]$$

where $A$, $B$ are two adjust parameters. The reason for our choice are: at first, the three lines vertex calculations are model dependent, but we hope that our investigation can be more general; secondly adjust the values of $A$, $B$ can made $g'_{\rho}$ be decreased or increased with density, then we can study the LG phase transition for two different cases. We can imagine that the Eq. (10) is a density expansion of effective coupling $g'_{\rho}$ at low density regions.

The results for density dependent $g'_{\rho}$ ansatz Eq. (10) are shown in Figs. 3–5. We see from Figs. 3 and 4 that the chemical potential of neutron $\mu_n$ increases rapidly with density. It passes through an inflection point and becomes monotonous when pressure increases. But the shape of $\mu_p$ vs. $\alpha$ curves change slowly. Then we will get a limit pressure $p_{\text{lim}}$, when $p > p_{\text{lim}}$, the rectangle cannot be found and the coexisted equations have no solution. The last rectangle in the chemical isobar vs. $\alpha$ curves for $A = 1, B = 0$, $T = 10$ MeV, and $p_{\text{lim}} = 0.130$ MeV fm$^{-3}$ is shown in Fig. 3 by dashed lines, where $\alpha_1 = 0.62$ and $\alpha_3 = 0.75$ correspond to the maximum and the minimum of $n$, respectively. The pair $\alpha_1 = 0.62$ and $\alpha_2 = 0.84$ form the end of the binodal surface, as shown in Fig. 5. The curve for $A = 1, B = 0$, $T = 10$ MeV, but $p = 0.145$ MeV fm$^{-3}$ ($p > p_{\text{lim}}$) is shown in Fig. 4. We see that $\mu_n$ becomes monotonous at this pressure, and no rectangle can be found. The relation between limit pressure and parameters $A$ and $B$ for a fixed temperature $T = 10$ MeV is shown in Table 1. We find from Table 1 when $A$ and $B$ increase, the effective coupling $g'_{\rho}$ decreases and the limit pressure decreases. If $A$ changes its sign to become negative, $g'_{\rho}$ will increase with density, and in this case, instead of $\mu_n$, $\mu_p$ becomes monotonous. The limit pressure is still existed but decrease when $A$ and $B$ decrease.

The section of binodal surface for $A = 1, B = 0$, $T = 10$ MeV is shown in Fig. 5. We see from Fig. 5 that the curve will cut off at limit temperature $p_{\text{lim}}$.
clearly. The total asymmetric parameter $\alpha$ is divided into four regions, namely, $[0, \alpha_1]$, $[\alpha_1, \alpha_2]$, $[\alpha_3, \alpha_4]$ and $[\alpha_2, \alpha_{\text{max}}]$. The physical behaviour of isothermal compression in different regions are different. We find:

1. If the system is initially prepared with $0 < \alpha < \alpha_1$, the process of isothermal compression is similar to that of the case with constant $g_I$. It begin at gas phase, suffers a second order LG phase transition and ends at liquid phase.

2. If the initial $\alpha$ is located at the region $\alpha_2 < \alpha < \alpha_{\text{max}}$, the system enters and leaves the two-phase region on the same branch, so the system remain in the same gas phase. As was pointed out by Müller and Serot [1], this retrograde condensation is unique to the binary system and does not occur in one-component systems.

3. Suppose that the initial $\alpha$ is located at the region $\alpha_1 < \alpha < \alpha_3$. Since $\alpha_1$ and $\alpha_3$ correspond to the maximum and minimum of $\mu_\alpha$, we find $(\partial\mu_\alpha/\partial\alpha)_{T,P} < 0$ in this region and the stability condition Eq. (8) will be destroyed. The system begins at gas phase, enters a two-phase region and becomes unstable at the limit pressure.

4. If the initial $\alpha$ is located at the region $\alpha_3 < \alpha < \alpha_2$, the behaviour of the system is similar to that of the case (3), except it will be ended to a stable phase at the limit pressure because the stability condition $(\partial\mu_\alpha/\partial\alpha)_{T,P} > 0$ is satisfied.

In summary, we have shown that the density dependence of effective NN coupling is important to the LG phase transition. A limit pressure $p_{\text{lim}}$ has been found for a fixed temperature and the LG phase transition cannot take place in asymmetric nuclear matter provided $p > p_{\text{lim}}$. Of course, for a fixed pressure, we can also get a limit temperature. This conclusion is similar to that of the Coulomb instability [2–4] of nuclei. The basic difference is that instead of finite nuclei, our conclusion has be found to be suitable for asymmetric nuclear matter. Finally, we would like to emphasize that the isospin is very important for the LG phase transition of asymmetric nuclear matter.

Acknowledgements

The work was support in part by the National Natural Science Foundation of China under contract No. 19975010, and the Foundation of Education Department of China.

References

Deeply virtual Compton scattering amplitude in the parton model

M. Penttinen\textsuperscript{a}, M.V. Polyakov\textsuperscript{a,b,*}, A.G. Shuvaev\textsuperscript{b}, M. Strikman\textsuperscript{c}

\textsuperscript{a} Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
\textsuperscript{b} Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia
\textsuperscript{c} Department of Physics, Pennsylvania State University, University Park, PA 16802, USA

Received 17 July 2000; revised 2 September 2000; accepted 15 September 2000

Abstract

We compute amplitude of deeply virtual Compton scattering in the parton model. We found that the amplitude up to the accuracy $O(1/Q)$ depends on new skewed parton distributions (SPDs). These additional contributions make the DVCS amplitude explicitly transverse.

\copyright 2000 Published by Elsevier Science B.V.

1. Introduction

Hard exclusive processes, such as deeply virtual Compton scattering (DVCS), owing to the QCD factorization theorems \cite{1-4} allow to probe so-called skewed parton distributions \cite{1,3,5-7}.

In this note we shall study DVCS amplitude in the parton model, our prime interest will be the question what kind of skewed parton distributions enter DVCS amplitude up to the order $O(1/Q)$. Our calculations follow closely ideas of Ref. \cite{8}. We compute the DVCS amplitude in the parton model, where the scattering of virtual photon occurs on a single parton on the mass-shell. In this approach the scattering amplitude is explicitly gauge invariant (transverse). We show that in the parton model in the case of pion target we reproduce results of Ref. \cite{8}. Also we give an expression for the DVCS amplitude for an arbitrary target.

We shall see that even in the simplest parton model the handbag contribution to the DVCS amplitude depend not only on the matrix elements of the light-ray operators with indices projected on the light cone direction but also on the operators projected on the transverse direction. The corresponding additional contributions make the DVCS amplitude explicitly transverse. The additional terms in the DVCS amplitude, noticed and discussed recently in Ref. \cite{9}, are proportional either to the transverse component of the momentum transfer or the transverse component of the target polarization, as it follows from the recent operator analysis of Ref. \cite{9}.

Let us emphasize that calculations in the parton model we perform here correspond to the QCD calculation of the handbag diagram in time ordered perturbation theory in the infinite momentum frame. Therefore, our calculations are model independent despite the use of the approximation of the parton model. The advantage of such formalism is that at each step of the
calculations the amplitude is transverse (e.m. gauge invariant). Of course, the calculations in the parton model do not allow us to determine the contributions of the operators of the type \( \bar{\psi} G \psi \) to the amplitude. In order to determine contributions of such operators one has to perform the QCD operator analysis.

2. DVCS amplitude

Here we give an expression for the handbag contribution to the DVCS amplitude computed in the parton model. The corresponding diagrams are shown in Fig. 1. It is convenient to introduce the light-cone decomposition of the momenta:

\[
p^\mu = (1 + \xi)\tilde{n}^\mu + (1 - \xi)\frac{M^2}{2}n^\mu - \frac{1}{2}\Delta^\mu,\]

\[
p^\mu = (1 - \xi)\tilde{n}^\mu + (1 + \xi)\frac{M^2}{2}n^\mu + \frac{1}{2}\Delta^\mu,\]

\[
q^\mu = -2\xi \tilde{n}^\mu + \frac{Q^2}{4\xi}n^\mu,\]

\[
\Delta^\mu = (p' - p)^\mu,\]

\[
\bar{M}^2 = M_N^2 - \frac{\Delta^2}{4}.\tag{1}
\]

Here \( \tilde{n}^\mu \) and \( n^\mu \) are arbitrary light-cone vectors such that \( \tilde{n} \cdot n = 1 \) and \( \tilde{n} \cdot \Delta = n \cdot \Delta = 0 \). The momentum \( k \) can be decomposed as follow:

\[
k^\mu = x\tilde{n}^\mu + \beta n^\mu + k^\mu,\tag{2}
\]

where \( \beta \) is fixed by the mass-shell conditions for the partons. Let us note that we do not need here the explicit expression for \( \beta \) because it contributes to the DVCS amplitude at the order \( O(\Delta^2/Q^2) \). We are interested here in the order \( O(\Delta^2/Q^2) \) and \( M^2/Q^2 \) one gets the following expression for the DVCS amplitude in the parton model:

\[
M^{\mu\nu} \propto \int_{-1}^{1} dx \int \frac{d^2k_\perp}{(2\pi)^2} \int d^4z \, \delta(n \cdot z) \times \frac{1}{(x - \xi)(x + \xi)} \exp[i(k \cdot z)] \times \left\{ \text{Tr} \left( \left( k + \frac{1}{2}\Delta \right) \Gamma^{\mu\nu} \left( k - \frac{1}{2}\Delta \right) \bar{\psi} \psi \right) \right\} \\
+ \frac{1}{x} \frac{1}{(x - \xi)(x + \xi)} \text{Tr} \left( \left( k + \frac{1}{2}\Delta \right) \Gamma^{\mu\nu} \left( k - \frac{1}{2}\Delta \right) \bar{\psi} \gamma_5 \psi \right) \times \left\langle p' | \tilde{\psi} (-z/2) \gamma_5 \psi (z/2) | p \right\rangle \tag{3}
\]

where

\[
\Gamma^{\mu\nu} = \frac{\gamma^\mu (k - \Delta/2 + \frac{\delta}{2}) \gamma^\nu}{(k - \Delta/2 + q)^2 + i0} + \frac{\gamma^\nu (k + \Delta/2 - \frac{\delta}{2}) \gamma^\mu}{(k + \Delta/2 - q)^2 + i0}. \tag{4}
\]

Obviously, the amplitude (3) is transverse, i.e. \( (q - \Delta)^\mu M^{\mu\nu} = q^\nu M^{\mu\nu} = 0 \). In the limit \( Q^2 \to \infty \) and \( \Delta^2 \ll Q^2 \) the expression (3) can be rewritten as:

\[
M^{\mu\nu} = \frac{1}{2} \int_{-1}^{1} dx \left\{ \alpha^+(x, \xi) \times \left[ (\tilde{n}^\nu n^\mu + n^\nu \tilde{n}^\mu - g^\mu\nu) - \frac{4\xi}{Q^2} \Delta_\perp \tilde{n}^\mu \right] n^\sigma F_\sigma \right. \\
+ i\alpha^-(x, \xi) \left[ \epsilon_\perp^{\mu\nu} \tilde{n}^\mu n^\nu \right] n^\sigma F_\sigma^{(5)} \right. \\
+ \left. \left[ n^\nu + \frac{8\xi^2}{Q^2} \tilde{n}^\nu \right] n^\sigma F_\sigma^{(5)} \right. \\
+ i\alpha^-(x, \xi) \left( \tilde{n}^\mu (\Delta_\perp \cdot F) \right) \\
+ \left. \left[ n^\mu + \frac{8\xi^2}{Q^2} \tilde{n}^\mu \right] (\Delta_\perp \cdot F) \right) \\
+ \left. \left[ \alpha^+(x, \xi) \right. \left( F^{\mu\nu} - \frac{4\xi}{Q^2} \tilde{n}^\nu (\Delta_\perp \cdot F) \right) \\
+ \left. i\alpha^-(x, \xi) \left( \epsilon_\perp^{\mu\nu} F_\rho^{(5)} - \frac{4\xi}{Q^2} \tilde{n}^\nu \epsilon_\perp^{\mu\rho} F_\rho^{(5)} \right) \right) \right. \right. \\
+ \left. \left. n^\mu \alpha^+(x, \xi) F^{\mu\nu} - \alpha^- (x, \xi) \epsilon_\perp^{\mu\nu} F_\rho^{(5)} \right) \right) \\
+ O(\Delta^2/Q^2). \tag{5}
\]
Note that the last line in the above equation give contribution to the amplitude to the order $O(\Delta^2/Q^2)$ when index $\mu$ is contracted with polarization vector of real outgoing photon.\footnote{We are grateful to M. Diehl for this remark.}

In Eq. (5) $\varepsilon^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} \not{n}_\alpha \not{n}_\beta$. We also defined:

$$\alpha^\pm(x, \xi) = \pm \frac{1}{x - \xi + i0} \frac{1}{x + \xi - i0}.$$  

Skewed distributions $F_\mu$ and $F_\mu^{(5)}$ are defined in terms of bilocal quark operators on the light-cone:\footnote{The path ordered gauge link is assumed here. Although in the parton model we do not have interactions with the gluon fields the corresponding $P$-exponential can be obtained using eikonal approximation for the quark propagator in the external gluon field.}

$$F_\mu = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi} \left( - (\lambda/2)n \right) \gamma_\mu \psi \left( (\lambda/2)n \right) | p \rangle,$$

$$F_\mu^{(5)} = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi} \left( - (\lambda/2)n \right) \gamma_\mu \gamma_5 \psi \left( (\lambda/2)n \right) | p \rangle.$$  

Generically, the expression (3) contains operators off the light-cone, but all of them can be reduced to the operators (6) using obvious operator identities like:

$$\bar{\psi}_N(x) = i e^{\alpha \beta \rho \sigma} \bar{\psi} \left( y_0 \right) \gamma_\rho \gamma_5 \gamma_\sigma \psi \left( y_0 \right),$$

which are satisfied owing to the QCD equation of motion $\bar{\psi} \gamma^\mu \psi \left( x \right).$

Note that two first terms in the expression (5) coincide exactly with the improved DVCS amplitude used in [10]. This part of the amplitude is characterized by leading-twist skewed parton distributions $n \cdot F$ and $n \cdot F^{(5)}$, other contributions are characterized by new additional skewed parton distributions $F_{\mu \pm}$ and $F_{\mu \pm}^{(5)}$. The latter functions are suppressed in differential cross section of the reaction $e + N \rightarrow e' + \gamma + N$ by only one power of the hard scale $1/Q$ [12,13], therefore their estimates are important for the analysis of DVCS observables. In principle, considering azimuthal angle and $Q$ dependencies of the various spin and charge asymmetries one should be able to disentangle the contributions of the additional SPDs from the leading twist ones [12]. Note also that in certain spin and azimuthal asymmetries the new functions enter at the same order in $1/Q$ as the leading twist one, as examples of such quantities are $\sin(2\phi)$ term in the lepton spin asymmetry, and $\text{const}$ and $\cos(2\phi)$ term in the lepton charge asymmetry. The measurements of such observables would give us an information on the size of new SPDs.

It is easy to check that the amplitude (5) explicitly satisfies the transversality conditions $(q - \Delta)^\mu M^{\mu\nu} = q^\nu M^{\mu\nu} = 0$. In the case of the pion target our expression (5) coincides with the result obtained by Anikin et al. [8]. For the nucleon target we can write for skewed parton distributions $F_\mu$ and $F_\mu^{(5)}$, for instance, the following decomposition:

$$F_\mu = \bar{u}(p') \left\{ \gamma^{\mu} H \left( x, \xi, \Delta^2 \right) + \frac{\sigma^{\mu\nu} \Delta_\nu}{2M_N} \not{E} \left( x, \xi, \Delta^2 \right) \right\} u(p),$$

$$F_\mu^{(5)} = \bar{u}(p') \left\{ \gamma^{\mu} \gamma_5 \tilde{H} \left( x, \xi, \Delta^2 \right) + \frac{\sigma^{\mu\nu} \Delta_\nu}{2M_N} \not{E} \left( x, \xi, \Delta^2 \right) \right\} u(p),$$

where ellipses stand for terms which do not contribute to the order $1/Q$. The two first functions in above equations coincide with the distributions introduced in Ref. [7]. The functions $G_i(x, \xi, \Delta^2)$ are additional functions parameterizing the nucleon matrix element of quark light-cone operator. Obviously, these new functions satisfy the following sum rule:

$$\int_{-1}^{1} dx \tilde{G}_i(x, \xi, \Delta^2) = 0,$$

$$\int_{-1}^{1} dx G_i(x, \xi, \Delta^2) = 0.$$  

The additional function $G_1(x, \xi, \Delta^2)$ receive a contribution from the so-called D-term in the double distributions parametrization of light-ray operators [11].
This contribution to the function $G_1(x, \xi, \Delta^2)$ has the form:

$$G_1(x, \xi, \Delta^2) = \frac{1}{2\xi} D \left( \frac{x}{\xi}, \Delta^2 \right).$$

In the forward limit the SPDs $F_\mu$ and $F_\mu^{(5)}$ are:

$$F_\mu \to 2\left\{ f_1(x) \bar{n}_\mu + M_{Q_3}^2 f_3(x) n_\mu \right\},$$

$$F_\mu^{(5)} \to 2\left\{ g_1(x) \bar{n}_\mu \left( n \cdot S + g_{T}(x) S_{\perp\mu} \right)
+ g_3(x) M_{Q_5}^2 n_\mu \right\}.$$

Hence, we observe that in the forward limit the SPDs $F_\mu$ and $F_\mu^{(5)}$ are reduced to the combinations of twist-2 parton distributions and higher twist distributions $f_3(x)$, $g_3(x)$ and $g_{T}^{\text{tw}3}(x)$. The functions $F_\mu$ and $F_\mu^{(5)}$ can be computed in the chiral quark–soliton model using methods of Ref. [14].

Very interesting sum rules can be derived if we consider the second Mellin moments of the distributions $G_1$ and $\tilde{G}_1$. With help of identities like (7) one can derive relations between the second Mellin moments. Here we restrict ourselves to the following relations:

$$\int_{-1}^{1} dx x G_3(x, \xi) = -\frac{1}{2} \int_{-1}^{1} dx \left[ H(x, \xi) + E(x, \xi) \right]$$
$$+ \frac{1}{2} \int_{-1}^{1} dx \tilde{H}(x, \xi),$$

$$\int_{-1}^{1} dx x \tilde{G}_2(x, \xi) = -\frac{1}{2} \int_{-1}^{1} dx \operatorname{H}(x, \xi)$$
$$+ \frac{\xi^2}{2} \int_{-1}^{1} dx \left[ H(x, \xi) + E(x, \xi) \right].$$

The first relation in the forward limit can be related to the quark orbital momentum because:

$$\lim_{\Delta^2 \to 0} \int_{-1}^{1} dx x G_3(x, \xi) = -J_q + \frac{1}{2} \Delta q = -L_q.$$  

The second relation (12) is the non-forward generalization of the Efremov–Leader–Teryaev sum rule [16], it reduces to the ELT sum rule in the forward limit, because in the forward limit the SPDs $\tilde{H}$ and $\tilde{G}_2$ are reduced to the spin distributions $g_1$ and $g_2$, correspondingly.

As the final remark we note that the new functions $G_1$ and $\tilde{G}_1$ can be related to the leading twist functions $H$, $\tilde{H}$ and $E$, $\tilde{E}$ if one neglects the contributions of operators of type $\bar{\psi} G \psi$. The method of derivation is analogous to derivation of the Wandzura–Wilczek type of relations between twist-2 and twist-3 rho-meson distributions amplitudes [15]. Recently such relations were derived in Ref. [13].

3. Discussion

We have demonstrated that the handbag contribution to the DVCS amplitude up to the order $1/Q$ depends not only on the leading twist skewed parton distributions $(n \cdot F)$ and $(n \cdot F^{(5)})$ but also on the additional functions $F_{\rho_1}$ and $F_{\rho_1}^{(5)}$. These additional contributions are suppressed by only one power of the hard scale $1/Q$ in the differential cross section. The estimates of these contributions are important for the extraction of the leading twist skewed parton densities $H$, $\tilde{H}$ and $E$, $\tilde{E}$ from observables [17] and measuring the Ji’s sum rule [7]. Also it is encouraging that the new functions can be also related to the spin structure of the nucleon, see Eqs. (12), (13). This can give additional possibility to extract quark orbital momentum from DVCS observables.

Note also that in our analysis we used the approximation $\Delta^2 \ll Q^2$, natural for the parton model. An account of the QCD evolution may lead effectively to a breakdown of this approximation and further complicate description of DVCS. Indeed, in the QCD ladder virtualities of the partons in the rungs of the ladder close to the nucleon are much smaller than $Q^2$. They are rather close to $Q_0^2$. Hence the accuracy of neglecting $\Delta^2$ in these propagators is $\sim \Delta^2 / Q_0^2$, not $\sim \Delta^2 / Q^2$. The effects due to evolution of distributions $F_{\mu \perp}$ and $F_{\mu \perp}^{(5)}$ will be studied elsewhere.
Acknowledgements

We are grateful to V.Yu. Petrov for sharing with us his ideas and valuable contributions to calculations. Discussions with L. Frankfurt, K. Goeke, N. Kivel, D. Müller, P.V. Pobylitsa, A. Radyushkin, A. Schäfer, J. Soffer, O. Teryaev, M. Vanderhaeghen are greatly acknowledged. M.V.P. is thankful to C. Weiss for many interesting discussions. Critical remarks of M. Diehl and N. Kivel were of big help for us.

References

Light fermion finite mass effects in non-relativistic bound states

Dolors Eiras\textsuperscript{a,a,}, Joan Soto\textsuperscript{a,b}

\textsuperscript{a} Departamento d’Estructura i Constituents de la Matèria and IFAE, U. Barcelona, Diagonal 647, E-08028 Barcelona, Catalonia, Spain
\textsuperscript{b} Department of Physics, University of California at San Diego, 9500 Gilman Drive, La Jolla, CA 92093, USA

Received 22 May 2000; received in revised form 6 July 2000; accepted 24 August 2000

Editor: R. Gatto

Abstract

We present analytic expressions for the vacuum polarization effects due to a light fermion with finite mass in the binding energy and in the wave function at the origin of QED and (weak coupling) QCD non-relativistic bound states. Applications to exotic atoms, $^{1\text{s}}s$ and $^{1\text{t\text{N}}}$ production near threshold are briefly discussed. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 11.10.St; 13.25.Gv; 36.10.-k

1. Introduction

It is well known that the vacuum polarization effects due to light fermions produce leading corrections to the Coulomb-like behavior of non-relativistic bound states both in QED [1] and in QCD [2]. If a light particle has a mass ($m_f$) of the order of the inverse Bohr radius, it can neither be approximated by a massless particle nor by a very heavy one. Hence any observable related to the bound state depends non-trivially on $m_f$. For a given bound state the $m_f$ dependence in physical observables is usually calculated numerically [1,3 – 8] and only a few analytical results are available [9 – 12].\textsuperscript{1} In this letter we present further analytical formulas with the exact light fermion mass dependence for the leading corrections to the energy shift for arbitrary quantum numbers $(n, l)$ and to the wave function at the origin for the ground state.

These formulas may be useful for quite a few physical systems of current interest. Any QED bound state built out of particles heavier than the electron may require them. For instance di-muonium, muonic hydrogen, pionium, pionic hydrogen and other simple hadronic atoms where the electron mass ($m_e$) is such that $m_e \sim \mu \alpha /n$, $\mu$ and $n$ being the reduced mass and the principle quantum number respectively. It is worth mentioning that simple hadronic atoms are experiencing a renewed interest because they may allow to extract important information on the QCD scattering lengths for several isospin channels [13]. In particular the measurement of the decay width of pionium [14] at the 10% level will allow to extract a combination of scattering lengths with sufficient

\textsuperscript{1} We learnt about the three last references after completing our calculations.
accuracy as to discern between the large and the small quark condensate scenario of QCD, namely between Chiral Perturbation Theory [15] and Generalized Chiral Perturbation Theory [16]. In fact, the leading corrections to the pionium decay width have been recently calculated in a systematic way using non-relativistic effective field theory techniques [9,17] (see [18], [5,19] and [20] for earlier relativistic, non-relativistic, and quantum mechanical calculations respectively). Fully analytic results have been obtained except for the contribution of the electron mass to the vacuum polarization where only a numerical result is available [5]. We shall fill this gap here and present the only remaining piece to have the full leading corrections to the pionium decay width in an analytic form.

For QCD non-relativistic bound states, \( \Upsilon(1s) \) seems to be the only one amenable to a weak coupling analysis [21]. Using the results of [9], analytic expressions for the binding energy shift due to the finite charm mass have been recently presented in [22]. We present here analytic results for these effects in the wave function at the origin.

An important non-relativistic weak coupling QCD system for the Next Linear Collider physics is the top quark–antiquark pair near threshold. The production cross-section has already been calculated at NNLO [24]. However, the calculations are done assuming the mass of the bottom and charm quarks zero. Our results may allow to include the leading effects of these masses.

The fact that our results can be applied to such a variety of a priori quite different physical systems can be easily understood in terms of modern effective field theories. Non-relativistic bound states have at least three dynamical scales: the hard scale (mass of the particles forming the bound state), the soft scale (typical relative momentum in the bound state) and the ultrasoft scale (typical binding energy in the bound state). Upon integrating out the hard scale local non-relativistic effective theories arise. These are NRQED for QED, NRQCD for QCD [25], and \( \text{NR}_{\chi L} \) for the chiral Lagrangian\(^3\) [26]. Upon integrating out the soft scale (see [28,29] for the precise statements) effective theories which are local in time but non-local in space arise. The non-local terms in space are nothing but the usual quantum-mechanical potentials and only ultrasoft degrees of freedom\(^4\) are left dynamical. The corresponding non-relativistic effective theories have been named pNRQED, pNRQCD and pNR\(\chi L\) for QED, QCD and the Chiral Lagrangian respectively [26,30], (the “p” stands for potential). Since the leading (mass independent) coupling of the photon field to the non-relativistic charged particles as well as the one of the gluon field to the non-relativistic quarks in the NR theories is universal, it produces the same potential in the pNR theories, and hence they all can be discussed at once. If there is a light (relativistic) charged particle in QED or a light (relativistic) quark in QCD whose mass is of the order of the soft scale, it must be integrated out keeping the mass dependence exact, which produces a light fermion mass dependent correction to the static potential.

When matching the NR theories to the pNR theories only the diagram of Fig. 1 gives rise to a potential which contributes to the leading effect. For QED (on-shell scheme) it reads,

\(\bar{h} h \to \bar{l} l \to h' h'\)

Fig. 1. Matching between the non-relativistic theory and the potential one.

---

\(^2\) The extraction of the bottom \(\overline{MS}\) mass from the \(\Upsilon(1s)\) mass also requires the bottom pole mass dependence on \(m_c\) [23].

\(^3\) We apologize to the authors of Ref. [5] for slightly modifying their previously given name, namely Non-Relativistic Chiral Perturbation Theory. The reason is that the calculation in the non-relativistic regime cannot be organized according to the chiral counting any longer [9].

\(^4\) In the language of the threshold expansions in QCD [27] these correspond to ultrasoft gluons and potential quarks.
\[ V_{\text{QED}}(x) = -\frac{\alpha(v)}{\pi |x|} \int_0^1 dv \frac{v^2 (1 - v^2/3)}{(1 - v^2)} e^{-\frac{m_{|x|}}{\sqrt{1 - v^2}}} \] (1)

and for QCD (MS):
\[ V_{\text{QCD}}(x) = -\frac{C_F T_F \alpha_s}{\pi |x|} \int_0^1 dv \frac{v^2 (1 - v^2)}{(1 - v^2)} e^{-\frac{m_{|x|}}{\sqrt{1 - v^2}}} + \frac{1}{3} \log \left( \frac{m_{|x|}^2}{v^2} \right), \]
\[ C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}, \quad T_F = \frac{1}{2} \] (2)

If \( N_f \) is the number of flavors lighter than \( m_1 \), the \( \alpha_s(v) \) above runs with \( N_f + 1 \) flavors. Notice that the difference between the QED and the QCD case is, apart from the trivial color factors \( C_F \)s = \( 1 \) and \( T_F \)s = \( 1 \), a term which can be absorbed in a redefinition of the Coulomb potential [31]. Hence for the actual calculation we shall only deal with (1) and use these facts to extend our results to the QCD realm.

2. Energy shift

For the energy shift we obtain (\( \xi := \frac{m_{|x|}}{\mu} \)):
\[ \delta E_{\text{nl}}(\xi) = -\frac{2\alpha}{3\pi} E_n \left\{ \frac{5}{3} - \frac{3\pi}{2} n \xi + (n(2n + 1) + (n + l)(n - l - 1)) \xi^2 \right. \]
\[ - \pi n \left( \frac{1}{3} (n + 1)(2n + 1) + (n + l)(n - l - 1) \right) \xi^3 \]
\[ \left. - \frac{1}{(2n - 1)!} \sum_{k=0}^{n-l-1} \left( \begin{array}{c} n - l - 1 \\ k \end{array} \right) \left( \begin{array}{c} n + l \\ 2l + 1 + k \end{array} \right) \xi^{2(n-l-1-k)} \times \frac{d^{2n-1}}{d\xi^{2n-1}} \left[ (2 - \xi^2)^{n/2}\right] F_1(\xi) \right\}, \] (3)
\[ F_1(\xi) := \begin{cases} \frac{1}{\sqrt{\xi^2 - 1}} \arccos \frac{1}{\xi} & \text{if } \xi > 1, \\ 1 & \text{if } \xi = 1, \\ \frac{1}{\sqrt{1 - \xi^2}} \text{log} \left[ \frac{1 + \sqrt{1 - \xi^2}}{\xi} \right] & \text{if } \xi < 1, \end{cases} \]

where \( E_n = -\mu^2/2n^2 \) is the Coulomb energy. For \( \xi \) large, namely \( m_l \gg \mu \alpha/n \), it reduces to
\[ \delta E_{\text{nl}}(\xi \to \infty) \to \frac{2\alpha}{3\pi} E_n \left\{ \frac{(n+l)!}{(n-l-1)!(2l+1)!} \right. \]
\[ \times \left. \left( \begin{array}{c} 2l + 2 \\ 2l + 3 \end{array} \right) \frac{(2l+2)(2l+4)}{(2l+3)(2l+5)} + O\left( \frac{1}{\xi^2} \right) \right\}, \] (4)

whereas for \( \xi \) small, namely \( m_l \ll \mu \alpha/n \), we obtain
\[ \delta E_{\text{nl}}(\xi \to 0) \to \frac{2\alpha}{3\pi} E_n \left\{ \frac{5}{3} + 2(\psi(n+l+1) - \psi(1)) - 2 \log \frac{2}{\xi} \right\} \]
\[
- \frac{3\pi}{2} n^2 + \frac{3}{2} (n(2n+1) + (n+l)(n-l-1)) \xi^2 \\
- \pi n \left( \frac{1}{3} (n(2n+1) + (n+l)(n-l-1)) \xi^3 + O(\xi^4) \right),
\]

where we have used (A.4). The key steps to obtain (3) are given in the Appendix B. We have done the following checks. For the 1S state (3) reduces to the formula (5.3) of Ref. [9]. The energy shifts for the 1S, 2S, 2P, 3S, where we have used (A.4). The key steps to obtain (3) are given in the Appendix B. We have done the following formula (2.8) of [28]). We also agree for large enormous cancellations occur in formula (3) and hence the analytic expansion (4) may prove very useful.

3. Wave function at the origin

The correction for the wave function at the origin (see Fig. 2) for \( n = 1 \) states reads

\[
\delta \Psi_{10}(0) = -\frac{\alpha}{\pi} \frac{\xi}{F_{10}(0)} \left[ \frac{5}{9} - \frac{\pi}{4} \frac{\xi}{\sin \theta} + \frac{1}{3} \xi^2 - \frac{\pi}{6} \xi^3 + \frac{1}{3} (\xi^4 + \xi^2 - 2) F_1(\xi) \right] \\
+ \frac{11}{18} - \frac{2}{3} \frac{\xi^2}{\sin \theta} + \frac{2\pi}{3} \xi^3 - \frac{1}{6} (12\xi^4 + \xi^2 + 2) F_1(\xi) \\
- \frac{1}{6} \frac{4\xi^4 + \xi^2 - 2}{(\xi^2 - 1)} (1 - \xi^2 F_1(\xi)) \\
+ \frac{2}{3} + \frac{\pi}{4} \frac{\xi}{\sin \theta} - \frac{1}{9} \xi^2 + \frac{13\pi}{18} \xi^3 - \frac{1}{9} (13\xi^4 - 11\xi^2 - 11) F_1(\xi) \\
- \frac{1}{3} (4\xi^3 + 3\xi) F_2(\xi) + \frac{1}{3} (4\xi^4 + \xi^2 - 2) F_3(\xi) + \frac{1}{3} \left( \frac{4\xi^2 + 11}{3} \right) \log \frac{\xi}{2},
\]

where \( \Psi_{10}(x) \) is the Coulomb wave function. The first bracket corresponds to the zero photon exchange and has already been calculated analytically in [5]. The second and third brackets correspond to the pole subtraction and multi-photon exchange contributions respectively. \( F_i(\xi), i = 2, 3 \) are defined as follows:

\[
F_2(\xi) = \int_0^{\pi/2} d\theta \log \frac{\sin \theta + \xi}{\sin \theta}, \quad F_3(\xi) = \int_0^{\pi/2} d\theta \frac{1}{\sin \theta + \xi} \log \frac{\sin \theta + \xi}{\sin \theta}.
\]

\( F_2(\xi) \) and \( F_3(\xi) \) can be expressed in terms of Clausen integrals and dilogarithms. We present the explicit formulas in Appendix A. The key steps in order to obtain (6) are given in Appendix B.

For \( \xi \) large, namely \( m_l \gg \mu \alpha, \) (6) behaves like

\[
\delta \Psi_{10}(0)_{\xi \to \infty} \to \frac{\alpha}{\pi} \frac{\pi}{16} \left[ \frac{3\pi}{16} + \frac{107}{225} \xi^2 + \frac{4}{15} \log \frac{\xi}{2} + O\left( \frac{1}{\xi^3} \right) \right].
\]

This result must be compatible with the one obtained by integrating out the light fermion first and then calculating the electromagnetic potential. In the case \( m \gg m_l \gg \mu \alpha/n \) (for simplicity, we are assuming \( h = h', m \) being

\( \epsilon_m = 0 \) in that reference and upon correcting an obvious misprint \( \kappa_l \to \kappa_m. \)
the mass of the non-relativistic particles) we expect that a local non-relativistic effective theory is obtained after integrating out the energy scale $m_I$ and the associated three momentum scale $\sqrt{m_I}$ for the non-relativistic particle. The leading term in (8) corresponds to the contribution that would be obtained from the local term induced by the diagram in Fig. 3. The logarithm in the subleading term corresponds to the iteration of two delta function potentials in quantum mechanics (see formula (5.5) in [9]). The second delta function is due to the contribution to the electromagnetic potential of the dimension six photon operator (see [32]) which arises after integrating out a heavy particle [33].

For $\xi$ small, namely $m_I \ll \mu \alpha$, we obtain

$$
\delta \psi_{10}(0)(\xi \rightarrow 0) \sim -\frac{\alpha}{\pi} \psi_{10}(0) \left[ \frac{3}{2} - \frac{2}{9} \log \frac{\mu}{\xi} - \frac{3}{2} \xi^2 + O(\xi^3) \right].
$$

(9)

We have made the following checks. For $\xi$ large and small, the leading term of (8) and (9) agree with formulas (22) and (23) of Ref. [4] respectively (the next-to-leading terms are not displayed in [4]). For $\xi$ small we can also compare with known results for massless quarks in QCD. We agree with the $O(\alpha_s)$ correction of formula (69) of Ref. [35]. We have also checked that the formula (6) reproduces the numerical results obtained for di-muonium and pionium in Refs. [11] and [5] respectively, and we also agree numerically with the analytical result in terms of a non-trivial integral of Ref. [4].

4. Applications

4.1. Exotic atoms

We have listed in Tables 1 and 2 the corrections to some energy splittings and to the wave function at the origin respectively of simple exotic atoms of current interest. This purely electromagnetic corrections must be conveniently taken into account if one wants to obtain precise information of the strong scattering lengths from hadronic atoms.
Table 1
Vacuum polarization induced energy splittings for some exotic atoms

<table>
<thead>
<tr>
<th></th>
<th>$\Delta E_{21}^{E_2} - \Delta E_{10}^{E_1}$</th>
<th>$\Delta E_{10}^{E_1} - \Delta E_{21}^{E_2}$</th>
<th>$\Delta E_{31}^{E_3} - \Delta E_{20}^{E_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pK^-$</td>
<td>0.44593</td>
<td>0.38629</td>
<td>-0.10453</td>
</tr>
<tr>
<td>$p\pi^-$</td>
<td>0.18103</td>
<td>0.15388</td>
<td>-0.056337</td>
</tr>
<tr>
<td>$p\mu^-$</td>
<td>0.13616</td>
<td>0.11548</td>
<td>-0.044443</td>
</tr>
</tbody>
</table>

Table 2
Vacuum polarization correction to the ground state wave function at the origin of some exotic atoms

<table>
<thead>
<tr>
<th></th>
<th>$\xi = m/e$</th>
<th>$\Delta \langle \Psi(0) \rangle$</th>
<th>$\Delta \langle \Psi'(0) \rangle$</th>
<th>$\Delta \langle \Psi''(0) \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^- p$</td>
<td>0.21648</td>
<td>0.34290</td>
<td>0.15454</td>
<td>0.09650</td>
</tr>
<tr>
<td>$K^+ K^-$</td>
<td>0.28369</td>
<td>0.29837</td>
<td>0.12958</td>
<td>0.08785</td>
</tr>
<tr>
<td>$\pi^- p$</td>
<td>0.57635</td>
<td>0.19613</td>
<td>0.07285</td>
<td>0.06166</td>
</tr>
<tr>
<td>$K^+ \pi^-$</td>
<td>0.64357</td>
<td>0.18237</td>
<td>0.06549</td>
<td>0.05741</td>
</tr>
<tr>
<td>$\mu^- p$</td>
<td>0.73738</td>
<td>0.16627</td>
<td>0.05703</td>
<td>0.05222</td>
</tr>
<tr>
<td>$\pi^+ \pi^-$</td>
<td>1.00344</td>
<td>0.13338</td>
<td>0.04052</td>
<td>0.04099</td>
</tr>
<tr>
<td>$\mu^+ \mu^-$</td>
<td>1.32550</td>
<td>0.10793</td>
<td>0.02876</td>
<td>0.03184</td>
</tr>
</tbody>
</table>

Table 3
Vacuum polarization correction to wave function at the origin in quarkonia. MS has been used

<table>
<thead>
<tr>
<th></th>
<th>$\xi = m/e$</th>
<th>$\Delta \langle \Psi(0) \rangle$</th>
<th>$\Delta \langle \Psi'(0) \rangle$</th>
<th>$\Delta \langle \Psi''(0) \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{b}b$</td>
<td>1.4</td>
<td>0.088</td>
<td>0.011</td>
<td>0.0019</td>
</tr>
<tr>
<td>$\bar{t}t$</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{t}t$</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2. $\Upsilon(1s)$ and $t\bar{t}$

The current calculations of heavy quarks near threshold assume that the remaining lighter quarks are massless. This approximation is difficult to justify a priori at least in two cases. For the $\Upsilon(1s)$ system the typical relative momentum $m_b\alpha_s/2 \sim 1.3$ GeV [34] is of the same order as the charm mass $m_c \sim 1.5$ GeV. The effects of a finite charm mass in the binding energy have been recently quantified in [22]. We give in Table 3 the size of these effects in the wave function at the origin. For the $t\bar{t}$ production near threshold at a relative momentum $m_t\alpha_s/2 \sim 18$ GeV the effects of a finite bottom mass $m_b \sim 5$ GeV should be noticeable. In order to estimate them, we also show in Table 3 the size of this effect, both for bottom and charm, in the wave function at the origin for the would-be-toponium ($1s$) state.

If the corrections were organized in a series of $\alpha_s/\pi$ multiplied by numbers of order 1, one may conclude that the leading effects of a finite quark mass are: (i) in the $\Upsilon(1s)$ system for charm more important than the next to leading corrections [35]; (ii) in the $t\bar{t}$ system near threshold for bottom (charm) as important as (less important than) the next to leading corrections in $\alpha_s$ [24]. However, this follows from assuming that the size of the leading and next to leading corrections is $\alpha_s/\pi$ and $\alpha_s^2/\pi^2$ multiplied by numbers of order 1, respectively. In practise, these
numbers turn out to be very large and the next to leading corrections are comparable to the leading ones, even for the \( t\bar{t} \) system (see [36] for a discussion). This makes the actual size of the finite mass effects smaller than the next to leading order corrections in all the cases above.

**Acknowledgements**

We are indebted to Antonio Pineda for providing us with useful references, independent checks of various results in the literature and a critical reading of the manuscript. We also thank the referee for appropriated remarks on the relative size of the corrections in heavy quark systems. We are supported by the AEN98-031 (Spain) and the 1998SGR 00026 (Catalonia). D.E. acknowledges financial support from a MEC FPI fellowship (Spain) and J.S. from the BGP-08 fellowship (Catalonia) respectively. J.S. thanks A. Manohar and High Energy Physics Group at UCSD for their warm hospitality while this work was written up.

**Appendix A**

\( F_2(x) \) in (7) can be expressed in terms of Clausen integrals. We get

\[
F_2(x) = \begin{cases} 
2C_2(\arcsin x) - \frac{1}{4} C_2(2\arcsin x) & \text{if } x < 1, \\
2 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} \sim 1.831932 & \text{if } x = 1, \\
-i \text{Li}_2(-x - \sqrt{x^2 - 1}) + i \text{Li}_2(i(-x + \sqrt{x^2 - 1})) & \\
-i \text{Li}_2(-x + \sqrt{x^2 - 1}) + i \text{Li}_2(-i(x + \sqrt{x^2 - 1})) & \\
\frac{1}{2} \left( i\pi + 4 \log(2) + 4 \log(1 + ix - i \sqrt{x^2 - 1}) - 4 \log(1 + ix + i \sqrt{x^2 - 1}) \right) & \\
\frac{1}{2} \left( 2 - i \log \left( \sqrt{\frac{1-x}{2}} - \sqrt{\frac{1-x}{2}} \right) \right) - 2i \arctan \left( \frac{1-x}{\sqrt{1-x^2}} \right) & \\
+ \log(1 + x - \sqrt{x^2 - 1}) - \log(1 + ix - i \sqrt{x^2 - 1}) & \\
+ \log(1 + ix + i \sqrt{x^2 - 1}) - \log(1 + x + \sqrt{x^2 - 1}) & \text{if } x > 1.
\end{cases}
\]

(A.1)

Recall that the Clausen integral is defined as

\[
\text{Cl}_2(x) := -\int_0^x d\theta \log(2 \sin \frac{\theta}{2}) = i \frac{\pi^2}{6} - \frac{i}{4} x^2 - x \log(i e^{-ix}) - i \text{Li}_2(e^{ix}).
\]

(A.2)

\( F_3(x) \) in (7) can be expressed in terms of dilogarithms. We get

\[
F_3(x) = \frac{1}{\sqrt{1-x^2}} \left[ \text{Li}_2 \left( \frac{1+a+b}{2b} \right) - \text{Li}_2 \left( \frac{1+a-b}{2b} \right) + \text{Li}_2 \left( \frac{2b}{1+a+b} \right) \right] - \text{Li}_2 \left( \frac{2b}{1+a+b} \right) - \text{Li}_2 \left( \frac{1+a+b}{2b} \right) + \text{Li}_2 \left( \frac{1+a-b}{2b} \right) - \text{Li}_2 \left( \frac{2b}{a+b} \right) + \text{Li}_2 \left( \frac{a+b}{2b} \right) - \text{Li}_2 \left( \frac{(a+b)}{1+a+b} \right) - \text{Li}_2 \left( \frac{(a+b)}{1+a-b} \right) + \log(bx) \log \left( \frac{1+a+b}{1+a-b} \right), \quad \text{if } x < 1,
\]

where \( a := x^{-1} \) and \( b := \sqrt{1-x^2}/x \);
\[ F_3(1) = 2 - \log 2, \]
\[ F_3(x) = \frac{1}{i\sqrt{x^2 - 1}} \sum_{n=1}^{\infty} \frac{\log(x + n) + \log(x - n)}{x^2 - 1} \]
\[ \frac{F}{F_0} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{\log(x + n) + \log(x - n)}{x^2 - 1} \]

where \( a := x^{-1} \) and \( b := \sqrt{x^2 - 1} \).

In order to make contact with the expressions found in the literature for the massless limit of (3) the following formula is useful
\[ \psi(2n) - \frac{(n + l)!}{(2n - 1)!} \sum_{k=0}^{n-l-1} \frac{2(n - l - k - 1)(2k + 1)!}{(n - l - k - 1)!} \log(2 + 2k)! = \psi(n + l + 1). \]  

**Appendix B**

We sketch here the main steps which lead to our analytic formulas. For the energy shift we have to calculate \((v = \sqrt{1 - x^2})\)

\[ \delta E_{nl} = \frac{2\alpha E_n}{3\pi} \sum_{l=1}^{\infty} \left(\frac{n - l}{n - l - 1}\right) \frac{(n - l)!}{(2l + 1)!} \int_0^1 dx \frac{x^{2l+1}}{(x + \xi)^{2n-2l-2}} \sqrt{1 - x^2(2 + x^2)} \times F\left(\frac{1}{l - n - 1}, \frac{1}{n - l - 1}, 2l + 2; \frac{x^2}{\xi^2}\right). \]  

Since \( l < n \) the hypergeometric function above reduces to a polynomial

\[ F\left(\frac{1}{l - n - 1}, \frac{1}{n - l - 1}, 2l + 2; z\right) = \sum_{j=0}^{n-l-1} \frac{(2l + 1)(n - l - 1)!}{(n - l - j - 1)!} (j + 2l + 1)! \times \]

\[ \delta E_{nl} = \frac{2\alpha E_n}{3\pi} \sum_{l=0}^{n-l-1} \left(\frac{n - l}{n - l - 1}\right) \frac{(n + l)}{(n - j - 1)} \frac{(2l + 1)!}{(n - l - j - 1)!} \frac{(n - l - j - 1)!}{(n - l - j - 1)!} \frac{1}{x^{2l+2j+1}} \sqrt{1 - x^2(2 + x^2)}. \]  

and making the change \( x \rightarrow \sin \theta \) we obtain (3).
For the wave function at the origin we have to calculate:

\[
\delta\psi_{00}(0)\psi_{00}(0) = \lim_{E \to E_n} \langle n|0\rangle \delta(x) \left( \frac{1}{E - H} - \frac{|n0\rangle\langle n0|}{E - E_n} \right) V_{\text{ph}}|n0\rangle.
\]  

(B.5)

Upon using the following representation for the Coulomb propagator [37]

\[
\langle x|E-H|y\rangle = \sum_{l=0}^{\infty} G_l(x, y, E) \sum_{m=-l}^{l} Y_l^m \left( \frac{x}{x} \right) Y_l^m \left( \frac{y}{y} \right).
\]  

(B.6)

\[
G_l(x, y, E) = -4k\mu(2kx)^l(2ky)^l e^{-k(x+y)} \sum_{n'=1}^{\infty} \frac{L_{n'-1}^{2l+1}(2kx)L_{n'-1}^{2l+1}(2ky)}{(n' + l - \frac{n'n'}{k})^2} \Gamma(n' + 2l + 1),
\]  

(B.7)

where \( k^2 = -2\mu E \). (B.5) can be split into three pieces

\[
\delta\psi_{00}(0) = \delta_{\text{ps}}\psi_{00}(0) + \delta_{\text{ph}}\psi_{00}(0) + \delta_{\text{mph}}\psi_{00}(0).
\]  

(B.8)

The first piece (pole subtraction) corresponds to the term \( n' = n \) in the sum (B.7) and it can be calculated using the formulas given above. The remaining pieces read

\[
\delta_{\text{ph}}\psi_{00}(0) + \delta_{\text{mph}}\psi_{00}(0)
\]

\[
= \frac{\alpha}{\pi} \psi_{00}(0) \int_0^1 dv \frac{v^2(1 - \frac{v^2}{\xi})^{n-1}}{(\xi + \sqrt{1 - v^2})^{n+1}} \sum_{n' = 1, n' \neq n}^{\infty} \frac{n'n'}{n'^2 - n^2} \left( \frac{\xi}{\xi + \sqrt{1 - v^2}} \right)^{n'-1} \times F \left( -(n'-1), -(n'-1); 2; \frac{1-v^2}{\xi^2} \right).
\]  

(B.9)

Again the hypergeometric function above reduces to a polynomial. For \( n = 1 \) it reduces in fact to 1 and the sum over \( n' \) can be carried out explicitly. We obtain:

\[
\delta_{\text{ph}}\psi_{10}(0) + \delta_{\text{mph}}\psi_{10}(0)
\]

\[
= \frac{\alpha}{\pi} \psi_{10}(0) \int_0^1 dv \frac{v^2(1 - \frac{v^2}{\xi})}{(\xi + \sqrt{1 - v^2})^2} \left( \frac{\xi}{\sqrt{1 - v^2}} \right)^2 + \log \left( \frac{\xi + \sqrt{1 - v^2}}{\sqrt{1 - v^2}} \right).
\]  

(B.10)

where the first term corresponds to the zero photon exchange and the second one to the multiphoton exchange. Again the change of variable \( v \to \cos \theta \) and a number of manipulations allow us to obtain (6) from the above.

References

BFKL gluon dynamics and resolved photon processes in $D^*$ and dijet associated photoproduction at HERA

S.P. Baranov $^a$, N.P. Zotov $^{a,b}$

$^a$ P.N. Lebedev Physics Institute, Leninsky prosp. 53, Moscow 117924, Russia
$^b$ D.V. Skobelzyn Institute of Nuclear Physics, M.V. Lomonosov Moscow State University, Moscow 119899, Russia

Received 16 June 2000; received in revised form 31 July 2000; accepted 6 September 2000

Abstract

In the framework of the semihard approach, we consider the production of $D^*$ mesons associated with two hadron jets at HERA conditions. The attention is focused on the variable $x_T$, which is the fraction of the photon momentum contributed to a pair of jets with largest $p_T$. We find that our theoretical calculations describe the data quite well, thus showing that the BFKL gluon dynamics can provide a reasonable explanation for the observed regularities.

© 2000 Elsevier Science B.V. All rights reserved.

PACS: 12.38.-t; 13.60.-r; 13.87.Ce

Keywords: Jet production; Semihard approach

1. Introduction

The experimental results on heavy flavour photoproduction processes obtained recently by the H1 [1] and ZEUS [2] collaborations at HERA provide a strong impetus for further theoretical studies. In due time, the experimental data have been compared with next-to-leading order (NLO) perturbative QCD calculations using the ‘massive’ and ‘massless’ schemes. The measured cross sections generally lie above the predicted level, and an agreement between the theoretical and experimental results can only be achieved using some extreme parameter values. In particular, the production rates of $D^*$ mesons in the NLO massive scheme [3] require as low quark mass as $m_c = 1.2$ GeV and as sharp charm fragmentation function as $\epsilon = 0.02$ (in the Peterson parametrization). However, even within this set of parameters, the shapes of the $D^*$ transverse momentum and rapidity distributions cannot be said well reproduced. A good agreement between the massless scheme [4] and the measured $p_T(D^*)$ (though not $\eta(D^*)$) spectrum was achieved upon introducing an additional charm excitation contribution assuming a remarkably large charm content in the photon structure functions [5]: $c(x) \approx u(x)$.

A different production scheme has been proposed in Ref. [6]. There, the production of $D^*$ mesons is considered as a nonrelativistic formation of a bound $c\bar{q}$ system (rather than as a $c$-quark fragmentation), with both $c\bar{c}$ and $q\bar{q}$ pairs produced perturbatively in an $O(\alpha_s^2)$ partonic subprocess. The authors took into account both colour-singlet and colour-octet $c\bar{q}$...
states, the latter transforming into a real $D^*$ meson via emitting an extra soft (nonperturbative) gluon. The probability for such a transition was regarded as free parameter fitted to the experimental data. The authors succeeded in describing the shape of the $D^*$ rapidity distribution, although the absolute normalization of the cross section remains rather uncertain because of its strong sensitivity to the light quark mass value, which is an ambiguous quantity.\(^\text{1}\)

The so-called $k_t$ factorization, or the semihard approach (SHA) \(\text{[7–10]}\), may give a reasonable solution for some of the above problems. For example, it leads to $p_T$ broadening effects in the $D^*$ \(\text{[11,12]}\) and $J/\psi$ \(\text{[13]}\) photoproduction spectra. These effects originate from the gluon transverse motion, which is the characteristic property of the semihard approximation. It has been also found in \(\text{[12]}\), in the framework of Monte-Carlo program CASCADE \(\text{[14]}\), that the CCFM \(\text{[15]}\) evolution scheme for the gluon densities leads to a satisfactory description of the $D^*$ pseudorapidity distributions.

To some extent, the BFKL \(\text{[16]}\) gluon dynamics includes the relevant effects of higher order contributions \(\text{[17,18]}\). It has been also demonstrated in \(\text{[19]}\) that the BFKL approach and the inclusion of the quark excitation via the resolved photon process in the LO DGLAP scheme lead to similar visible effects in the production of forward jets in DIS.

As a further test of the underlying parton dynamics, the ZEUS collaboration has measured the associated charm and dijet production \(\text{[2]}\). In these measurements, the quantity of interest is the fraction of the photon momentum contributing to the production of two jets with highest $E_T$, which is experimentally defined as

\[
x_y = \frac{E_{1T} \exp(-\eta_1) + E_{2T} \exp(-\eta_2)}{2E_y} \quad (1)
\]

with $E_{1T}$ and $\eta_i$ being the transverse energy and rapidity of these hardest jets.

\(\text{2. The semihard QCD approach}\)

The idea that the BFKL gluon evolution includes already a significant part of higher-order contributions is illustrated in more detail in Fig. 1. Shown there is a schematical representation of a typical three-jet photoproduction process viewed in three different approaches.

Fig. 1a represents the fixed-order perturbative chromodynamics; the upper part of the diagram (above the dash-dotted line) corresponds to the basic partonic subprocess $\gamma g \rightarrow q\bar{q}g$; the lower part of the diagram describes the evolution of gluon densities in a proton. As the incoming gluon and photon are assumed to have no transverse momentum in the parton model, the transverse momentum distributions of the produced jets are totally determined by the properties of the QED/QCD $O(\alpha s^2)$ matrix element.

\(\text{Fig. 1. A schematical representation of the three-jet photoproduction process in three different approaches: (a) fixed-order perturbative chromodynamics; (b) resolved photon; (c) semihard approach.}\)

---

\(^{1}\) The theoretical grounds for this kind of calculations seem a bit doubtful. The light quark mass value neither encourages making use of perturbative expansions in $\alpha_s$ nor justifies the nonrelativistic approximation for the $c\bar{q}$ bound state.
From the point of view of the ‘resolved photon’ mechanism (Fig. 1b), the basic parton interaction is the flavour excitation \( qg \rightarrow qg \), where the initial quark is regarded as a parton-like constituent of the photon with its momentum distribution given by the photon structure function. The properties of the final gluon jet are again determined by the hard \( O(\alpha_s^2) \) matrix element.

Finally, in the semihard approach (Fig. 1c), the underlying partonic subprocess is \( \gamma g \rightarrow q\bar{q} \), which is formally of order \( O(\alpha_s \alpha_s) \). Some extra powers of \( \alpha_s \) are hidden in the gluon evolution equations represented by the part of the diagram shown below the dash-dotted line. In contrast with the collinear approximation, the semi-hard parton interaction block, its evolution equation where the collinear gluon density is now determined by the properties of the evolution equation only. It is important to point out that the gluon radiated close to the quark box can even have larger transverse momentum than any of the two quarks involved in the hard subprocess [12].

All the considered approaches represent different ways of describing the same physical process, and it is the point of interest to see which of them is the closest to reality. Since the fixed-order QCD and the ‘resolved photon’ contributions have been already studied elsewhere [3,4] we concentrate on the semi-hard approach.

The relevant off-shell matrix elements for the photon–gluon fusion \( \gamma g \rightarrow c\bar{c} \) refer to the usual two Feynman diagrams:

\[
\mathcal{M} = \bar{u}(p_c) \left\{ \gamma \gamma (p_c - \bar{q} + m_c) \gamma \gamma [q^2 - 2(p_c \bar{q})]^{-1} \right\} u(p_c) 
\]

(2)

with \( q, k, p_c \) and \( p_c \) being the 4-momenta of the virtual photon, virtual gluon, charm quark and charm antiquark, respectively, and \( \epsilon_\gamma \) and \( \epsilon_g \) the photon and gluon polarization vectors.

When calculating the spin average of the matrix element squared, we substitute the full lepton tensor for the photon polarization matrix:

\[
\overline{\epsilon_\gamma^\mu \epsilon_g^{\nu*}} = \left[ 8 p_c^\mu p_c^{\nu} - 4(p_c \bar{q}) g^{\mu\nu} \right] / (q^2)^2
\]

(3)

(including also the photon propagator factor). The virtual gluon polarization matrix is taken in the form [7]

\[
\overline{\epsilon_g^\mu \epsilon_g^{\nu*}} = p_c^\mu p_c^{\nu} / |k_t|^2 = k_t^\mu k_t^{\nu} / |k_t|^2,
\]

(4)

where \( p_c \) and \( p_D \) are the 4-momenta of the incoming electron and proton. The evaluation of trace over the Dirac indices is straightforward and is done using the algebraic manipulation system FORM [20].

Another important ingredient of the semi-hard approach is the so called unintegrated gluon distribution \( F(x, k_t^2, Q_0^2) \), which determines the probability to find a gluon carrying the longitudinal momentum fraction \( x \) and transverse momentum \( k_t \). To parameterize the unintegrated structure functions, we use the prescriptions of Ref. [21]. The proposed method lies upon a straightforward perturbative solution of the BFKL equation where the collinear gluon density \( x G(x, \mu^2) \) is used as the boundary condition in the integral form:

\[
\int_0^\mu^2 F(x, k_t^2, Q_0^2) \, dk_t^2 = x \, G(x, \mu^2, Q_0^2).
\]

(5)

Technically, the unintegrated gluon density is calculated as a convolution of the collinear gluon density with universal weight factors [21]:

\[
F(x, k_t^2, \mu^2) = \int_{x} G(\eta, k_t^2, \mu^2) x \, G(x, \mu^2, Q_0^2) \, d\eta.
\]

\[
G(\eta, k_t^2, \mu^2) = \frac{\alpha_s}{\eta k_t^2} J_0 \left( 2 \sqrt{\frac{\alpha_s}{\eta}} \ln(1/\eta) \ln(\mu^2/k_t^2) \right),
\]

(6)

\[
G(\eta, k_t^2, \mu^2) = \frac{\alpha_s}{\eta k_t^2} I_0 \left( 2 \sqrt{\frac{\alpha_s}{\eta}} \ln(1/\eta) \ln(k_t^2/\mu^2) \right),
\]

(7)

where \( J_0 \) and \( I_0 \) stand for Bessel functions of real and imaginary arguments, respectively, and \( \alpha_s = \alpha_s / 4 \pi \). The latter parameter is connected with the Pomeron trajectory intercept: \( \Delta = \Delta s \sum_{2} \ln 2 \) in the LO,
and $\Delta = \bar{a}_s 4 \ln 2 - N \bar{a}_s^2$ in the NLO approximations, respectively, where $N$ is a number [22].

The multiparticle phase space

$$\prod d^3 p_i / (2E_i) \delta^4 \left( \sum p_{\text{in}} - \sum p_{\text{out}} \right)$$

of the reaction $e^+ e^- \rightarrow \ell^+ \ell^- X$ is parameterized in terms of transverse momenta, rapidities and azimuthal angles:

$$\frac{d^3 p_i}{2E_i} = \frac{\pi}{2} \frac{p_T^2_i d\gamma_i d\phi_i}{2\pi}$$

Let $s = (p_e + p_p)^2$ and $y_c, \phi_c, y\bar{c}$ and $\phi\bar{c}$ be the rapidity and the azimuthal angle of the charm quark and charm antiquark, respectively. Then, the fully differential cross section reads:

$$d\sigma (e^+ e^- \rightarrow \ell^+ \ell^- X)$$

$$= \frac{\alpha_s \alpha_s^2 e^2}{4s^2 x} \left( \sum \frac{1}{N_c} \sum \text{colours} \right) |\mathcal{M}(\gamma g \rightarrow c\bar{c})|^2$$

$$\times \mathcal{F} \left( x, k_T^2, \mu^2 \right) dK_T^2 dp_T^2 dp_T\gamma d\gamma d\phi \phi$$

$$\times \frac{d\phi_\gamma d\phi_c d\phi_\bar{c}}{2\pi 2\pi 2\pi}. \quad (8)$$

The phase space physical boundary is determined by the inequality

$$G(\hat{s}, \hat{t}, m_c^2, q^2, k_T^2, m_{\ell}^2) \leq 0. \quad (9)$$

where $\hat{s} = (k + q)^2$, $\hat{t} = (q - p_e)^2$, and $G$ is the standard kinematical function [23].

The gluon momentum fraction $x$ is calculated from the independent integration variables using the energy-momentum conservation in the light cone projection:

$$(k + q)_{E + P} = 2E_P x$$

$$= m_t \exp(y_t) + m_{\ell} \exp(y_\ell). \quad (10)$$

$m_t = (m_c^2 + |p_1|^2)^{1/2}$. The multidimensional integration in (8) has been performed by means of Monte-Carlo technique, using the routine VEGAS [24]. Finally, the charm quarks were converted into $D^*$ mesons using the Peterson fragmentation function.

In the previous work [11] we used the standard GRV parametrization [25] for the collinear gluon density, from which the unintegrated gluon distribution was developed according to Eqs. (5)–(7). Some other essential parameters were chosen as follows: the charm-quark mass $m_c = 1.5$ GeV, the Peterson fragmentation parameter $\epsilon = 0.06$, the overall $c \rightarrow D^*$ fragmentation probability 0.26. The Pomeron intercept $\Delta$ was regarded as free parameter, and then the value $\Delta = 0.35$ has been extracted from a fit to the experimental $p_T(D^*)$ spectrum measured by the ZEUS collaboration [2]. Close estimates for $\Delta$ have also been obtained by many other authors, see, e.g. [26,27].

Since the agreement with the data achieved within this set of parameters was really good, we continue using it in the present calculations.

3. Numerical results

In Fig. 2 we present the results of theoretical calculations made within the BFKL approach together with the experimental data collected by ZEUS. The simulation procedure consists in generating a photon–gluon fusion event using the off-shell matrix elements and the unintegrated gluon distribution functions described in Section 2.

The basic $2 \rightarrow 2$ partonic subprocess gives rise to two high-energy quarks, which can further evolve into hadron jets. Actually, as the matter of some reasonable
approximation, the calculations were restricted to parton level, and so the produced quarks (with their known kinematical parameters) were taken to play the role of the final jets: $E_T^{\text{jet}_{1,2}} = E_T(q, \bar{q})$.

The two quarks are accompanied by a number of gluons radiated in the course of the gluon evolution. It has been mentioned already that, on the average, the gluon transverse momentum decreases from the hard interaction block towards the proton. As an approximation, we assume that the gluon $k'_t$ closest to the quark block compensates the whole transverse momentum of the virtual gluon participating in the hard interaction: $k'_t \simeq -\hat{k}_t$, while all the other emitted gluons are collected together in the ‘proton remnant’, which is assumed to carry only a negligible transverse momentum compared to $k'_t$. This gluon gives rise to a third hadron jet with $E_T = |k'_t|$.

From among the three hadron jets represented by the quark, antiquark and gluon we choose the two carrying the largest transverse energies, and then calculate the quantity $x_y$ according to its definition given by Eq. (1). The weight $d\sigma$ of the corresponding generated event is given by Eq. (8). In a significant fraction of events, the gluon radiated from the BFKL cascade appears to be harder than one or even both of the quarks produced in hard parton interaction. In fact, the specified events are responsible for the wide plateau seen in the $x_y$ distribution in Fig. 2. In the ZEUS collaboration analysis [2], the existence of this plateau was attributed to the charm excitation from a ‘resolved photon’ and was interpreted as a likely signature of the photon structure. On the contrary, we have not included the resolved photon contribution explicitly in our calculations and have described the experimental data in terms of the proton structure only. Thus the $k_t$ factorization effectively imitates the anomalous coupling of the resolved photon.

4. Conclusions

We have considered the production of $D^*$ mesons associated with two hadron jets at HERA conditions. Our interest was focused on the kinematical variable $x_y$, which was expected to be a sensitive indicator of the underlying parton dynamics. The calculations were based on the leading-order QCD matrix elements combined with the BFKL evolution of gluon densities. The results of the simulations show reasonably good agreement with data, and an ad hoc contribution from the resolved photon is no longer needed. The semihard approach on its own can provide a rather natural explanation for all essential details of the production dynamics.

Acknowledgements

The authors express their thanks to Hannes Jung, Leif Jönsson and Leonid Gladilin for many valuable discussions. The work was supported by the Royal Swedish Academy of Sciences.

References


The full FORTRAN code is available from the authors on request. This code is identical to that used in [11], with the exception that an additional histogram is now introduced for the variable $x_y$.2
To extract information about the polarized gluon distribution, $\Delta G(x, Q^2)$, in the nucleons, we propose $\Lambda_c^+$ productions in polarized $pp$ scattering, $p + \bar{p} \rightarrow \Lambda_c^+ + X$, which will be observed at RHIC experiment starting soon. For this process, we have calculated the spin correlation differential cross section, $d\sigma/dp_T$, and the spin correlation asymmetry defined by $A_{LL} = d\sigma/dp_T / [d\sigma/dp_T]$. We have found that the $A_{LL}$ is sensitive to the polarized gluon distribution in the nucleon and thus the process is promising for testing $\Delta G(x, Q^2)$.

PACS: 13.88.+e; 14.20.Lg; 13.85.Ni
p + p → Λ_c^+ + X (Fig. 1), which would be observed in the forthcoming RHIC experiment. In this process, Λ_c^+ is dominantly produced via fragmentation of a charm quark originated from gluon–gluon fusion. The reason we focus on this process is as follows. The Λ_c^+ is composed of a heavy quark c and antisymmetrically combined light u and d quarks. Hence, the Λ_c^+ spin is basically carried by a charm quark which is produced via gluon–gluon fusion at the lowest order in this process as shown in Fig. 2.1 Therefore, observation of the spin of the produced Λ_c^+ gives us information about the polarized gluons in the proton. In order to study this attractive process:

\[
p(p_A) + \bar{p}(p_B) \rightarrow \Lambda_c^+ (p_{\Lambda_c^+}) + X,
\]

whose subprocess at the lowest order is

\[
g(p_a) + \bar{g}(p_b) \rightarrow \bar{c}(p_c) + \bar{c}(p_c),
\]

where \( p_i \) denotes the four-momenta of the \( i \)-particle\(^2\) and the over-arrow means that the particle polarization is either provided (g) or measured (c), we introduce two useful observables which will be measured by the forthcoming RHIC experiment: one is the spin correlation differential cross section, \( d\sigma/dp_T \), and the other is the spin correlation asymmetry, \( A_{LL} \). They are defined as follows:

\[
\frac{d\Delta\sigma}{dp_T} = \frac{d\sigma(++)-d\sigma(+-)+d\sigma(-+)-d\sigma(--)}{d\sigma(++)+d\sigma(+-)+d\sigma(-+)+d\sigma(--)}/dp_T,
\]

\[
A_{LL} = \frac{d\Delta\sigma}{dp_T} = \frac{d\sigma(++)+d\sigma(+-)+d\sigma(-+)+d\sigma(--)}/d\sigma(++)/U
\]

where \( d\sigma(+-)/dp_T \), for example, denotes the spin-dependent differential cross section with the positive helicity of the target proton and the negative helicity of the produced \( \Lambda_c^+ \).

Let us consider the process in the proton–proton c.m. frame and take the four-momenta of \( p_i \) as follows:

\[
p_{A,B} = \sqrt{s} \left( 1, \pm \beta, 0 \right) \quad \text{with} \quad \beta \equiv \sqrt{1 - \frac{4m_T^2}{s}},
\]

\[
p_{\Lambda_c^+} = \left( E_{\Lambda_c^+}, p_L, \vec{p}_T \right) = \left( \sqrt{m_{\Lambda_c^+}^2 + p_T^2 \csc^2 \theta_c}, p_T \cot \Theta, \vec{p}_T \right),
\]

\[
\cos \theta_{c,b} = \frac{x_{a,b} p_{A,B}}{z}, \quad p_c = \frac{p_{\Lambda_c^+}}{z},
\]

where the first, second and third components in parentheses are the energy, the longitudinal momentum and the transverse momentum, respectively.\(^3\) \( \Theta \) and \( m_i \) represent the c.m. scattering angle of the produced \( \Lambda_c^+ \) and mass of the \( i \)-particle, respectively. \( x_{a,b} \) and \( z \) are the momentum fraction of the proton carried by the gluon and the one of the charm quark carried by \( \Lambda_c^+ \), respectively. Notice that we assume that the scattering angle of the \( \Lambda_c^+ \) produced in the final state and the angle of the charm quark in the subprocess are almost same, \( \theta_{c} \simeq \theta_{\Lambda_c^+} \equiv \Theta \). This assumption is not unreasonable, because the momentum of \( \Lambda_c^+ \) is almost carried by a charm quark whose mass is much larger than

\(^1\) Since charm quarks are not main constituents of the proton, the gluon–gluon fusion process is dominant for charm quark production.

\(^2\) For example, if \( i = \Lambda_c^+ \), \( p_i \) indicates the four-momenta of \( \Lambda_c^+ \).

\(^3\) Here we do not neglect the masses of all particles, though we basically follows the method of Ref. [3].
the other constituents of $A_1^+$. Then, using Eq. (6), we define the following Lorentz invariant variables:

\[ \tilde{s} = s - 2m_p^2, \]
\[ \tilde{t} = t - m_p^2 - m_{A_1}^2, \]
\[ \tilde{u} = u - m_p^2 - m_{A_1}^2, \]

where $s$, $t$, and $u$ are conventional Mandelstam variables, $s = (p_A + p_B)^2$, $t = (p_A - p_B)^2$, and $u = (p_{A_1} - p_A)^2$, respectively. Furthermore, for the subprocess, we define

\[ \tilde{t}_1 \equiv \tilde{t} - m_{A_1}^2 = \frac{x_b}{z}, \]
\[ \tilde{u}_1 \equiv \tilde{u} - m_{A_1}^2 = \frac{x_a}{z}, \]

with the subprocess Mandelstam variables \( \tilde{s} = (p_u + p_b)^2, \tilde{t} = (p_u - p_b)^2 \) and \( \tilde{u} = (p_u - p_a)^2 \).

Using Eqs. (6)–(8) the spin correlation differential cross section (Eq. (4)) can be expressed as

\[
\frac{d\Delta\sigma}{dp_T} = \int_{\min}^{\max} \int_{\min}^{\min} \int_{\min}^{\min} G_{PA \to x_a, Q^2}(x_a, Q^2) \\
\times \Delta G_{\bar{P}_B \to x_b, Q^2}(x_b, Q^2) \Delta D_{c \to A_1^+}(z) \\
\times \frac{d\Delta\hat{\sigma}}{dt} J d\phi_a d\phi_b d\phi, \tag{9}
\]

where $G_{PA \to x_a, Q^2}$, $\Delta G_{\bar{P}_B \to x_b, Q^2}$, and $\Delta D_{c \to A_1^+}(z)$ represent the unpolarized gluon distribution function, the polarized gluon distribution function, and the spin-dependent fragmentation function of the outgoing charm quark decaying into a polarized $A_1^+$, respectively. Moreover $J$ is the Jacobian which transform the variables $z$ and $\tilde{t}$ into $\phi$ and $p_T$. Unfortunately, at present there is no established spin-dependent fragmentation functions because of lack of experimental data. However, since a charm quark is much heavier than other constituents of $A_1^+$, it is expected to be very rare for a charm quark to change its spin alignment during fragmentation process. Therefore, it is not unreasonable to substitute $D_{c \to A_1^+}$ for $\Delta D_{c \to A_1^+}$. In this work, we use the model by Peterson et al. [4] for both $D_{c \to A_1^+}$ and $\Delta D_{c \to A_1^+}$. For the subprocess, the spin correlation differential cross section, $d\Delta\hat{\sigma}/dt$ is calculated to be

\[
d\Delta\hat{\sigma}/dt = \frac{\pi\alpha_s^2}{s} \left[ m_c^2 \left\{ \frac{9\tilde{t} - 19\tilde{u}_1}{24} \frac{\tilde{u}}{\tilde{u}_1} + \frac{3}{\tilde{u}_1^2} \right\} \\
+ \frac{\hat{s}}{6} \left\{ \tilde{t}_1 - \tilde{u}_1 \right\} - \frac{3}{8} \left( \frac{2\tilde{t}_1}{\tilde{u}} + 1 \right) \right]. \tag{10}
\]

and the Jacobian, $J$, is given by

\[
J = \frac{2\pi\beta p_T^2 \operatorname{cosec}^2 \Theta}{\hat{s}^2 m_{A_1}^2 + p_T^2 \operatorname{cosec}^2 \Theta}. \tag{11}
\]

where $z$ is

\[
z = \frac{x_1}{x_a} + \frac{x_2}{x_b} \tag{12}
\]

with $x_1 = -\tilde{t}/\tilde{s}$ and $x_2 = -\tilde{u}/\tilde{s}$. The minimum of $x_a$, $x_b$ are given by

\[
x_a^{\min} = \frac{x_1}{1-x_2}, \quad x_b^{\min} = \frac{x_a x_2}{x_a-x_1}. \tag{13}
\]

The unpolarized differential cross sections were calculated by Babcock et al. [5].

For numerical calculation, we use as input parameters, $m_c = 1.5$ GeV, $m_p = 0.938$ GeV and $m_{A_1} = 2.28$ GeV [6]. We limit the integration region of $\Theta$ and $p_T$ of produced $A_1^+$ as $\pi/6 \leq \Theta \leq 5\pi/6$ and $3$ GeV $\leq p_T \leq 20$ GeV for $\sqrt{s} = 200$ GeV and $500$ GeV, in order to get rid of the contribution of the diffractive $A_1^+$ production and also the $A_1^+$ production through a single charm quark production via $W$ boson exchange and $W$ boson production. As for the gluon distributions, we take the GS96 (set-A and -B) [7] and the GRVS96 [8] parameterization models for the polarized gluon distribution function and the GRV95 [9] model for the unpolarized one. Note that the GS96 and GRVS96 models can excellently reproduce experimental results for the polarized structure function of nucleons, though behavior of these polarized gluon distributions is quite different. In other words, the data on polarized structure functions of nucleons and deuteron alone are not enough to distinguish the
model of gluon distribution functions. We are interested in the sensitivity of those observables of Eqs. (4) and (5) on the polarized gluon distributions in the nucleon in this process.

We show the $p_T$ distribution of $d\Delta\sigma/dp_T$ and $A_{LL}$ in Fig. 3 for $\sqrt{s} = 200$ GeV and in Fig. 4 for $\sqrt{s} = 500$ GeV, respectively. Notice that in Fig. 4 the absolute value of $d\Delta\sigma/dp_T$ is presented, since the negative value of $d\Delta\sigma/dp_T$ cannot be depicted in the figure which has an ordinate with logarithmic scale. Actually, for the case of $\sqrt{s} = 500$ GeV, the value of $d\Delta\sigma/dp_T$ becomes negative for $p_T$ smaller than the value corresponding to the sharp dip shown in Fig. 4. On the other hand, $d\Delta\sigma/dp_T$ at $\sqrt{s} = 200$ GeV is positive for all $p_T$ regions as shown in Fig. 3. To understand this quite different behavior of $d\Delta\sigma/dp_T$ depending on $\sqrt{s}$, some comments are in order. As seen from Eqs. (9)–(13), the sign of $d\Delta\sigma/dp_T$ is determined by the sign of $d\Delta\hat{\sigma}/d\hat{t}$ because all variables except for $d\Delta\hat{\sigma}/d\hat{t}$ are positive in whole kinematical regions of $x_a$, $x_b$ and $\theta$. Since both $\hat{t}_1$ and $\hat{u}_1$ are negative and smaller than $\hat{s}$ in magnitude, the sign of $d\Delta\hat{\sigma}/d\hat{t}$ becomes negative for $p_T$ smaller than the value corresponding to the sharp dip shown in Fig. 4.
mainly originates from the second term of the right-hand side of Eq. (10), in which \( \hat{t}_1 - \hat{u}_1 \) is calculated to be

\[
\hat{t}_1 - \hat{u}_1 \simeq x_b \sqrt{s} \left( x_a \sqrt{s} - \frac{2}{c} \left( m_{A_2}^2 + p_T^2 \csc^2 \theta \right) + p_T \cot \theta \right),
\]

(14)
in the limit of the massless proton. Furthermore, in the same limit, \( x_a^{\min} \) reduces to

\[
x_a^{\min} \simeq \frac{\sqrt{s} - \left( m_{A_2}^2 + p_T^2 \csc^2 \theta + p_T \cot \theta \right)}{\sqrt{s} - \left( m_{A_2}^2 + p_T^2 \csc^2 \theta - p_T \cot \theta \right)}
\]

(15)
from Eq. (13). Eq. (15) shows that \( x_a \) becomes very small in some region of \( p_T \) and \( \theta \) when \( \sqrt{s} \) is large and hence leads to large values of the gluon distributions, \( G_{pA \to x_A} (x_A, \sqrt{s}) \). Thus when Eq. (14) becomes negative due to very small \( x_a \), the negative contribution of \( d\Delta \hat{t}/d\hat{t} \) to \( d\Delta \sigma/dp_T \) becomes larger for \( \sqrt{s} = 500 \text{ GeV} \) than for \( \sqrt{s} = 200 \text{ GeV} \) because \( G_{pA \to x_A} (x_A, \sqrt{s}) \) is larger at smaller \( x_a \). Therefore, for large \( \sqrt{s} \) such as \( \sqrt{s} = 500 \text{ GeV} \), even after integration, \( d\Delta \sigma/dp_T \) is negative for \( p_T \) smaller than the value corresponding to the dip, as shown in Fig. 4. It is very interesting to note that the behavior of \( A_{\perp} \) strongly depends on \( \sqrt{s} \) as shown in Figs. 3 and 4. In any case, both figures show that \( A_{\perp} \) is sensitive to the polarized gluon distribution function. Finally, to examine if our predictions can be tested at the forthcoming RHIC experiment, we estimate the statistical sensitivity of the spin correlation asymmetry, \( \delta A_{\perp} \), according to the method given in Ref. [10]. The value of \( \delta A_{\perp} \) is estimated by

\[
\delta A_{\perp} \simeq \frac{1}{P} \frac{1}{\sqrt{b_{A_2}^{\perp} \epsilon \mathcal{L} T \sigma}}
\]

(16)

We can estimate the statistical sensitivity, \( \delta A_{\perp} \), for \( T = 100 \)-day experiments by using the parameters of the beam polarization (\( P = 70\% \)), a luminosity (\( \mathcal{L} \equiv 8 \times 10^{31} (2 \times 10^{32}) \text{ cm}^{-2} \text{sec}^{-1} \) for \( \sqrt{s} = 200(500) \text{ GeV} \)), the trigger efficiency (\( \epsilon \equiv 10\% \)) of detecting high \( p_T \) charm production events through their semi-leptonic decays and a branching ratio (\( b_{A_2}^{\perp} \equiv \text{Br}(A_2 \to pK^-\pi^+) \approx 5\% \)). \( \delta A_{\perp} \) denotes the unpolarized cross section integrated over suitable \( p_T \) region. The results are summarized in the right panels of Figs. 3 and 4. As shown in these figures, \( \delta A_{\perp} \) is smaller than the difference of the model predictions at moderate \( p_T \) region and thus our predictions are expected to be tested in the RHIC experiment, though the differential cross section, \( d\sigma/dp_T \), becomes rapidly smaller with increasing \( p_T \), and thus in the larger \( p_T \) region, \( \delta A_{\perp} \) becomes too large to distinguish the model of polarized gluon distribution functions. Furthermore, note that \( \delta A_{\perp} \) is expected to be small if a trigger efficiency \( \epsilon \) is improved and/or another decay modes of \( A_2 \) are additionally taken into account. Moreover, the statistics should be twice by using the \( A_2^{\perp} \) data as well, since high \( p_T \) charm production is dominated by gluon fusion, as long as the \( A_2^{\perp} \) decays satisfy CP invariance.

In summary, to extract the polarized gluon distribution \( \Delta G(x, Q^2) \), we proposed a new polarized process, \( p+\bar{p} \to A_2^\perp+X \), which could be observed in the forthcoming RHIC experiment. We calculated the spin correlation differential cross section, \( d\Delta \sigma/dp_T \), and the spin correlation asymmetry, \( A_{\perp} \), and found that \( A_{\perp} \) is quite sensitive to the polarized gluon distribution functions; we can rather clearly distinguish the model of polarized gluon distribution functions at moderate \( p_T \) region. It is also remarkable that the \( A_{\perp} \) shows very different behavior in the region of \( \sqrt{s} = 200–500 \text{ GeV} \), covered by RHIC. Therefore, the process looks promising for testing the model of polarized gluon distribution functions, though the present analysis is based on the leading order calculation. In order to get more profound information on the behavior of polarized gluons, we need the next-to-leading order calculation. Furthermore, we also need further investigation of the polarized fragmentation functions, \( \Delta D_{c \to A_2^{\perp}} (z) \), to get more reliable predic-

5 In the region of \( s \) much larger than \( m_{\gamma}^2 \), where we are now focusing, this limit with \( s = \tilde{s} \) and \( \hat{p} = 1 \) is a good approximation.

6 Here, we simply assumed the efficiency for reconstructing the \( pK^-\pi^+ \) decay of \( A_2^{\perp} \) to be 100\% by quoting only the decay branching fraction. Although the actual efficiency might be more likely to be less than 10\%–20\% rather than more than 50\% (almost perfect, background free measurement), this should be studied by experimentalists in the forthcoming RHIC experiment.
tion. Although these subjects are interesting and important in their own right, they are out of scope in this work.

We hope our prediction will be tested in the forthcoming RHIC experiment.

Acknowledgements

One of the authors (T.M.) would like to thank for the financial support by the Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, Japan (No. 11694081).

References

[2] For a review see:
B. Lampe, E. Reya, hep-ph/9810270;
The $\eta$ and $\eta'$ mesons in QCD

UKQCD Collaboration

C. McNeile, C. Michael

Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

Received 21 June 2000; received in revised form 25 August 2000; accepted 7 September 2000

Editor: P. V. Landshoff

Abstract

We study the flavour singlet pseudoscalar mesons from first principles using lattice QCD. We explore the quark content of the $\eta$ and $\eta'$ mesons and we discuss their decay constants.

$\copyright$ 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

There is considerable interest in understanding hadronic decays involving $\eta$ and $\eta'$ in the final state. The phenomenological study of hadronic processes involving flavour singlet pseudoscalar mesons makes assumptions about their composition. Here we address the issue of the nature of the $\eta$ and $\eta'$ from QCD directly, making use of lattice techniques.

Lattice QCD directly provides a bridge between the underlying quark description and the non-perturbative hadrons observed in experiment. The amplitudes to create a given meson from the vacuum with a particular operator made from quark fields are measurable, an example being the determination of $f_\pi$. It also allows a quantitative study of the disconnected quark contributions that arise in the flavour singlet sector. The lattice approach provides other information such as that obtained by varying the number of quark flavours and their masses.

In the case of pseudoscalar mesons, the chiral perturbation theory approach also provides links between a quark description and the hadronic states. For the pion, this has the well known consequence that the decay constant $f_\pi$ describes quantitatively both the $\mu\nu$ and $\gamma\gamma$ decays. For the flavour singlet states ($\eta$ and $\eta'$), the situation is more complicated [1]. The axial anomaly now involves a gluonic component and the definition of decay constants is not straightforward. From the chiral perturbation theory description, one expects the mixing of $\eta$ and $\eta'$ to be most simply described in a quark model basis. In the flavour singlet sector, for pseudoscalar mesons, we then have contributions to the mass squared matrix with quark model content $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ (which we label as $nn$ and $ss$ respectively):

$$\begin{pmatrix}
m_{nn}^2 + 2x_{nn} & \sqrt{2}x_{ns} \\
\sqrt{2}x_{ns} & m_{ss}^2 + x_{ss}
\end{pmatrix}.$$

Here $m$ corresponds to the mass of the flavour non-singlet eigenstate and is the contribution to the mass coming from connected fermion diagrams while $x$ corresponds to the contribution from disconnected fermion diagrams. In the limit of no mixing ($x = 0$,
the OZI suppressed case), then we have the quenched QCD result that the $\eta$ is degenerate with the $\pi$ meson and the $\eta'$ would correspond to the $s \bar{s}$ pseudoscalar meson. This is not the case, of course, and the mixing contributions $x$ are important.

Using as input $m_{nn}$, $m_{ss}$, $m_{\eta}$ and $m_{\eta'}$, the three mixing parameters $x$ cannot be fully determined. It is usual to express the resulting one parameter freedom in terms of a mixing angle, here defined by

$$\eta = \eta_{nn} \cos \phi - \eta_{ss} \sin \phi,$$
$$\eta' = \eta_{nn} \sin \phi + \eta_{ss} \cos \phi.$$  \hspace{1cm} (2)

We show the resulting values of the mixing parameters $x$ in Fig. 1 (the input value for $m_{ss}$ will be discussed later).

The $\eta$ and $\eta'$ mesons are often described in an SU(3) motivated quark basis, namely,

$$\eta_8 = (u \bar{u} + d \bar{d} - 2s \bar{s})/\sqrt{6},$$
$$\eta_1 = (u \bar{u} + d \bar{d} + s \bar{s})/\sqrt{3}.$$

The mixing angle $\theta$ in this basis would be given by $\phi - 54.7^\circ$ in a lowest order chiral perturbation theory. In order to have $f_8 \neq f_\eta'$, one needs higher order terms in the chiral perturbation theory treatment and then the mixing scheme becomes more complicated [1] in this basis with more than one angle needed.

In the SU(3) symmetric limit, $m_{nn} = m_{ss} = m$ and $x_{nn} = x_{ss} = x$, so that only one mixing parameter is relevant and the mixing matrix simplifies considerably to a diagonal form with elements $m^2$ (octet) and $m^2 + 3x$ (singlet). Previous lattice studies [2] have used degenerate quarks, so have explored this case and have found that the mixing parameter $x$ is of a magnitude which can explain qualitatively the observed splitting between the $\eta$ and $\eta'$ mesons.

Here we undertake a non-perturbative study in QCD from first principles which will be able to establish the values of the mixing parameters $x$, including the pattern of SU(3) breaking. This more comprehensive study would take into account the different masses of the light ($u$ and $d$) quarks and the heavier $s$ quark. Within the lattice approach, it is not at present feasible to evaluate using quarks as light as the nearly massless $u$ and $d$ quarks and also it is more tractable to use an even number of degenerate quarks in the vacuum. As we shall show, despite these restrictions, a thorough study of the mixing between $\eta$ and $\eta'$ is possible.

Our lattice study uses dynamical configurations with $N_f = 2$ flavours of sea quarks of type 1 and we consider the properties of pseudoscalar mesons made of either quark 1 or quark 2, where quark 2 corresponds to a heavier quark. Thus quark 2 is treated as partially quenched. Here we have in mind exploring a situation which will be relevant to treating strange quark propagation (quark 2) in a vacuum containing only lighter quarks (quark 1). We focus here on the results of lattice evaluations, for background to the methods used see Ref. [3]. We address three topics where lattice input permits us to construct a firm foundation for the $\eta$, $\eta'$ mixing:

- From comparing pseudoscalar meson masses with valence quarks of two different masses (namely meson masses $m_{11}$, $m_{12}$ and $m_{22}$), we can estimate the mass $m_{ss}$ of the unmixed $\bar{s}s$ meson, given the observed $m_{ns}$ and $m_{nn}$ masses (i.e., $K$ and $\pi$, respectively).

- From measuring the mixing parameters $x_{11}$, $x_{12}$ and $x_{22}$ between initial and final flavour singlet states consisting of either quark 1 or 2 with different masses as above, we can establish the pattern of SU(3) breaking in the mixing.

Fig. 1. The mass mixing parameters $x$ in GeV $^2$ versus $\eta$, $\eta'$ mixing angle $\phi$ in the $\eta_{nn}$, $\eta_{ss}$ basis. The horizontal dotted lines give the allowed range from the lattice determination of $x_{ss}$. The vertical line illustrates our preferred solution.

124

• For \( N_f = 2 \) degenerate flavours of quark, we determine the pseudoscalar decay constants for the flavour singlet (\( P_0 \)) and non-singlet (\( P_1 \)) meson. This input allows us to discuss the relation between the observed \( \gamma \gamma \) decay modes of \( \pi^0, \eta \) and \( \eta' \) and the underlying quark content.

2. Lattice results

2.1. The \( s\bar{s} \) pseudoscalar mass

Chiral symmetry considerations lead to the expectation that the pseudoscalar meson composed of quarks of mass \( M_q \) has mass squared \( m^2 \) which behaves linearly with \( M_q \) at small quark mass. However, at large quark mass (\( c \) and \( b \) quarks, for instance), one expects the meson mass to vary approximately linearly with the quark mass. Here we are not concerned with the region of very small quark mass where chiral logs are important [1], so we summarise this behaviour by

\[
m^2 = bM_q + cM^2_q + O(M^3_q).
\]  

(3)

For a pseudoscalar meson made of two different quarks of mass \( M_{q_1} \) and \( M_{q_2} \), we shall assume its mass only depends on \((M_{q_1} + M_{q_2})/2\) and not on \((M_{q_1} - M_{q_2})/2\) as found in lattice studies [4] and in lowest order chiral perturbation theory. If Eq. (3) were valid with just the linear term in the quark mass (i.e., \( c = 0 \)), then one directly obtains the required mass of the pseudoscalar meson composed of \( s \) quarks, \( m^2_{ss} = 2m^2_{ss} - m^2_{pp} \) that is \( 2K^2 - \pi^2 \), leading to \( m_{ss} = 0.687 \) GeV.

This can be explored on a lattice by measuring the pseudoscalar meson mass for valence quarks in combinations 11, 22 and 12. Then, for small \( c/b \), we have

\[
c = \frac{4(bm^2_{11} + bm^2_{22}) - bm^2_{12}}{(bm^2_{22} - bm^2_{11})^2}.
\]  

(4)

This has been studied in the quenched approximation giving evidence [5] for a positive coefficient \( c \) in Eq. (3). In the quenched approximation, however, the chiral behaviour at small quark mass is anomalous since the theory is not unitary. A better way to study this issue on the lattice is then to use dynamical configurations with sea quarks of type 1 and to consider the propagation of mesons made of either quark 1 or quark 2, where quark 2 corresponds to a heavier quark.

We present results from UKQCD configurations [6] with \( N_f = 2 \) flavours of sea quark with SW-clover coefficient \( C_{SW} = 1.76 \), lattice size 12\(^3 \) · 24, and with sea quarks having \( \kappa = 0.1398 \), corresponding to sea quarks of mass around the strange quark mass \((m_p/m_V = 0.67)\). Then we take the heavier valence quark (with \( \kappa = 0.1380 \)) as corresponding to approximately twice the strange mass \((m_p/m_V = 0.81)\).

The fits with two states to a \( 4 \times 4 \) matrix of mesonic correlators for \( t \) range 3 to 10 give results for the spectrum shown in Table 1.

Taking account of the correlation among the errors, we obtain the dimensionless ratio

\[
m_{11}^2 c
\]

\[
\frac{4b^2} = 0.011(3),
\]  

(5)

which indicates a statistically significant curvature from the \( c \) term. Setting the scale [6] using \( a^{-1} = 1.47 \) GeV then the value of \( c/b^2 \) in physical units can be obtained from \( m_{11} = 698 \) MeV.

Applying this value of \( c \) to the determination of the \( m_{ss} \) mass from the \( \pi \) and \( K \) masses, gives a relative shift upwards due to the curvature term \((c) \) of 1.1(3)%%, corresponding to a value of \( m_{ss} = 0.687 + 0.008 \) GeV.

Note that this value also helps us to identify the meson mass ratio corresponding to strange quarks, namely \( m_p/m_V = m_{ss}/m_\phi = 0.682 \).

This lattice study thus answers the question of the likely deviation in the pseudoscalar mass formula from the result given by the lowest order chiral expression.

2.2. Flavour-singlet mixing

The mass splitting between flavour non-singlet and singlet mesons can be measured using lattice evaluation of disconnected quark propagators. This is not an easy task: the contamination from excited states is difficult to remove and the statistical errors turn out to be relatively large. Initial studies have been in the quenched approximation [2,3,7]. Here, although there is no flavour splitting of the masses, the mass splitting matrix element \( x \) can be evaluated. It is, however, preferable to be able to study the mass splitting directly and hence we focus on results from full QCD simulations [8,9].
The study of the mass spectrum of flavour singlet ($P_0$) and non-singlet pseudoscalar meson ($P_1$) using $N_f = 2$ flavours of sea quark 1 leads to singlet mass $m_0 = (m_{11}^2 + 2x_{11})^{1/2}$ and non-singlet mass $m_1 = m_{11}$ respectively which allows $x_{11}$ to be extracted. We shall also be interested in the dependence of $x$ on quark masses and on non-diagonal mixings. These can be studied with a little less rigour as we discuss later.

We use the UKQCD lattices introduced in the previous section. The disconnected diagrams were evaluated using a variance reduction method [8] which uses all the data available with no dilution from the stochastic method used. We use as many different operators for the pseudoscalar meson as possible to have the largest basis in which to extract the ground state — local and non-local (fuzzed) in space with both $\gamma_5$ and $\gamma_5\gamma_2$ spin structure. Unfortunately, even with this basis of four operators, we are unable to determine the singlet mass precisely. For example for both valence and sea quarks having $\kappa = 0.1398$, as shown in Table 1, we obtain $am_0 = 0.56(4)$ from a one state fit to $t = 3$ to 7 with a $4 \times 4$ matrix of meson correlators. A two state fit to a wider $t$ range (2–7) gives a similar mass value. The corresponding non-singlet pseudoscalar mass is also given in Table 1, so the determination of $x$ from $m_1^2 + 2x_{11} = m_0^2$ has relatively large errors ($x_{11} = 0.10(4)$ GeV$^2$ using $a^{-1} = 1.47$ GeV). As an alternative, we also fit the ratio of the singlet to non-singlet correlators directly to a ground state mass difference. Using the $t$ range 2–7 and local and fuzzed pseudoscalar operators, we obtain $am_0 - am_1 = 0.12(7)$. This method gives a slightly larger mass value (indicating $x_{11} = 0.13(8)$ GeV$^2$) but has even larger errors. These results indicate that much larger ensembles of gauge configurations will be needed to make more precise this approach of determining $x$ from masses.

If one studies correlations of meson operators made from valence quarks of type 2 in a sea of quarks of type 1, one will find the ground state pseudoscalar meson to be that composed of quarks of type 1 (assuming type 2 quarks are heavier than type 1). Because of this, we need to explore in more detail to study the SU(3) breaking of the mixing parameters.

To get a first look at this issue, we consider a quenched lattice and measure the ratio of the disconnected to connected diagrams for pseudoscalar meson propagation. We present results for $\beta = 5.7$, $C_{SW} = 1.57$, 12$^3 \times 24$ with 100 configurations with $\kappa = 0.14077$ and 0.13843. The non-singlet spectrum at these parameters was studied previously [10] giving $m_V/m_P$ values of 0.65 and 0.78 which correspond approximately to strange quarks and quarks twice as heavy as strange. The scale was set as $a^{-1} = 1.2$ GeV.

The disconnected meson correlator was determined using a stochastic method with variance reduction [8].

Assuming dominance by ground state meson contributions, the ratio of disconnected to connected diagrams at time separation $t$ is

$$\frac{D_{ij}}{C_{ij}} = \frac{N_f x_{ij}(t+1)}{2(m_{ii}m_{jj})^{1/2}}$$

with flavour non-singlet pseudoscalar mass $m_{ii}$ for quarks of type $i$. The factor of $t + 1$ comes from the number of lattice sites at which the disconnected diagram can be split. To clarify the pattern of SU(3) breaking, we also study the non-diagonal case where we also measure the disconnected to connected ratio

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\kappa_1$</th>
<th>$\kappa_2$</th>
<th>$m_1 a$</th>
<th>$a f_1 / Z$</th>
<th>$m_0 a$</th>
<th>$a f_0 / Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1398</td>
<td>0.1398</td>
<td>0.1398</td>
<td>0.477(5)</td>
<td>0.171(6)</td>
<td>0.56(4)</td>
<td>0.196(12)</td>
</tr>
<tr>
<td>0.1398</td>
<td>0.1380</td>
<td>0.1398</td>
<td>0.563(5)</td>
<td>0.182(4)</td>
<td>0.56(5)</td>
<td>0.176(18)</td>
</tr>
<tr>
<td>0.1398</td>
<td>0.1380</td>
<td>0.1380</td>
<td>0.640(4)</td>
<td>0.190(4)</td>
<td>0.190(4)</td>
<td></td>
</tr>
</tbody>
</table>
from the propagation of mesons with different masses can make an additional correction for the contribution. 

For the propagation of quarks with different masses, we get the values of $x$ needed to reproduce the known $\eta$ and $\eta'$ masses for each mixing angle $\phi$. The lattice determination of $x_{ss}$ is shown by the dotted horizontal band. Keeping close to this band while satisfying the other lattice constraints is possible for the mixing illustrated by the vertical line. This has $x_{nn} = 0.292$, $x_{ns} = 0.218$, $x_{ss} = 0.13$ GeV$^2$ which gives a description of the observed $\eta$ and $\eta'$ masses while being consistent with our QCD inspired evidence about the mixing strengths. This assignment corresponds to a mixing angle $\phi$ in the $\eta_{nn}$, $\eta_{ss}$ band.

### Table 2

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.089(9)</td>
<td>0.073(6)</td>
<td>0.058(6)</td>
</tr>
<tr>
<td>3</td>
<td>0.089(12)</td>
<td>0.072(10)</td>
<td>0.063(10)</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.100(9)</td>
<td>0.072(6)</td>
<td>0.054(5)</td>
</tr>
<tr>
<td>3</td>
<td>0.112(12)</td>
<td>0.083(11)</td>
<td>0.063(9)</td>
</tr>
<tr>
<td>4</td>
<td>0.106(16)</td>
<td>0.077(15)</td>
<td>0.059(9)</td>
</tr>
<tr>
<td>4F</td>
<td>0.093(13)</td>
<td>0.073(12)</td>
<td>0.052(11)</td>
</tr>
</tbody>
</table>

(here $C_{ij}$ is taken as $(C_{ij}C_{jj})^{1/2}$). In extracting $x_{12}$ we can make an additional correction for the contribution from the propagation of mesons with different masses $m_{11}$ and $m_{22}$ although, in practice, this correction is very small.

Using local meson operators, we obtain for $x$ the values in Table 2. The largest $t$ value has least contributions from excited state contamination and the consistency of the results versus $t$ suggests that such contamination is small. As was found previously [2], $x$ increases as the quark mass is decreased. Moreover, we can check to see if there is a factorisation of $x$ as expected in some chiral perturbation theory descriptions [1], namely, $x_{12}^2 = x_{11}x_{22}$, and we find that $x_{12}$ lies somewhat below the value given by this assumption.

We now revert to discussing the more realistic (partially quenched) case: with heavier quarks of type 2 in a sea of two flavours of quarks of type 1. The method described above for the quenched case can be applied here too. In principle this method is now only valid for small $N_f$ at the propagation of quark 1. From this analysis of the measured $D/C$ values, we get the $x$ values shown in Table 3. The values of $x_{11}$ are similar to those obtained above (with larger errors) directly from the rigorous method of using the mass differences. This suggests that the strong assumptions made in determining $x$ directly from $D/C$ are actually reasonable in practice. This, and the consistency of values from different $t$ and different mesonic operators, gives us confidence to use the $x$ values from quarks of type 2 (which are partially quenched anyway) as a guide to the quark mass dependence of $x$. The $x$ values again show an increase with decreasing quark mass and also approximate factorisation.

Setting the quark mass to strange (since $m_P/m_V = 0.682$ in nature for $s$ quarks) in both quenched and $N_f = 2$ evaluations leads to a consistent lattice estimate of $x_{ss}$ in the range 0.09 to 0.13 GeV$^2$. This value is also consistent with that reported from a study of $N_f = 2$ by the CP-PACS collaboration [11] with $m_P/m_V = 0.69$ and $a^{-1} = 1.29$ GeV giving values of $x_{ss} = 0.10$ GeV$^2$ and 0.14 GeV$^2$ (depending on using $t_{\text{min}} = 2, 3$ in fits, respectively). These lattice values are obtained at quite coarse lattice spacings and there may be some additional systematic error arising from the extrapolation to the continuum limit. We have, however, chosen to use a clever improved fermion action [6] to minimise this extrapolation error.

We are unable to determine the mixing strengths $x$ for lighter quarks than strange. So we assume that the value of $x$ continues to increase as the quark mass is decreased below strange in a similar way to the decrease we see from twice strange (type 2) to strange (type 1).

Consider now the consequence of this determination of the mixing. We use input masses $m_{nn} = 0.137$ GeV, $m_{ss} = 0.695$ GeV (as discussed above) and aim to have $x$ values in line with our results above, namely $x_{ss} \approx 0.12$ GeV$^2$, $x_{ns}^2 \approx x_{nn}x_{ss}$ and we also expect, though with big errors from the extrapolation, $x_{nn}/x_{ss} \approx 2$. The figure shows the $x$ values needed to reproduce the known $\eta$ and $\eta'$ masses for each mixing angle $\phi$. The lattice determination of $x_{ss}$ is shown by the dotted horizontal band. Keeping close to this band while satisfying the other lattice constraints is possible for the mixing illustrated by the vertical line. This has $x_{nn} = 0.292$, $x_{ns} = 0.218$, $x_{ss} = 0.13$ GeV$^2$ which gives a description of the observed $\eta$ and $\eta'$ masses while being consistent with our QCD inspired evidence about the mixing strengths. This assignment corresponds to a mixing angle $\phi$ in the $\eta_{nn}$, $\eta_{ss}$ band.
sis of 44.5°. Note that this is almost maximal which implies that the quark content (apart from the relative sign) of the η and η′ meson is the same. The corresponding mixing angle in the ηK, ηK basis (modulo comments above) is a value of θ of −10.2°.

2.3. Flavour-singlet decay constants

The decays of π⁰, η and η′ to γγ are expected to proceed via the quark triangle diagram. The quark model gives a decay proportional to Q_i^2 for the contribution from a quark of charge Q_i. Thus for the π⁰ meson and the flavour-singlet nn and ss mesons, the quark charge contributions to the decay amplitudes would be in the ratio 1 : 5/3 : \sqrt{2}/3. The experimental [12] reduced decay amplitudes for π⁰, η, and η′ are in the ratio 1.0 : 1.00(10) : 1.27(7). This information can be used to analyse the quark content of the pseudoscalar mesons subject to a quantitative understanding of the decay mechanisms.

The conventional approach assumes that the decay constants for the decays of the three mesons are the same and then the relative decay amplitudes give information on the quark content. This suggests a mixing angle of θ ≈ −20° is preferred [1,12].

We now address the issue of determining these decay constants directly from QCD using lattice methods. Our study uses 2 flavours of degenerate quark and we define the decay constants by

\[
\langle 0| A^\mu | P_t(q) \rangle = f_1 q^\mu, \\
\langle 0| A^\mu | P_0(q) \rangle = f_0 q^\mu.
\] (7)

For the isospin 1 state P_t (π-like), this is on a firm footing because of the anomaly hence f_1 will be scale invariant. For the flavour singlet pseudoscalar meson P_0, the decay constant defined as above will not be scale invariant because of gluonic contributions to the anomaly [1]. In this exploratory study we determine the decay constants with lattice regularisation and we shall compare the singlet and non-singlet values.

These decay constants can be thought of as giving the quark wave function at the origin of the pseudoscalar meson. Since the mass splitting between singlet and non-singlet is not reproduced directly in quenched QCD, it is essential to use lattice studies that do include sea quark effects in this study of decay matrix elements.

Results were obtained using fits to full (connected and disconnected) meson propagation with 4 different types of meson creation and destruction operators. These are local and fuzzed operators with either γ_5 or γ_5γ_5 couplings, so giving 4 × 4 matrix of pseudoscalar correlators. We used the N_f = 2 UKQCD configurations [6] referred to above. For the disconnected correlators, the variance reduction technique [8] is essential to get a reasonable signal to noise ratio, particularly for the operators involving the γ_5γ_5 factor.

The lattice result for f with various valence quark masses with fixed sea quark mass (quark 1) as above is shown in Table 1. For the non-singlet results using a⁻¹ = 1.47 GeV and the tadpole-improved perturbative value of Z of 0.81 (and of c_A which is involved in mixing of the lattice pseudoscalar and axial currents but has a very small effect in practice) we get f_11 = 198(8) MeV. Since this corresponds to strange quarks, it is in reasonable agreement with experiment [12] assuming a steady increase from f_{nn} = 131 MeV and f_{ns} = 160 MeV to f_{ss}. We do see evidence for this increase in f with quark mass directly on the lattice going from quarks of type 1 (strange) to type 2 (twice strange) as shown in Table 1.

The flavour singlet results are shown in Table 1. They are determined by fits to the appropriate (connected plus disconnected) meson correlators which are a 4 × 4 matrix at each t value. Despite this extensive data set, the determinations of f have relatively large statistical errors and the systematic error from changing the type of fit is also comparable. For our case with N_f = 2 degenerate quarks, the comparison of the flavour singlet and non-singlet shows that the singlet decay constants appear to be somewhat larger, though the errors are too big to substantiate this.

Combining the mass dependence we find in the flavour non-singlet sector with the near equality of singlet and non-singlet decay constants, we can deduce properties of the physical case with three light quarks. Thus, in terms of the conventional treatment [12], we would expect f_{nn}/f_π > 1 and f_{ss}/f_π > 1. One way to minimise the effects of mixing is to consider X = (a² + a²)/(a²) where a refers to the reduced decay amplitude. Using the conventional formulae for the decay amplitudes would then give a value of X = 3r² (where r is a suitably weighted average of f_{ns}/f_π and f_{ss}/f_π which are both greater than 1). Thus the conventional treatment gives X > 3 which is significantly larger.
than the experimental value [12] of 2.64(24). Thus it appears unlikely that the conventional treatment (with the decay to $\gamma\gamma$ being given by the analogue of the formula for pions) is correct for any mixing angle.

We conclude that there is no support for the conventional assumption that the singlet decays are given by a similar expression to the non-singlet. As has been pointed out by many authors [1], this is plausible for at least two reasons: (i) The $\eta$ and $\eta'$ mesons are heavier and therefore less likely to dominate the axial current or, equivalently, higher order corrections to chiral perturbation theory will be more important, (ii) the flavour-singlet axial anomaly has a gluonic component which will give additional contributions to any hadronic process.

3. Conclusion

From our careful non-perturbative study of mass formulae for flavour non-singlet pseudoscalar mesons made of different quarks, we deduce that the $ss$ state lies at 695 MeV. We then determine the pattern of mixing for the flavour singlet sector, obtaining $x_{ss} \approx 0.12$ GeV$^2$, $x_{nn}/x_{ss} \approx 2$ and $x_{ns}^2 \approx x_{nn}/x_{ss}$. These conditions are indeed consistent and point to a mixing close to maximal ($\phi = 45 \pm 2^\circ$) in the $nn$, $ss$ basis (this corresponds to a conventional ($\eta_8$, $\eta_1$) mixing $\theta$ of $-10 \pm 2^\circ$). We are able to explore the decay constants for singlet pseudoscalar mesons for the first time. Our results show similar decay constants for singlet and non-singlet states of the same mass but with quite large errors.

We have not addressed here the issue of the origin of these mixing parameters $x$. Lattice studies [3] have the capability to relate them to topological charge density fluctuations or to other vacuum properties.

Our lattice studies have been hampered by two constraints. One is that the disconnected quark diagrams needed for a study of singlet mesons are intrinsically noisy. Much larger data sets (tens of thousands of gauge configurations) will be needed to increase precision. Another constraint is that we are unable to work with sea quarks substantially lighter than strange. We have also not attempted a continuum limit extrapolation of our lattice results. Although we are using a lattice formalism that should improve this extrapolation, it would be safer to test it directly. The lattice non-perturbative results do, however, show clearly the structure of the mixing in the singlet pseudoscalar mesons.

References

Analytical result for dimensionally regularized massless master
double box with one leg off shell

V.A. Smirnov

Nuclear Physics Institute of Moscow State University, Moscow 119899, Russia

Received 1 August 2000; accepted 6 September 2000
Editor: P.V. Landshoff

Abstract

The dimensionally regularized massless double box Feynman diagram with powers of propagators equal to one, one leg off the mass shell, i.e. with nonzero $q^2 = p_1^2$, and three legs on shell, $p_i^2 = 0$, $i = 2, 3, 4$, is analytically calculated for general values of $q^2$ and the Mandelstam variables $s$ and $t$. An explicit result is expressed through (generalized) polylogarithms, up to the fourth order, dependent on rational combinations of $q^2, s$ and $t$, and a one-dimensional integral with a simple integrand consisting of logarithms and dilogarithms. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

Massless four-point Feynman diagrams contribute to many important physical amplitudes. They are much more complicated than two- and three-point diagrams because depend on many parameters: the Mandelstam variables $s$ and $t$ and the values of the external momenta squared, $p_i^2$, $i = 1, 2, 3, 4$. In the most general case, when all the legs are off the mass shell, $p_i^2 \neq 0$, there exists an explicit analytical result [1] for the master (i.e. with powers of the propagators equal to one) double box diagram (see Fig. 1) strictly in four dimensions. Still no similar results are available for pure off shell four point diagrams with ultraviolet, infrared and/or collinear divergences.

In the opposite case, when all the end-points are on shell, i.e. for $p_i^2 = 0$, $i = 1, 2, 3, 4$, the problem of the analytical evaluation of such diagrams, in expansion in $\epsilon = (4 - d)/2$ in the framework of dimensional regularization [2] with the space–time dimension $d$ as a regularization parameter, was completely solved during
last year in [3–5]. Among intermediate situations, when some legs are on shell and the rest of them off shell, the case of one leg off shell, \(q^2 = p_i^2 \neq 0\) and three legs on shell is very important because of the relevance to the process \(e^+e^- \rightarrow 3\) jets (see, e.g., [6]). The purpose of this paper is to analytically evaluate the master double box diagram of such type, as a function of \(q^2, s\) and \(t\), and thereby demonstrate that the NNLO analytical calculations for this process are indeed possible.

One of the ways to evaluate the four point diagrams with one leg off shell is to expand them in the limit \(q^2 \rightarrow 0\) and compute as many terms of the resulting expansion as possible. We explain how to do this, following the strategy of regions [7,8], in the next section and present the leading power term in this expansion which provides a very nontrivial check of the subsequent analytical result.

To analytically evaluate the considered diagram we straightforwardly apply the method of Ref. [3]: we start from the alpha-representation of the double box and, after expanding some of the involved functions in Mellin–Barnes (MB) integrals, arrive at a six-fold MB integral representation with gamma functions in the integrand.

\[
\text{The dimensionally regularized master massless double box Feynman integral with one leg off shell, } q^2 = p_i^2 \neq 0, \text{ and three legs on shell, } p_i^2 = 0, \quad i = 2, 3, 4, \text{ can be written as}
\]

\[
F(s, t, q^2; \epsilon) = \int \int \frac{d^d k \, d^d l}{(k^2 + 2 p_i k + q^2)(k^2 - 2 p_2 k) k^2 (k - l)^2} \times \frac{1}{(l^2 + 2 p_1 (l + q^2)(l^2 - 2 p_2 l)(l + p_1 + p_3)^2)},
\]

where \(s = (p_1 + p_2)^2\), \(t = (p_1 + p_3)^2\), and \(k\) and \(l\) are respectively loop momenta of the left and the right box. Usual prescriptions, \(k^2 = k^2 + i 0\), \(s = s + i 0\), etc., are implied. To expand the given diagram in the limit \(q^2 \rightarrow 0\) one can apply the so-called strategy of regions [7,8] based on the analysis of various regions in the space of the loop integration momenta. Taylor expanding the integrand in the parameters that are considered small in the given region and extending resulting integrations to the whole integration domain in the loop momenta. When applying this strategy all integrals without scale are by definition put to zero.

Let us choose, for convenience, the external momenta as follows:

\[
p_1 = \tilde{p}_1 - \frac{q^2}{Q} \tilde{p}_2, \quad p_2 = \tilde{p}_2, \quad \tilde{p}_{1,2} = (\mp Q/2, 0, 0, Q/2),
\]

where \(s = -Q^2\). The given limit \(|q^2| \ll |s|, |t|\) is closely related to the Sudakov limit so that it is reasonable to consider each loop momentum to be one of the following types:

- hard (h): \(k \sim Q \sim \sqrt{t}\),
- 1-collinear (1c): \(k_+ \sim q^2/Q, \quad k_- \sim Q, \quad k \sim \sqrt{-q^2}\),
- 2-collinear (2c): \(k_+ \sim Q, \quad k_- \sim q^2/Q, \quad k \sim \sqrt{-q^2}\).
Here $k_\pm = k_0 \pm k_3$, $k = (k_1, k_2)$. We mean by $k \sim Q$, etc., that any component of $k_\mu$ is of order $Q$.

It turns out that the (h–h), (1c–h) and (1c–1c) are the only nonzero contributions to the leading power behaviour in the limit $q^2 \to 0$. Any term originating from the (h–h) contribution is given by the expansion of the integrand in Taylor series in $q^2$ and expressed through on-shell double boxes in shifted dimensions and can be analytically evaluated by the algorithm presented in [4]. The (1c–1c) contribution is obtained by expanding propagators number 2, 4 and 7 in a special way. In particular, propagators number 2 and 4 are expanded, respectively, in $t^2$ and $k^2$. (See [8] for instructive 2-loop examples of expansions in limits of the Sudakov type.)

The (1c–h) and (1c–1c) contributions are evaluated with the help of a two-fold (respectively, one-fold) MB representation. Still this program of the evaluation of a large number of terms of the expansion looks very complicated because one needs, for phenomenological reasons, the values of $q^2$ greater than $s$ and $t$ so that a reliable summation of a resulting series, using Padé approximants, requires the knowledge of at least first 20–30 terms. Such a great number of terms can be hardly evaluated since a lot of irreducible structures appear. This asymptotic expansion is however very useful for comparison with the explicit result derived below.

The leading power terms of the asymptotic expansion calculated in expansion in $\epsilon$, up to a finite part, are

$$F(s, t, q^2; \epsilon) = \frac{(i \pi)^{d/2} e^{-\nu(\epsilon)}}{(-s)^{2+2\epsilon}(-t)^{2+2\epsilon}} \sum_{i=0}^{4} \frac{g_i(X, Y)}{\epsilon^i} + O(q^2 \ln^3(q^2/s)) + O(\epsilon),$$

where $X = q^2/s$, $Y = t/s$ and

$$g_4(X, Y) = -1,$$

$$g_3(X, Y) = -2(\ln X - \ln Y),$$

$$g_2(X, Y) = \frac{11}{12} \pi^2 + 3 \ln X \ln Y - \frac{3}{2} \ln^2 Y,$$

$$g_1(X, Y) = 2\ln Y \, \text{Li}_2(-Y) - 2 \, \text{Li}_3(-Y) + \frac{7}{2} \ln^3 Y - \frac{1}{2} \ln X \ln^2 Y - \frac{3}{2} \ln^2 X \ln Y - \frac{1}{2} \ln X \ln^3 Y + \ln^2 Y \ln(1 + Y) + \pi^2 \left[ \frac{3}{2} \ln X - \frac{19}{18} \ln Y + \ln(1 + Y) \right] + \frac{60}{7} \xi(3),$$

$$g_0(X, Y) = 26 \, \text{Li}_4(-Y) - 2 S_{2,2}(-Y) - 2 \left( \ln X + 6 \ln Y + \ln(1 + Y) \right) \text{Li}_3(-Y) + 2 \ln Y \, \text{Li}_3 \left( \frac{Y}{1 + Y} \right) + (\ln^2 Y + 2 \ln X \ln Y + 4 \pi^2) \, \text{Li}_2(-Y) - \frac{1}{2} \ln^4 X + \frac{1}{2} \ln^3 X \ln Y + \frac{1}{2} \ln^2 X \ln^2 Y + \frac{1}{2} \ln X \ln^3 Y + \frac{7}{2} \ln^4 Y + \ln(1 + Y) \ln \ln X \ln^2 Y - \frac{3}{2} \ln^3 Y + \frac{1}{2} \ln^2 Y \ln(1 + Y) - \frac{1}{2} \ln Y \ln^2(1 + Y)] + \pi^2 \left[ - \frac{7}{4} \ln^2 X - \frac{1}{4} \ln X \ln Y + \frac{25}{6} \ln^2 Y + \ln X \ln(1 + Y) - 2 \ln Y \ln(1 + Y) + \frac{1}{2} \ln^2(1 + Y) \right] + \xi(3) \left[ \frac{19}{18} \ln X - \frac{19}{18} \ln Y + 2 \ln(1 + Y) \right] + \frac{8}{729} \pi^4.$$  \hspace{1cm} (3)

Here $\text{Li}_a(z)$ is the polylogarithm [9] and

$$S_{a,b}(z) = \frac{(-1)^a + b - 1}{(a - 1)! b!} \int_0^1 \ln^{a-1}(t) \ln^b(1-zt) \, dt$$

the generalized polylogarithm [10].

3. From alpha parameters through MB representation to analytical result

The alpha representation of the double box looks like:

$$F(s, t, q^2; \epsilon) = - \Gamma(3 + 2\epsilon) \left( \frac{i \pi}{d/2} \right)^2 \int_0^\infty d\alpha_1 \cdots \int_0^\infty d\alpha_7 \delta \left( \sum \alpha_i - 1 \right) D^{1+3\epsilon} A^{3-2\epsilon},$$

\hspace{1cm} (5)
where

\[ D = (a_1 + a_2 + \alpha_7)(a_3 + a_4 + a_5) + a_6(a_1 + a_2 + a_3 + a_4 + a_5 + a_7), \]

\[ A = [a_1a_2(a_3 + a_4 + a_5) + a_3a_4(a_1 + a_2 + a_7) + a_6(a_1 + a_3)(a_2 + a_4)](-s) \]

\[ + a_5a_6a_7(-t) + a_5[(a_1 + a_3)a_6 + a_3(a_1 + a_2 + a_7)](-q^2). \]

As it is well-known, one can choose a sum of an arbitrary subset of \( \alpha_i, \ i = 1, \ldots, 7, \) in the argument of the delta function in (5), and we use the same choice as in [3].

Starting from (5) we perform the same change of variables as in [3] and apply seven times the MB representation

\[ \frac{1}{(X + Y)^v} = \frac{1}{\Gamma(v)} \int_{-i\infty}^{+i\infty} dw \frac{Y^w}{X + w} \Gamma(v + w)\Gamma(-w) \]

in order to separate terms in the functions involved to make possible an explicit parametric integration. The two extra MB integrations arise form the extra term with \( q^2. \) After such integrations we are left with a 7-fold MB integral of a ratio of gamma functions. Fortunately, one of the integrations can be explicitly taken using the first Barnes lemma and we arrive at the following nice 6-fold MB integral:

\[ F(s, i, q^2; \epsilon) = -\frac{(i\pi^{d/2})^2}{\Gamma(-1 - 3\epsilon)(-s)^{\epsilon + 2\epsilon}} \frac{1}{(2\pi i)^d} \]

\[ \times \int dv \, dw \, dw_2 \, dw_3 \, dz_1 \left( \frac{q^2}{s} \right)^v \left( \frac{l}{s} \right)^w \Gamma(1 + w)\Gamma(1 + v + w)\Gamma(-v)\Gamma(-w) \]

\[ \times \Gamma(1 - w_3 + v)\Gamma(1 - 2\epsilon - w - w_2)\Gamma(1 - 2\epsilon - w - w_3) \]

\[ \times \Gamma(1 - w_2 + z_1)\Gamma(1 - w_3 + z_1)\Gamma(1 - w_2 + w_3 + z - z_1) \]

\[ \times \Gamma(1 - z + z_1)\Gamma(1 + w + w_2 + w_3 - z) \]

\[ \times \Gamma(-2 - 3\epsilon - w - w_2 - w_3 + z_1 - z) \Gamma(z_1 - z) \]

\[ \times \Gamma(3 + 2\epsilon + w + z). \]

It differs from its analog for \( q^2 = 0 \) by the additional integration in \( v. \) This variable enters only four gamma functions in the integrand. The integral is evaluated in expansion in \( \epsilon, \) up to a finite part, by resolving singularities in \( \epsilon \) absolutely by the same strategy as in the case \( q^2 = 0 \) [3]. Note that the infrared and collinear poles are a little bit softer than in the pure on-shell case, the integration variable \( v \) playing the role of an infrared regulator. The two key gamma functions that are responsible for the generation of poles in \( \epsilon \) are the same as in the previous case:

\[ \Gamma(\epsilon + w + w_2 + w_3 - z_1) \Gamma(-2 - 3\epsilon - w - w_2 - w_3 + z_1 - z). \]

The labeling of resulting terms is therefore the same: the initial integral is decomposed as \( J = J_{00} + J_{01} + J_{10} + J_{11}, \) etc. (Only arguments of some gamma functions are shifted by \( v. \) The applied strategy makes it possible to perform all the integrations apart from the last two, in \( v \) and \( w. \) We obtain four groups of terms with 26 terms in each group: the terms without MB integration, with MB integration in \( v \) or \( w \) and, finally, with a two-fold integration in \( v \) and \( w. \) The one-fold integrals are explicitly evaluated by closing contour and summing up series, using formulae from [11].

The contribution of the resulting two-fold MB integral takes the form

\[ \frac{2(i\pi^{d/2})^2}{-s^3} \frac{1}{(2\pi i)^d} \int dv \, dw \left( \frac{q^2}{s} \right)^v \left( \frac{l}{s} \right)^w \Gamma(1 + v + w)\Gamma(-v)\Gamma(1 + w)\Gamma(-w)^2 \]

\[ \times \left[ \Gamma(1 + v + w)\Gamma(-v - w) \left( \frac{1}{\epsilon} - y \right) - 2\ln(-s) - \frac{5}{1 + w} - \frac{1}{1 + v + w} + \psi(1 + v) - 2\psi(-v - w) \right] \]
\[-3\psi(-w) + 2\psi(1+w) + \psi(1+v+w) - \Gamma(1+v)\Gamma(-v)\Gamma(1+w)\Gamma(-w)]

The integration contours are straight lines along imaginary axes with \(-1 < \text{Re } v\). Re \(w\). Re \(v + w < 0\). By closing contours it is possible to convert this integral into a two-fold series where each term is identified as a derivative of the Appell function \(F_2\) in parameters, up to the third order. The \(1/e\) part is then explicitly summed up with a result in terms of polylogarithms. (In fact, it is proportional to the \(e\) part of the master one-loop box.)

The so obtained result can be transformed into a one-dimensional integral with a simple integrand. To present the final result let us turn to the variables \(x = s/q^2\) and \(y = t/q^2\) keeping in mind typical phenomenological values of the involved parameters relevant to the process \(e^+e^- \rightarrow 3\) jets:

\[
F(s, t, q^2; \epsilon) = \frac{(\pi^{d/2}e^{-\epsilon\pi})^2}{-s^{d/2}(-q^2)^{d/2}} \sum_{i=0}^{4} \frac{f_i(x, y)}{\epsilon^i} + O(\epsilon). 
\]

We obtain:

\[
f_0(x, y) = -1, \tag{12}
f_3(x, y) = 2(\ln x + \ln y), \tag{13}
f_2(x, y) = 3L_{12}(x) + L_{12}(y) - 2(\ln x + \ln y)^2 + 3\ln(1-x)\ln x + \ln(1-y)\ln y - \frac{5\pi^2}{2}, \tag{14}
f_1(x, y) = 2\left[ L_{13}\left(\frac{-x}{1-x-y}\right) + L_{13}\left(\frac{-y}{1-x-y}\right) - L_{13}\left(\frac{-xy}{1-x-y}\right) - \ln x L_{12}\left(\frac{y}{1-x}\right) - \ln y L_{12}\left(\frac{x}{1-y}\right) \right] 
+ 2\ln(1-x-y)\left[ -\frac{1}{6}(\ln^2(1-x-y) + \pi^2) + \ln(1-x)\ln x + \ln(1-y)\ln y - \ln x \ln y \right] 
+ 3 L_{13}(x) - 8 L_{12}(y) + 4 L_{13}\left(\frac{-x}{1-x-2x}\right) - 2 L_{13}\left(\frac{-y}{1-y}\right) - (3 \ln x + 4 \ln y) L_{12}(x) + 3 \ln y L_{12}(y) 
+ \frac{2}{3} \ln^3 x - \frac{2}{3} \ln^3(1-x) + \ln^2(1-x) \ln x - \frac{2}{3} \ln(1-x) \ln^2 x + \frac{1}{6} \pi^2 (5 \ln x - 4 \ln(1-x)) 
+ \frac{2}{3} \ln^3 y + \frac{1}{3} \ln^3(1-y) - 2 \ln^2(1-y) \ln y - \ln(1-y) \ln^2 y + \frac{1}{6} \pi^2 (5 \ln y + 2 \ln(1-y)) 
+ 4 \ln x \ln y (\ln x - \ln(1-x) + \ln y) + \frac{25}{6} \zeta(3). \tag{15}
\]

The \(e^0\) part involves a one-dimensional integral:

\[
f_0(x, y) = \int_0^1 dx \left\{ -x \ln(1-x) \ln(1-y) \ln(1-x) + \ln(1-y)\ln(1-x) + 2 L_{12}(z) 
+ 2(6 \ln(1-z) - \ln z) L_{12}(-(1-y-xz))/y \right\] 
- 5 L_{14}(x) + 14 L_{14}\left(\frac{-x}{1-y}\right) - 6 L_{14}\left(\frac{-xy}{1-x(1-y)}\right) 
+ 8 L_{14}\left(\frac{-x}{1-x-y}\right) + 2 L_{14}(y) - 2 L_{14}(1-y) + 8 L_{14}\left(\frac{-y}{1-y}\right) - 2 L_{14}\left(\frac{y}{1-x}\right) 
- 8 L_{14}(1-x) - 8 L_{14}\left(\frac{-y}{1-x-y}\right) - 20 L_{14}\left(\frac{1-x-y}{1-y}\right) + 10 L_{14}\left(\frac{1-x-y}{1-x}\right) 
- 3 S_{2,2}(x) - 8 S_{2,2}(y) - 6 S_{2,2}\left(\frac{x}{1-y}\right) + (2 \ln y - 2 \ln x - 3 \ln(1-x)) L_{13}(x) 
\]
\[+ 2(16 \ln(1 - y) - 11 \ln y - 2 \ln(1 - x - y) + \ln x) \text{Li}_3\left(\frac{x}{1 - y}\right)\]
\[- (8 \ln y + 2 \ln(1 - x) + 3 \ln x) \text{Li}_3\left(\frac{-x}{1 - x}\right)\]
\[- 2(4 \ln(1 - y) - 4 \ln y - \ln(1 - x) + \ln x) \text{Li}_3\left(\frac{xy}{(1 - x)(1 - y)}\right)\]
\[+ (14 \ln(1 - y) - 18 \ln y + 4 \ln(1 - x - y)) \text{Li}_3\left(\frac{-x}{1 - x - y}\right) + 2 \ln y \text{Li}_3\left(\frac{-xy}{1 - x - y}\right)\]
\[+ (7 \ln y - 8 \ln(1 - y)) \text{Li}_3(y) + (8 \ln(1 - y) + \ln y + 2 \ln x) \text{Li}_3\left(\frac{-y}{1 - y}\right)\]
\[- 4(2 \ln y + 7 \ln(1 - x) + 2 \ln(1 - x - y) - 8 \ln x) \text{Li}_3\left(\frac{y}{1 - x}\right)\]
\[- 2(\ln y + 5 \ln(1 - x) + 8 \ln(1 - x - y) - 7 \ln x) \text{Li}_3\left(\frac{-y}{1 - x - y}\right)\]
\[\frac{1}{2}(\text{Li}_2(x))^2 - (\text{Li}_2\left(\frac{x}{1 - y}\right))^2 - \frac{3}{2}(\text{Li}_2(y))^2 + 4\left(\text{Li}_2\left(\frac{y}{1 - x}\right)\right)^2\]
\[+ [\ln^2(1 - x) - 4 \ln^2 y + 2 \ln y(4 \ln(1 - x) - \ln x) - 2 \ln(1 - y) \ln x - 3 \ln(1 - x) \ln x\]
\[+ \frac{7}{4} \ln^2 x + \frac{3}{2} \pi^2] \text{Li}_2(x)\]
\[+ [12 \ln^2(1 - y) + 15 \ln^2 y + 2 \ln(1 - y)(\ln(1 - x - y) + \ln x - 9 \ln y)\]
\[+ 2 \ln y(\ln(1 - x - y) - 4 \ln(1 - x) + 4 \ln x) - 2(\ln^2(1 - x - y) + \ln^2 x)] \text{Li}_2\left(\frac{x}{1 - y}\right)\]
\[+ [4 \ln^2(1 - y) - 8 \ln^2 y + 2 \ln y(4 \ln(1 - x) - 3 \ln x) + 2 \ln(1 - y)(4 \ln y - \ln x)\]
\[+ \ln^2 x - \frac{1}{4} \pi^2] \text{Li}_2(y)\]
\[+ [8 \ln^2 y - 8 \ln y \ln(1 - x) - 10 \ln^2(1 - x) + 8 \ln^2(1 - x - y) - 8 \ln(1 - x - y) \ln x\]
\[+ 8 \ln(1 - x)(\ln(1 - x - y) - 2 \ln x) + 2 \ln(1 - y) \ln x + \ln^2 x - \frac{7}{4} \pi^2] \text{Li}_2\left(\frac{y}{1 - x}\right)\]
\[+ [\ln^2(1 - x) - 4 \ln^2(1 - y) - 8 \ln^2 y + 2 \ln y(4 \ln(1 - x) - 3 \ln x) + 2 \ln(1 - y)(4 \ln y - \ln x)\]
\[- 2 \ln(1 - x) \ln x + 2 \ln^2 x] \text{Li}_2\left(\frac{xy}{(1 - x)(1 - y)}\right)\]
\[+ 2 \ln^2(1 - x - y) + \frac{1}{4} \ln^2(1 - x - y)[2 \ln y - 3 \ln(1 - y) - 9 \ln x - 11 \ln(1 - x)]\]
\[+ \ln^2(1 - x - y)\left[\pi^2 - 3 \ln^2(1 - y) + 3 \ln^2 y + 6 \ln^2(1 - x) - \ln(1 - y)(\ln y + 10 \ln x)\right]\]
\[+ 4 \ln(1 - x) \ln x - 2 \ln^2 x - \ln y(5 \ln(1 - x) + \ln x)\]
\[+ \frac{1}{4} \ln(1 - x - y)\left[7 \ln^2(1 - y) - 2 \ln(1 - y)(5 \pi^2 + 12 \ln^2 y) + 7 \pi^2 \ln(1 - x) + \pi^2 \ln x\right]\]
\[+ 6 \ln^2 y \ln x - 4 \ln^3(1 - x) + 15 \ln^2(1 - y)(\ln y - 2 \ln x) - 21 \ln^2(1 - x) \ln x\]
\[+ 3 \ln(1 - x) \ln^2 x + \ln y(2 \pi^2 + 9 \ln^2 x + 15 \ln^2(1 - x) - 6 \ln(1 - x) \ln x)\]
\[- \frac{7}{4} \ln^3(1 - x) - \frac{7}{4} \ln^4 x + \frac{3}{4} \ln^3(1 - x) \ln x - \frac{13}{4} \ln^2(1 - x) \ln^2 x + \frac{7}{4} \ln(1 - x) \ln^3 x\]
\[- \frac{1}{4} \pi^2(3 \ln^3(1 - x) - 10 \ln(1 - x) \ln x + 5 \ln^2 x) - \ln^4(1 - y) - \frac{7}{4} \ln^4 y + \frac{13}{4} \ln^3(1 - y) \ln y\]
\[+ 5 \ln^2(1 - y) \ln^2 y + \frac{7}{4} \ln(1 - y) \ln^3 y + \pi^2(9 \ln^2(1 - y) - \ln(1 - y) \ln y - 5 \ln^2 y)\]
\[+ \frac{1}{4} \ln(1 - x) \ln(1 - y)(\ln^2(1 - x) - 4 \ln^2(1 - y)) - \frac{7}{4} (\ln^2 x + \ln^2 y) \ln x \ln y\]
\[+ 3 \ln^3(1 - y) \ln x + \ln^2(1 - y) \ln y(4 \ln(1 - x) + \ln x) - 2 \ln(1 - x) \ln x\]
\begin{align*}
&+ \frac{1}{3} \ln y \left[ - \ln^3(1-x) - 9 \ln^2(1-x) \ln x - 12 \ln y \ln^2 x + 6 \ln(1-x) \ln x(2 \ln y + 3 \ln x) \right] \\
&- \ln(1-y) \left[ 8 \ln^2 y \ln(1-x) + \ln(1-x) \ln x(\ln(1-x) - 2 \ln x) \right] \\
&+ \ln y \left[ -8 \ln^2(1-x) + 6 \ln(1-x) \ln x + \ln^2 x \right] \\
&+ \frac{1}{2} \pi^2 \left[ \ln y (4 \ln(1-x) - 5 \ln x) - \ln(1-y) \ln x \right] \\
&+ \zeta(3) \left[ 12 (\ln(1-x) - \ln(1-y)) + 13 \ln(1-x) - \frac{25}{3} (\ln x + \ln y) \right] + \frac{23}{180} \pi^4. \quad (16)
\end{align*}

One may hope that the one-dimensional integral that is left can also be evaluated in terms of polylogarithms. To do this it is necessary to complete the table of integrals derived in [9].

This result is in agreement with the leading power behaviour when $q^2 \to 0$ (3). When performing this comparison it is reasonable to start with (10), take minus residue at $v = 0$ (the first pole of $\Gamma(-v)$), integrate in $w$ by closing the contour to the right, and take into account the three other contributions (without MB integration, and with integration in $v$ or $w$) that were not presented above. Eqs. (12)–(16) also agree with results based on numerical integration in the space of alpha parameters [12] (where the 1% accuracy for the $1/e$ and $e^0$ parts is guaranteed).

Acknowledgements

I am grateful to Z. Kunszt for involving me into this problem and for kind hospitality during my visit to ETH (Zürich) in April–May 2000 where an essential part of this work was performed. I am thankful to T. Binoth and G. Heinrich for comparison of the presented result with their results based on numerical integration. Thanks to A.I. Davydychev and O.L. Veretin for useful discussions. This work was supported by the Volkswagen Foundation, contract No. I/73611, and by the Russian Foundation for Basic Research, project 98–02–16981.

References

Neutrino oscillations in electromagnetic fields

A.M. Egorov, A.E. Lobanov, A.I. Studenikin

Department of Theoretical Physics, Moscow State University, 119899 Moscow, Russia

Received 22 March 2000; received in revised form 20 June 2000; accepted 6 September 2000

Editor: P.V. Landshoff

Abstract

Oscillations of neutrinos $\nu_L \leftrightarrow \nu_R$ in presence of an arbitrary electromagnetic field are considered. We introduce the Hamiltonian for the neutrino spin-evolution equation that accounts for possible effects of interaction of neutrino magnetic $\mu$ and electric $e$ dipole moments with the transversal (in respect to the neutrino momentum) and also the longitudinal components of electromagnetic field. Using this Hamiltonian we predict the new types of resonances in the neutrino oscillations $\nu_L \leftrightarrow \nu_R$ in the presence of the field of an electromagnetic wave and in combination of an electromagnetic wave and constant magnetic field. The possible influence of the longitudinal magnetic field on neutrino oscillations is emphasized. © 2000 Elsevier Science B.V. All rights reserved.

The electromagnetic properties of neutrinos are among the most interesting issues in particle physics. Studies of the neutrino electromagnetic properties could provide an important information about the structure of theoretical model of particle interaction. For instance, the discovery of the nonvanishing neutrino magnetic moment, as well as the neutrino mass, would clearly indicate that the Standard Model has to be generalized.

The nonvanishing neutrino magnetic moment has also crucial consequences in astrophysics. As it has been shown in plenty of studies (see, for example, [1 – 18]) that have emerged during past decades, the neutrino conversions and oscillations produced under the influence of transversal constant or constant and twisting (in space) magnetic fields could be important for evolution of astrophysical object, like the Sun and neutron stars, or could result in sufficient effects while neutrinos propagate through interstellar galactic media.¹

In the previously performed studies of neutrino spin precession only effects of the neutrino magnetic (or flavour transition) moment interaction with transversal constant or twisting magnetic fields were considered (see, for example, [3 – 6, 8 – 15]). The influence of the longitudinal component of magnetic field is usually neglected because in the relativistic limit it is suppressed. In the presence of magnetic fields the neutrino evolution equation accounting for the magnetic moment interaction can be received on the basis of relativistic wave equations. The usually discussed Hamiltonians for the neutrino time evolution Schrödinger equation can be derived by expanding the exact Dirac

¹ It should be noted here that the neutrino helicity flip could be caused not only by the interaction with an external magnetic field (or, as it is shown below with an electromagnetic wave) but also by the scattering with charged fermions in the background (see, for example, [19] and references therein).
Hamiltonian in powers of the neutrino kinetic energy. In the lowest order there is no dependence on the longitudinal to the neutrino momentum component of the magnetic field, because the transversal components of the field acquire a factor $\gamma = (1 - \beta^2)^{-1/2}$ in the rest frame of neutrino ($\beta = \bar{v}/c$, where $v$ is the neutrino speed).

The purpose of this paper is to generalize the Hamiltonian describing the neutrino spin evolution for the case of the neutrino motion in an arbitrary configuration of electromagnetic fields. We derive [20,21] the Hamiltonian that accounts not only for the transversal to the neutrino momentum components of electromagnetic field but also for the longitudinal components. With the using of the proposed Hamiltonian it is possible to consider neutrino spin precession in an arbitrary configuration of electromagnetic fields including those that contain strong longitudinal components. We also consider the new effect of the neutrino spin precession that could appear when neutrinos propagate in matter under the influence of a field of electromagnetic wave and the superposition of electromagnetic wave and constant longitudinal magnetic field. The new types of resonances in the neutrino oscillations $\nu_L \leftrightarrow \nu_R$ in such field configurations are predicted. The influence of the longitudinal component of the magnetic field on the neutrino oscillations is discussed.

The equation for the neutrino spin evolution in electromagnetic field $F_{\mu\nu}$ is obtained on the basis of the Bargmann–Michel–Telegdi (BMT) equation [22] for the spin vector $S^\mu$ of a neutral particle that has the following form

$$\frac{dS^\mu}{dt} = 2\mu \left\{ F_{\mu\nu} S^\nu - u^\mu (\bar{u}_\nu F^{\nu\lambda} S^\lambda) \right\} + 2\epsilon \left\{ \tilde{F}_{\mu\nu} S^\nu - u^\mu (\bar{u}_\nu \tilde{F}^{\nu\lambda} S^\lambda) \right\}. \tag{1}$$

We suppose that the particle is moving with constant speed, $\beta = \text{const}$, in presence of an electromagnetic field $F_{\mu\nu}$. Here $\mu$ is the fermion magnetic moment and $\tilde{F}_{\mu\nu}$ is the dual electromagnetic field tensor. Eq. (1) covers also the case of a neutral fermion having static nonvanishing electric dipole moment, $\epsilon$. Note that the term proportional to $\epsilon$ violates $T$ invariance.

Let us underline that Eq. (1) accounts for the direct interaction of a neutral fermion with electromagnetic field $F_{\mu\nu}$. It should be noted here that there could be an indirect influence of electromagnetic field on a neutral fermion via one-loop finite-density contributions to the particle self-energy in electromagnetic field.

The BMT equation (1) is derived in the frame of electrodynamics. However, neutrino participates also in weak interaction in which, contrary to the electromagnetic interaction, $P$ invariance is not conserved. Clearly, this fact has to be reflected in the form of equation that describes the neutrino spin evolution in an electromagnetic field. Our goal is to modify Eq. (1) and to derive the new one which is appropriate for description of the neutrino spin evolution in electromagnetic fields.

We obtain [20,21] the Lorentz invariant generalization of Eq. (1) for the case of $P$ invariance violating theory demanding that the equation has to be linear over spin vector $S_\mu$ and electromagnetic field $F_{\mu\nu}$. Nonconservation of $P$ invariance implies existence of a preferred direction in space in any reference frame. The only choice of this direction is given by vector $\bar{\eta} = \bar{\beta}/\beta$. Thus, the Lorentz invariant generalization of Eq. (1) can be obtained by the substitution of the electromagnetic field tensor $F_{\mu\nu} = (\bar{e}, \bar{B})$ in the following way:

$$F_{\mu\nu} \rightarrow F_{\mu\nu} + G_{\mu\nu}, \tag{2}$$

where the antisymmetric tensor $G_{\mu\nu}$ is constructed on the base of the vector $\bar{\eta}$ in a way which is analogous to one by which the electromagnetic tensor $F_{\mu\nu}$ is constructed on the base of the polar vector $\bar{E}$ and axial vector $\bar{B}$:

$$G_{\mu\nu} = (\xi \bar{\eta}, \rho \bar{\eta}). \tag{3}$$

Here $\rho$ and $\xi$ are scalars. This substitution (2) in the case of the constant velocity $\bar{\beta} = \text{const}$, implies that the magnetic $\bar{B}$ and electric $\bar{E}$ fields are shifted by the vectors $\rho \bar{\eta}$ and $\xi \bar{\eta}$, respectively:

$$\bar{B} \rightarrow \bar{B} + \rho \bar{\eta}, \quad \bar{E} \rightarrow \bar{E} + \xi \bar{\eta}. \tag{4}$$

We finally arrive [20,21] to the following equation for the three-dimensional neutrino spin vector $\bar{S}$:

$$\frac{d\bar{S}}{dt} = \frac{2\mu}{\gamma} \left[ \bar{S} \times (\bar{B}_0 + \rho \bar{\eta}) \right] + \frac{2\epsilon}{\gamma} \left[ \bar{S} \times (\bar{E}_0 + \xi \bar{\eta}) \right]. \tag{5}$$

The derivative in the left-hand side of Eq. (5) is taken with respect to time $t$ in the laboratory frame, whereas the values $\bar{B}_0$ and $\bar{E}_0$ are the magnetic and electric fields in the neutrino rest frame.
\[ \tilde{B}_0 = \gamma \left( \tilde{B}_\perp + \frac{1}{\gamma} \tilde{B}_1 + \sqrt{1 - \frac{1}{\gamma^2} \left[ \tilde{E}_\perp \times \tilde{n} \right]} \right), \]
\[ \tilde{E}_0 = \gamma \left( \tilde{E}_\perp + \frac{1}{\gamma} \tilde{E}_1 - \sqrt{1 - \frac{1}{\gamma^2} \left[ \tilde{B}_\perp \times \tilde{n} \right]} \right), \]
\[ \tilde{F}_\perp = \tilde{F} - \tilde{n}(\tilde{F}\tilde{n}), \quad \tilde{F}_|| = \tilde{n}(\tilde{F}\tilde{n}), \quad \tilde{F} = \tilde{B} \text{ or } \tilde{E}, \tag{7} \]

where

\[ \tilde{F}_\perp = \tilde{F} - \tilde{n}(\tilde{F}\tilde{n}), \quad \tilde{F}_|| = \tilde{n}(\tilde{F}\tilde{n}), \quad \tilde{F} = \tilde{B} \text{ or } \tilde{E}, \tag{7} \]

are the transversal and longitudinal, in respect to the direction of the neutrino motion components of magnetic and electric fields in the laboratory frame.

It should be noted here that the values of scalars \( \rho \) and \( \xi \) describe all kinds of interactions in which neutrinos participate except the direct electromagnetic interaction of the neutrino magnetic and electric moments with the external electromagnetic field that are given by terms proportional to \( B_0 \) and \( E_0 \). The explicit expressions for the values \( \rho \) and \( \xi \) depend on the considered model of the neutrino interaction. Let us introduce the spin tensor \( [23] \)
\[ S = \tilde{\sigma} \tilde{S}, \quad \tilde{\sigma} = (\sigma_1, \sigma_2, \sigma_3), \tag{8} \]
that can be used for description of the neutrino spin states (here \( \sigma_i \) are the Pauli matrices). The behavior of \( S = S(t) \) is given by the evolution operator \( U \):
\[ S(t) = US(t_0)U^+. \tag{9} \]

For this operator one can get the Schrödinger type equation
\[ \frac{1}{i} \frac{dU}{dt} = Hu \tag{10} \]
with the Hamiltonian given by
\[ H = \langle \tilde{\sigma} \tilde{n} \rangle \left( \frac{\Delta m^2}{4E} - \frac{V}{2} - \frac{1}{\gamma} (\mu B_\parallel + \epsilon E_\parallel) \right) - \mu \tilde{\sigma} \left( \tilde{B}_\perp + [\tilde{E}_\perp \times \tilde{n}] \right) - \epsilon \tilde{\sigma} \left( \tilde{E}_\perp - [\tilde{B}_\perp \times \tilde{n}] \right). \tag{12} \]

The two parameters, \( A = A(\theta) \) being a function of vacuum mixing angle and \( V = V(n_{\text{eff}}) \) being the difference of neutrino effective potentials in matter depend on the nature of neutrino conversion processes in question. For specification of \( A \) and \( V \) for different types of the neutrino conversions see, for example, in Refs. [13,17]. Then, taking into account \( C \), \( P \), and \( T \) transformation properties of different terms of the Hamiltonian we come from (11), (6) and (12) to the following identification:
\[ \mu \rho + \epsilon \xi \to \gamma \left( \frac{V}{2} - \frac{\Delta m^2 /4E}{\gamma} \right). \tag{13} \]

Finally we get the effective Hamiltonian that determines the evolution of the system \( \nu = (\nu_R,\nu_L) \) in presence of electromagnetic field with given components \( B_\parallel, E_\parallel \) in the laboratory frame:
\[ H = \langle \tilde{\sigma} \tilde{n} \rangle \left( \frac{\Delta m^2 A}{4E} - \frac{V}{2} - \frac{1}{\gamma} (\mu B_\parallel + \epsilon E_\parallel) \right) - \mu \tilde{\sigma} \left( \tilde{B}_\perp + [\tilde{E}_\perp \times \tilde{n}] \right) - \epsilon \tilde{\sigma} \left( \tilde{E}_\perp - [\tilde{B}_\perp \times \tilde{n}] \right). \tag{14} \]

In this expression for the Hamiltonian the terms proportional to \( 1/\gamma^2 \) and higher corrections in powers of \( 1/\gamma \) are omitted. It should be noted that the difference of neutrino effective potentials in matter, \( V \), may contain also contributions [24] from the medium polarization by the longitudinal magnetic field.

As it follows from Eq. (14) the effective Hamiltonian depends on the transversal \( B_\perp \) and longitudinal \( B_\parallel \), \( E_\parallel \) components of the magnetic and electric fields. Terms proportional to \( B_\parallel \), \( E_\parallel \) are suppressed by a factor of \( 1/\gamma \ll 1 \) for the case of relativistic neutrinos. However, for electromagnetic field configurations with strong enough components \( B_\parallel \) and \( E_\parallel \) these terms can be important. In particular, as it will be shown below the longitudinal component of the magnetic field could affect the resonance condition in the neutrino oscillations \( \nu_L \leftrightarrow \nu_R \) (as well as in the helicity preserving neutrino oscillations) through the direct interaction of the neutrino magnetic moment with \( B_\parallel \). This effect appears in addition to the influence of the longitudinal magnetic field on neutrino oscillations due to magnetic polarization of medium [24,25].
Let us use the Hamiltonian (14) for considering the neutrino spin precession in a field of electromagnetic wave with frequency $\omega$. Here we suppose that the neutrino velocity is constant. We denote by $\tilde{e}_3$ the axis that is parallel with $\vec{n}$ and by $\phi$ the angle between $\tilde{e}_3$ and the direction of the wave propagation. For simplicity we shall neglect terms proportional to the neutrino electric dipole moment $\epsilon$. In this case the magnetic field in the neutrino rest frame is given by

$$\vec{B}_0 = \gamma \left[ B_1 (\cos \phi - \beta) \tilde{e}_1 + B_2 (1 - \beta \cos \phi) \tilde{e}_2 - \frac{1}{\gamma} B_1 \sin \phi \tilde{e}_3 \right], \tag{15}$$

where $\tilde{e}_{1,2,3}$ are the unit orthogonal vectors. For the electromagnetic wave of circular polarization propagating in matter it is easy to get:

$$B_1 = B \cos \psi, \quad B_2 = B \sin \psi, \tag{16}$$

where $B$ is the amplitude of the magnetic field in the laboratory frame and the phase of the wave at the point where the neutrino is located at given time $t$ is

$$\psi = g \omega t \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right). \tag{17}$$

The phase depends on the wave speed $\beta_0$ in matter ($\beta_0 \ll 1$). The values $g = \pm 1$ correspond to the two types of the circular polarization of the wave.

The difference of the two terms, $(\cos \phi - \beta)$ and $(1 - \beta \cos \phi)$ of Eq. (15) is proportional to $1/\gamma^2$. Neglecting this difference, we obtain

$$\vec{R} = \left( -\frac{V}{2} + \frac{\Delta m^2 A}{4E} + \frac{1}{\gamma} \mu B_1 \sin \phi \right) \tilde{e}_3 + \mu B (1 - \beta \cos \phi) (\tilde{e}_1 \cos \psi - \tilde{e}_2 \sin \psi). \tag{18}$$

The exact solution of Eq. (10) with the Hamiltonian determined by Eq. (18) for the neutrino spin evolution in the electromagnetic wave can be obtained in the case of $\sin \phi = 0$ (parallel or antiparallel propagation of the electromagnetic wave in respect to the neutrino momentum). Moreover, it is possible to show that for any direction of the wave propagation ($\sin \phi \neq 0$) the presence of the term $\gamma^{-1} B_1 \sin \phi$ leads to insufficient changes of the type of the solution because $B_1$ according to the definition (16) is an oscillating function of time and there is also a suppression by a factor of $\gamma^{-1}$. If we neglect this term the solution of Eq. (10) for the evolution operator $U(t)$ can be written in the form

$$U(t) = U_{\tilde{e}_3} (\psi - \psi_0) U_{\tilde{e}} (\chi - \chi_0). \tag{19}$$

$$U_{\tilde{e}_3} (\psi) = \exp \left( i \frac{\psi}{2} \right), \tag{20}$$

$$U_{\tilde{e}} (\chi) = \exp \left( i \frac{(\vec{\sigma} \vec{l}) \chi}{l} \right). \tag{21}$$

The evolution operator $U(t)$ is a combination of the operator which describes rotation on the angle $\chi - \chi_0 = 2l (t - t_0)$ around the axis $\vec{l}$, and the rotation operator on the angle $\psi - \psi_0$ around the axis $\tilde{e}_3$ (the initial conditions for some time $t_0$ are fixed by the angles $\psi_0$ and $\chi_0$). For the vector $\vec{l}$ we get

$$\vec{l} = \left( \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\psi}{2} \right) \tilde{e}_3 - \mu B (1 - \beta \cos \phi) (\tilde{e}_1 \cos \psi - \tilde{e}_2 \sin \psi). \tag{22}$$

From the exact expression (21) for vector $\vec{l}$ one can straightforwardly get probabilities of conversions $\nu_L \leftrightarrow \nu_R$ between different types of neutrinos with change of helicity. It can be seen from Eq. (21) that the amplitude of probability of conversion ($\sin^2 \theta_{\text{eff}}$) between the two neutrino helicity states could become sufficient, i.e. $\sin^2 \theta_{\text{eff}} \sim 1$, when vector $\vec{l}$ is orthogonal or nearly orthogonal to the axis $\tilde{e}_3$. This happens when the condition

$$\left| \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\omega}{2} \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right) \right| \ll \mu B (1 - \beta \cos \phi) \tag{23}$$

is satisfied. It follows that the probability amplitude of conversion $\nu_L \leftrightarrow \nu_R$ in the electromagnetic wave could get its maximum value ($\sin^2 \theta_{\text{eff}} = 1$) for any strength of the field $B$ (it is supposed that $\mu B (1 - \beta \cos \phi) \neq 0$) when the resonance condition is fulfilled:

$$\left| \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\omega}{2} \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right) \right| = 0. \tag{24}$$

Eq. (23) represents the new type of the resonance conditions for the neutrino conversion processes $\nu_L \leftrightarrow \nu_R$ under the influence of the field of the electromagnetic wave specified by the amplitude of the magnetic field $B$, the frequency $\omega$, the polarization $g = \pm 1$, the direction of propagation (given by the angle $\phi$) in respect to the neutrino momentum, and the speed...
of propagation in matter $\beta_0 \leq 1$. As it follows from Eq. (23) for the fixed values of $\cos \phi$, $\beta$, and $\beta_0$ the resonance condition for the particular conversion process $\nu_L \leftrightarrow \nu_R$ can be satisfied only for one of the two possible wave polarizations.

If for the particular conversion process $\nu_L \leftrightarrow \nu_R$ the resonance condition is not satisfied then from the inequality (22) we can get (in a way similar to our analysis for the case of the constant (and twisting) magnetic field $|12,13,14|$ the critical strength of the magnetic field of the electromagnetic wave

$$B_{cr} = \frac{1}{\mu (1 - \beta \cos \phi)} \frac{\gamma}{2} \sqrt{\frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{g\omega}{2} \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right)}.$$  

which determines a lower bound of the magnetic field for which the oscillation amplitude is close to unity (i.e., at least it is not less then 1/2).

Now let us consider the neutrino conversion $\nu_L \leftrightarrow \nu_R$ in the case when in addition to electromagnetic wave given by Eqs. (16) and (17), a constant longitudinal magnetic field, $\vec{B}_1 = (0, 0, B_1)$ is superimposed.

The effective magnetic field in the neutrino rest frame is given by

$$\vec{B}_0 = \gamma \left[ B_1 (\cos \phi - \beta) \vec{e}_1 + B_2 (1 - \beta \cos \phi) \vec{e}_2 \right. + \left. (B_1 - B_1 \sin \phi) \vec{e}_3 / \gamma \right].$$  

Applying the results of the previous consideration for the case when $\vec{B}_0$ is determined by Eq. (25) we get

$$\vec{R} = \left( -\frac{\gamma}{2} \sqrt{\frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\mu}{\gamma} (B_1 - B_1 \sin \phi)} \right) \vec{e}_3 + \mu B_1 (1 - \beta \cos \phi) (\vec{e}_1 \cos \psi - \vec{e}_2 \sin \psi).$$  

The solution of Eq. (10) is given by Eqs. (19) and (20), where for vector $\vec{l}$ we get

$$\vec{l} = \left( \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\psi}{2} + \frac{1}{\gamma} \mu B_1 \right) \vec{e}_3 - \mu B_1 (1 - \beta \cos \phi) (\vec{e}_1 \cos \psi_0 - \vec{e}_2 \sin \psi_0).$$  

Therefore, the amplitude of the probability of conversion between the two neutrino helicity states in presence of the electromagnetic wave and longitudinal magnetic field could become sufficient when the following condition is satisfied,

$$\left| \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\psi}{2} + \frac{1}{\gamma} \mu B_1 \right| \ll \mu B (1 - \beta \cos \phi).$$  

The corresponding resonance condition now is:

$$\left| \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{g\omega}{2} \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right) + \frac{1}{\gamma} \mu B_1 \right| = 0.$$  

It follows that the longitudinal component of the magnetic field $B_1$ could affect the resonance condition in the neutrino oscillations $\nu_L \leftrightarrow \nu_R$ through the direct interaction of the neutrino magnetic moment with $B_1$. This modification of the resonance condition under the influence of $B_1$ exists in addition to the indirect effect of $B_1$ that can arise due to the polarization of medium in longitudinal magnetic field $|15,24|$. The latter effect gives contribution to the difference of neutrino effective potentials in matter, $V_{mat}$. Equations (28) and (29) bind together the properties of neutrinos ($\mu$, $\Delta m^2$, $E$, $\theta$) and medium ($V$), as well as the direction of propagation, $\phi$, and other characteristics of the electromagnetic wave (the frequency $\omega$, the polarization $g$, the speed in matter $\beta_0$, the strength of the field $B$) and the strength of the superimposed longitudinal magnetic field $B_1$. Using the condition (29) we predict the new type of resonances in the neutrino oscillations $\nu_L \leftrightarrow \nu_R$ that can exist in presence of the combination of electromagnetic wave and constant longitudinal magnetic field.

Finally, we should like to emphasize the role of the direct interaction of the neutrino magnetic (transition) moment with longitudinal component of magnetic field. Consider neutrino moving in the presence of constant magnetic field $\vec{B}$. As it follows from the derived Hamiltonian (14) even in vacuum ($V = 0$) left handed and right handed neutrino states are not maximally mixed by the presence of a magnetic field $\vec{B} = B_1 + B_\parallel$ unless $B_\parallel$ vanishes exactly. Let us also discuss possibility for the neutrino resonance condition to be realized in the electromagnetic wave under the influence of the superimposed longitudinal magnetic field, $B_\parallel$. Suppose that the term $V/2 - \Delta m^2 A/(4E)$ can be neglected in Eqs. (28) and (29). Then the critical field strength of the electromagnetic wave is given by

$$B_{cr} = \frac{1}{\mu (1 - \beta \cos \phi)}.$$
In the case when the neutrino is propagated along the electromagnetic wave \((\cos \phi = 1)\) and \(g = \text{sign}(B_1)\) the corresponding resonance condition can be written in the form
\[
\omega = 4\gamma \mu B_1
\]
(we also take \(\beta_0 = 1\)). If the condition (31) is fulfilled then \(B_{cr} \to 0\). Thus we conclude that the left handed and right handed neutrino states are maximally mixed even for very low strength of the field of the electromagnetic wave. Let us choose the frequency of the electromagnetic wave to be equal to the frequency of the microwave background radiation, \(\omega \sim 2.5 \times 10^{-4} \text{ eV}\). Then from Eq. (31) we get the following expression for \(B_1\):
\[
B_1 = 2.5 \times 10^{-10} \gamma^{-1} \mu_0 B_\nu / \mu,
\]
where \(\mu_0 = \frac{1}{2} e / m_e\) is the Bohr magneton and \(B_\nu = m_\nu^2 / e = 4.41 \times 10^{13} \text{ G}\). If one choose \(m_\nu = 1 \text{ eV}\) and \(E_\nu = 1 \text{ GeV}\) it follows that \(\gamma = 10^9\) and for the value of the neutrino magnetic moment \(\mu = \mu_0 \times 10^{-10}\) from Eq. (32) it is possible to get estimation \(B_1 \sim 10^5 \text{ G}\).

The presence of the term \(\frac{1}{2} \mu B_1\) (that describes the direct interaction of neutrinos with \(B_1\)) in the diagonal elements of the corresponding Hamiltonians for neutrino conversions will also shift the resonance condition in the case of neutrino oscillations without change of helicity. These phenomena may contribute to the mechanisms of the neutron star motion proposed previously (see Refs. [16,17]). The effects discussed above can have important consequences for neutrino oscillations in the other astrophysical environment. This issue will be considered in detail elsewhere [26].

In conclusion we argue that the effective Hamiltonian for neutrino oscillations (14) can be used for description of neutrino oscillations under the influence of an arbitrary electromagnetic fields given by their components in the laboratory frame.

Acknowledgements

We should like to thank Samoil Bilenky, Angelo Della Selva and Lev Okun for helpful discussions.

References

Neutrino mass, bulk majoron and neutrinoless double beta decay

R.N. Mohapatra a,*, A. Pérez-Lorenzana a,b, C.A. de S. Pires a

a Department of Physics, University of Maryland, College Park, MD, 20742, USA
b Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N., Apdo. Post. 14-740, 07000, México, D.F., Mexico

Received 18 August 2000; accepted 4 September 2000

Abstract

A new economical model for neutrino masses is proposed in the context of the brane-bulk scenarios for particle physics, where the global $B-L$ symmetry of the standard model is broken spontaneously by a gauge singlet Higgs field in the bulk. This leads to a bulk singlet majoron whose Kaluza–Klein excitations may make it visible in neutrinoless double beta decay for some parameter range if the string scale is close to a TeV.

PACS: 14.60.Pq; 14.60.St

1. Introduction

One of the major phenomenological challenges for models with large extra dimensions and low string scale [1] is to understand the small mass of neutrinos. The basic problem arises due to the fact that the effective theory below the string scale will have nonrenormalizable operators which are suppressed by powers of the string scale $M$. The operator relevant for neutrino masses is of the form $LHLH/M$, where $L$ and $H$ are the lepton and Higgs doublets of the standard model, respectively. After symmetry breaking, it leads to neutrino masses which are much too large. In general, higher dimensional operators can also create other phenomenological difficulties for such models, e.g., rapid proton decay via operators of the form $QQQL/M$; however, it has been suggested that operators that involve different matter fields such as the ones that lead to proton decay, can be suppressed by using the idea of “fat” branes [2] where different matter fields are located at different points in the brane. This idea, however, does not help in the case of the neutrino mass operator above since it involves only one matter field and since the Higgs field needs to be “spread out” rather than localized for all fermions to have mass. One must therefore seek other ways to suppress the effects of this operator.

A simple way to understand small neutrino masses in these models, suggested early on, is to assume the existence of a global $B-L$ symmetry and include only bulk neutrinos in addition to the standard model particles [3]. This leads to small neutrino masses for natural values of all parameters, due to suppressed overlap of the wave function between the brane and the bulk fields. The neutrinos in this model are Dirac particles. A second suggestion is to use a local $B-L$ symmetry [4], which generically requires the

* Corresponding author.
E-mail addresses: rmohapat@physics.umd.edu
(R.N. Mohapatra), aplorenz@glue.umd.edu (A. Pérez-Lorenzana), cpires@physics.umd.edu (C.A. de S. Pires).
string scale to be intermediate rather than TeV type. The neutrinos in this model can be either Dirac or Majorana particles. By now, the phenomenology of the former case has been studied extensively in several papers [5].

More recently an alternative suggestion has been put forth [6] where the global $B–L$ symmetry of the standard model is assumed to be an exact symmetry of the complete model so that all undesirable higher dimensional terms contributing to neutrino mass are forbidden. However, instead of adding extra neutrinos to the bulk, a scalar field in a separate brane is used to break the $B–L$ symmetry spontaneously. This leads to a singlet majoron [7], which practically decouples from the theory even though the scales are very small. The smallness of neutrino masses in this model arise from the Yukawa like suppression (called “shining” [8]) that has its origin in the propagator of a massive bulk field (denoted by $\chi$). In order to implement this picture, one needs the number of large extra dimensions to be at least three, preferably more. The reason for at least three extra dimensions is that the desired suppression takes place only in these cases. Furthermore, four or more are preferable because in case of three extra dimensions, the relation

$$M_{\ell}^2 = M^4 R^3,$$  \hspace{1cm} (1)

implies that the size of the extra dimension is $R \lesssim (\text{keV})^{-1}$ for $M \gtrsim 1$ TeV. Since to get small neutrino masses via the “shining” effect one needs $m_\nu \ll R^{-1}$, the bulk field must have a tiny mass, much less than a keV. If the number of extra dimensions is four or more, this constraint on the parameter $m_\nu$ becomes much weakened, requiring less “fine tuning” in the theory. The other point is that even for three extra dimensions, their sizes become so small that the current gravity experiments lose their usefulness as ways to search for their existence.

In this paper, we like to pursue the idea that spontaneous breaking of $B–L$ symmetry may indeed be the origin of neutrino masses in models with large extra dimensions but with sizes of extra dimensions in the millimeter range and furthermore, we will work only with one large extra dimension. As far as the string scale $M$ goes, we will assume that $M \sim 30$ TeV so as to satisfy the SN1987A bounds [9] for the case of one large extra dimension. Sizes of other dimensions will be accordingly adjusted so as to satisfy the generalized version of the relation in Eq. (1). In order to get nonvanishing neutrino masses, we will assume that the $B–L$ symmetry is spontaneously broken by a gauge singlet scalar field $\chi$ in the bulk. This scalar field $\chi$ which is assumed to carry two units of lepton number ($B–L$), is the only extra field in the model, thus making it the most economical extension of the standard model to date that leads to neutrino masses. (Note in contrast that in the bulk neutrino alternative, one needs a minimum of three bulk fermions to get a realistic mass pattern.)

An interesting feature of this model is that in some extreme domains of the parameter space of the model, the singlet majoron has a chance to be visible in processes such as neutrinoless double beta decay due to its many Kaluza–Klein excitations. A disadvantage is that to get neutrino masses in the eV range, some suppression of the strengths of the higher dimensional operators or a small value for $m_\nu^2$ is needed, unlike the bulk neutrino models. The required suppressions are at the same level as that needed for example in the triplet majoron model [10,11].

2. Bulk singlet and neutrino mass

Our scenario consists of standard model in the brane, to which we add only one gauge singlet complex scalar field $\chi$ propagating in the bulk. We assume that (i) the field $\chi$ carries two units of $B–L$ quantum number; (ii) the model respects global $B–L$ symmetry prior to symmetry breaking by vacuum and (iii) it includes in the Lagrangian operators of all dimensions that conserve $B–L$ symmetry. A list of some of the leading operators are:

$$\mathcal{L}(x) = \int dy \left[ 2 \left( \frac{f}{M^{5/2}} (LH)^2 \chi(x, y) + \frac{f'}{M^{3/2}} Q^6 \tilde{H}^2 \chi^*(x, y) \right) \delta(y) \right],$$  \hspace{1cm} (2)

where $\tilde{H} = i\tau_2 H^*$. We have not included any operator that could be suppressed by appropriate “fattening” of the brane [2]. The second operator in Eq. (2) is also not suppressed in the fat brane scenario and is a $\Delta B = 2$ operator that can give rise to the process of neutron-anti-neutron oscillation [12]. We will discuss this later.
In order to implement spontaneous breaking of $B-L$ symmetry, let us write down the bulk scalar potential for $\chi$. The part of the potential important here is

$$V(\chi) = -\frac{m_\chi^2}{2} \chi^\dagger \chi + \frac{\lambda}{4 M^2}(\chi^\dagger \chi)^2.$$  

Minimizing this potential, we find that at its minimum, the singlet field has the vev

$$\langle \chi \rangle_B = \frac{m_\chi M^{1/2}}{\sqrt{\lambda}}.$$  

Using this, we find that after electroweak symmetry breaking, i.e., $\langle H \rangle = \nu_{wk}$, the neutrinos acquire a Majorana mass given by:

$$m_\nu = \frac{f}{M^{5/2}} \nu_{wk}(\chi).$$

For $M = 30$ TeV, we can generate neutrino mass of order of eV if we take $\langle \chi \rangle = 5 \times (10^{-3}/f)$ GeV$^{3/2}$. This leads to $m_\chi \simeq 90 \times (10^{-3}/f)$ MeV (for $\lambda \sim 10$). Thus we need somewhat of a strong fine tuning of the parameters of the bulk fields to get the right order for neutrino masses. This fine tuning is at the same level as that required in the case of the triplet majoron model [10]. Despite this feature, we consider these models to be of interest since they appear rather economical and embody a new phenomenon not hitherto discussed in the context of neutrino masses. Furthermore, the small $m_\chi^2$ values could perhaps be made natural if there is supersymmetry, while keeping the other features unaffected. As we discuss below, the extreme small mass range of $\chi$ has one advantage that it makes the associated Goldstone boson, the majoron more visible in certain low energy processes.

It is clear that due to spontaneous breaking of $B-L$ symmetry in the bulk, this model has the massless particle, majoron (the CP-odd part of the singlet $\chi$, denoted by $J$), which is a bulk field. In four dimensions, the majoron has a tower of partners with masses separated by a tiny amount ($\sim 10^{-3}$ eV) for millimeter extra dimensions. They will be produced as a whole tower in any process where majoron is produced. Furthermore, the real part of the field $\chi$ (to be denoted by $\sigma_\chi$) also has a mass $m_\chi^2/2$. Since in this model $m_\chi$ has a value in the range of few MeV or less, it and its tower could also be produced in processes that have enough phase space. We give the example of the neutrinoless double beta decay in the next section, where only the majoron is produced unless the $\sigma_\chi$ has a mass in the sub MeV range. In processes such as muon decay however, both particles will be produced although the amplitude for it is highly suppressed.

The neutrino mass texture in this model arises purely from the flavor profile of the higher dimensional coupling $f_{ij}$. Experimental data on neutrino oscillations will fix this profile. As an example, which embodies the so-called bimaximal neutrino mixing pattern and nearly degenerate neutrino masses, we provide the following $f$ matrix:

$$f = \begin{pmatrix} c_\nu & \frac{1}{\sqrt{2}} \delta_S & \frac{1}{\sqrt{2}} \delta_S \\ \frac{1}{\sqrt{2}} \delta_S & m_0 + \delta_A/2 & \delta_A/2 \\ \frac{1}{\sqrt{2}} \delta_S & \delta_A/2 & m_0 + \delta_A/2 \end{pmatrix},$$

where $m_0$ is the common mass, $c = \cos \theta$ and $s = \sin \theta$; $\delta_{A,S}$ are responsible for the mass splittings that explain the atmospheric and solar neutrino data. For $c = s = 1/\sqrt{2}$, we get the bimaximal pattern. An advantage of the mass degeneracy is that it enhances the contribution to the neutrinoless double beta decay.

3. Neutrinoless double beta decay with majoron emission

One of the primary experimental manifestation of the majoron idea is in the process of neutrinoless double beta decay with majoron emission, a fact which was first noted for the case of the triplet majoron [10] in Ref. [11]. Note that the original singlet majoron coupling to neutrinos is so weak that it is generally not visible in this process. However, the bulk majoron, though a gauge singlet, is different. It can be produced in neutrinoless double beta decay via the diagram in Fig. 1.

The differential decay width for this process can be written as:

$$\frac{d^2 \Gamma}{d\epsilon_1 d\epsilon_2} = A_{\text{Nuc}}^2 \frac{G_F^4 p_F^2}{8\pi^3} \left( \frac{f^2 \nu_{wk}}{M^5} \right) \times (E - \epsilon_1 - \epsilon_2)^2 \epsilon_1 k_1 \epsilon_2 k_2,$$

where $\epsilon_{1,2}$ and $k_{1,2}$ are the electron energy and momenta, respectively, $A_{\text{Nuc}}$ is a dimensionless nuclear factor, whose value we take from nuclear calculations for the single majoron decay mode [13]; $p_F$ is
Fig. 1. Majoron emission in neutrinoless double beta decay in the bulk majoron model.

the Fermi momentum in nuclei (which we take to be 100 MeV); $E$ is the available energy for electrons and the majoron in the decay [13]. Note that there are two powers of the factor $(E - \epsilon_1 - \epsilon_2)$ (the power of this factor in the differential decay distribution is called in the literature as spectral index [14]) above in contrast with a single power in the triplet majoron model and generally odd powers in most theoretical models [15]. This is the effect of the tower of majoron KK modes.

The lower limits on the lifetime for the process $\Delta B = 0$ from various nuclei are now at the level of $7.2 \times 10^{20}$ years for $^{48}$Ca [16] to $7.2 \times 10^{21}$ yr for $^{130}$Xe [18] and $^{76}$Ge [17]. For $A_{\text{Nucl}} \sim 0.1$ and $f \sim 1$, using the best of the above experimental limits [13] for majoron emission in $\beta \beta_{0
u}J$, we get a lower limit on $M \geq 1$ TeV. This bound can be improved once the search for the majoron emitting double beta decay is carried out at higher precision level by experiments such as, for instance, the proposed GENIUS [19] experiment as well as others.

4. Neutron–antineutron oscillation

Let us now turn to the second operator in Eq. (1), which leads to $\Delta B = 2$ transitions. The strength of this operator is given by

$$G_{\Delta B=2} \sim \frac{f(\chi)_R v_{\text{wk}}^2}{M^{17/2}}.$$  

For the values of $M \simeq 30$ TeV and $\langle \chi \rangle_R = 10-0.01$ GeV$^{3/2}$ discussed above, we get $G_{\Delta B=2} \sim 10^{-32}$–$10^{-35}$ GeV$^{-5}$. This translates into an oscillation time for $N = \bar{N}$ from $10^{10}$ to $10^{15}$ s after allowing for uncertainties in the hadronic matrix elements. The lower values are in the range accessible to a proposed experiment [20].

The same operator also leads to a novel process where an infinity tower of KK majorons are emitted in the transition $NN \rightarrow \chi$ or in terms of actual nuclear transmutation $(Z, A) \rightarrow (Z, A-2) + \chi$. The width for this process is given by:

$$\Gamma_{\Delta B=2} \sim M^{-17}(\chi)_R^{14} v_{\text{wk}}^4 \cdot 10^{-6} \text{ GeV},$$

where we have used the factor of $10^{-6}$ to denote the hadron dressing of the six quark operator. For $M = 30$ TeV, the nuclear instability life times implied by this are $\sim 10^{39}$ yr. However, if we ignored other constraints on $M$ and chose it to be of order 10 TeV or so, we would expect majoron emitting modes of the above type with a life time of about $10^{32}$ years which looked for perhaps even in existing data.

5. Conclusion

In this brief note, we have suggested a new model for neutrino masses in theories with large extra dimensions using spontaneous breaking of lepton number symmetry by a bulk scalar field. The resulting bulk majoron may be visible in neutrinoless double beta decay experiments for certain domains of parameters, if the string scale is indeed close to a TeV. The model also predicts a novel baryon number violating process where two neutrons in a nucleus disappear with the emission of a majoron which would lead to missing energy proton decay events.

Acknowledgements

The work of RNM is supported by a grant from the National Science Foundation under grant number PHY-9802551. The work of APL is supported in part by CONACyT (México). The work of CP is supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP).

References


A. Lukas, A. Romanino, hep-ph/0004130;


For a recent limit from the NEMO-2 experiment, see: R. Arnold et al., ITEP preprint, 15/00.


Natural mass generation for the sterile neutrino

K.S. Babu a,*, T. Yanagida b, c

a Department of Physics, Oklahoma State University, Stillwater, OK 74078, USA
b Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
c Research Center for the Early Universe, University of Tokyo, Tokyo 113-0033, Japan

Received 17 August 2000; accepted 2 September 2000

Editor: M. Cvetic

Abstract

We point out that there is a serious cosmological problem in the supersymmetric standard model if a sterile neutrino is responsible for the solar neutrino oscillation, and propose a possible solution to this problem. We show that our solution induces naturally a mass of order $10^{-4}$ eV for the sterile neutrino, which is deeply related to the mechanism of supersymmetry breaking.

© 2000 Elsevier Science B.V. All rights reserved.

Two flavor active–sterile neutrino oscillation seems to be disfavored by recent Superkamiokande data on both atmospheric and solar neutrino experiments [1]. However, a recent global analysis of the solar neutrino data [2] suggests that both the small angle MSW oscillation and the quasi-vacuum oscillation (corresponding to $\Delta m^2 \simeq 10^{-7} - 10^{-9}$ eV$^2$) are still consistent solutions. It has also been pointed out recently [3] that an energy-independent active–sterile neutrino oscillation is well consistent with the present solar neutrino experiments with the exception of the $^{37}$Cl results. Thus, the sterile neutrino is still interesting, since it may explain all neutrino oscillation data including LSND experiments [4]. In this short Letter we propose a natural mechanism for generating a small mass for the sterile neutrino $\nu_s$, which induces a $\nu_e-\nu_s$ oscillation together with the conventional see-saw mechanism [5]. This new mechanism is deeply related to the dynamics of supersymmetry (SUSY) breaking.

Before describing the model let us discuss a cosmological difficulty due to the presence of the sterile neutrino in the SUSY standard model. Since the sterile neutrino is a gauge singlet and its Yukawa coupling constant is very small ($y_s \simeq 10^{-15}$), the scalar partner of $\nu_s$ has a very flat potential. Thus, it is quite natural to consider that it has a large value of order the Planck scale ($M_G \simeq 2.4 \times 10^{18}$ GeV) at the end of inflation and its coherent oscillation dominates the early universe like moduli fields in string theory. The lifetime of the scalar sterile neutrino is estimated as $\tau \simeq 10^4$ s with the above small Yukawa coupling constant $y_s$ and a mass $m \simeq 1$ TeV. It is well known that such late decays of massive heavy particles destroy the success of the big-bang nucleosynthesis [6].

1 This small Yukawa coupling, $W = y_s L H S$, induces a small Dirac-type mass for the sterile neutrino of order $10^{-4}$ eV, which is required for the solar neutrino quasi-vacuum oscillation. $y_s$ should be similar in magnitude for the small angle MSW solution as well.

2 The lifetime should be shorter than $0.1$ s to avoid this problem [7].
A solution to the above problem is easily given by introducing the following superpotential:

$$W = h \frac{Z}{M_G} L_i H S,$$

(1)

where $L_i$ ($i = 1, 2, 3$), $H$ and $S$ are supermultiplets of three families of lepton doublets, a Higgs doublet and the sterile neutrino, and $Z$ is a supermultiplet responsible for SUSY breaking. Then, we get an $A$-term,

$$\mathcal{L} = hm_{3/2} \tilde{L}_i H \tilde{S},$$

(2)

where $\tilde{L}_i$ and $\tilde{S}$ are scalar components of the supermultiplets $L_i$ and $S$, respectively and $H$ is the Higgs boson. We have used that the gravitino mass $m_{3/2}$ is given by the vacuum-expectation value (vev) of the $F$-component of the supermultiplet $Z$, that is,

$$m_{3/2} = \frac{1}{\sqrt{3} M_G} \langle \mathcal{F}_Z \rangle.$$  

(3)

Then, the lifetime of the scalar sterile neutrino becomes $\tau \approx 10^{-26}$ s and the scalar sterile neutrino is cosmologically harmless. Here, we have assumed $m_{3/2} \approx 1$ TeV.

We now discuss the SUSY breaking sector. We adopt the SUSY breaking model found in Ref. [10], which is based on an $SU(2)$ gauge theory with four quark doublets, $Q_i^a$ ($a = 1, 2$ and $i = 1, 2, 3, 4$). We introduce six gauge-singlet supermultiplets $Z_a$ ($a = 1, \ldots, 6$) and assume the following superpotential:

$$W = \sum_{i<j} \delta_{ab} Q_i^a Q_j^b Z_a,$$

(4)

It is shown in Ref. [10] that the effective low-energy superpotential is given by

$$W_{\text{eff}} = \lambda A^2 Z.$$  

Here, $Z$ is a linear combination of $Z_a$ supermultiplets and $A$ denotes the dynamical scale of the $SU(2)$ gauge interactions. The Kahler potential takes, on the other hand,

$$K = ZZ^* - \frac{k}{2 A^2} (ZZ^*)^2 + \cdots,$$

(5)

where $k$ is a real constant and the ellipsis denotes higher-order terms of $ZZ^*$. If the coupling constant $k$ is positive, we have a unique vacuum

$$\langle Z \rangle = 0, \quad \langle F_Z \rangle = \lambda A^2.$$  

(6)

Thus, SUSY is dynamically broken and the sterile neutrino remains massless.

A crucial point observed in Ref. [11] is that the superpotential effects induce a small shift of the vacuum and the $A$-component of the $Z$ has a small nonvanishing vev:

$$\langle Z \rangle \approx \frac{A^2}{\sqrt{3} k M_G} \approx \frac{m_{3/2}}{\lambda k}.$$  

(7)

Substituting this result, Eq. (7), into Eq. (1) we obtain a Dirac-type $v_e - v_s$ mass ($i = e, \mu, \tau$) as

$$\mathcal{L} \approx h \frac{m_{3/2}}{\lambda k M_G} \langle H \rangle v_i v_s \text{ h.c.}.$$  

(8)

The Dirac-type mass is of order $10^{-4}$ eV for $\lambda k / h \approx 1$. It is now clear that if the Majorana mass for the active electron neutrino induced by the see-saw mechanism is of order $10^{-4}$ eV, the present model will naturally reproduce the solar $v_e - v_s$ oscillation. Since the mechanism suggested here generates a $v_e - v_s$ mass term, and no direct $v_{\mu} - v_{\tau}$ mass term, it turns out that the lighter eigenstate is predominantly in $v_s$, and not in $v_e$. The MSW resonance condition will not be satisfied for solar neutrinos in this case.

---

3 This superpotential is also discussed in Refs. [8,9].

4 We wish to remark that the operator of Eq. (1) can provide a possible solution to the moduli problem that is generic in string theory. If $S$ is identified as one of the moduli fields and $L_i$ is $H$ in Eq. (1), the cosmological problem associated with the moduli will be solved, very much in analogy to the scalar neutrino.

5 For the large angle $v_e - v_s$ quasi-vacuum oscillation, it is possible to evade the cosmological limit by choosing $y_s \sim 3 \times 10^{-13}$, so that the $v_e - v_s$ mass term is of order 0.1 eV. If the direct $v_e - v_s$ mass term arising from the see-saw mechanism is of order $10^{-7}$ eV, the required $\Delta m^2$ for solar neutrinos will be generated. Such a scenario is not realized by the mechanism suggested in this Letter as long as all relevant Yukawa couplings are $O(1)$.

6 This is an important dynamical assumption in this Letter. If the $k$ is negative, the $A$-component of the $Z$ has a vev of order the dynamical scale $A$, which induces too large a Dirac-type mass for the sterile neutrino ($m_{\nu_s} \sim 1$ keV).

7 We assume the standard Yukawa coupling, $W = f L H S$ to exactly vanish and consider the case where $S$ has only gravitationally suppressed nonrenormalizable interactions. This will be the case if $S$ is one of the moduli fields of string theory. For neutrino mixing with modulino fields, see, e.g., Ref. [12].

8 If one identifies the sterile neutrino with a right-handed neutrino and keeps exact lepton-number conservation, one may have a light Dirac neutrino as discussed in Ref. [13], see also Ref. [8].
Our scenario will prefer the quasi-vacuum oscillation solution [2] with the inclusion of the Chlorine experiment, or the energy-independent solutions advocated in Ref. [3] excluding the Chlorine experiment. In either case, the other two active neutrinos together with the electron neutrino may explain the atmospheric and LSND neutrino oscillations. While $\nu_e \leftrightarrow \nu_x$ MSW resonance does not occur for supernova neutrinos, $\bar{\nu}_e \leftrightarrow \nu_x$ resonance will occur within the supernova [14]. However, vacuum oscillations on its way from supernova to the Earth will regenerate $\nu_x$, but with its flux reduced by half.\footnote{If $\bar{\nu}_e$ mixes also with $\bar{\nu}_{\mu,\tau}$, the supernova $\bar{\nu}_{\mu,\tau}$ are also converted into $\bar{\nu}_e$ through the MSW resonances. In this case the $\bar{\nu}_e$ flux is enhanced by factor 3/2 instead.} Such a reduction is not inconsistent with $\bar{\nu}_e$ data from SN1987A, but may be testable with future supernova neutrinos. It is interesting to note that the neutrino data from supernova alone makes the large $\nu_e \leftrightarrow \nu_x$ mixing preferable in our scenario, independent of solar neutrino data.

If one supposes a superpotential term $W = (SSZZ/M_G)$, in addition to Eq. (1), a direct Majorana mass term for the $\nu_x$ of order $m_{1/2}^2/M_G \sim 10^{-3}$ eV will result. In this case the small angle $\nu_e \leftrightarrow \nu_x$ MSW oscillation may become relevant for solar neutrinos. However, this operator is less motivated (compared to the one in Eq. (1)) from the point of view of cosmology.

Acknowledgements

One of the authors (T.Y.) is grateful to F. Borzumati, H. Murayama and Y. Nomura for useful discussions. K.B. acknowledges the Theory Group at Tokyo University for its warm hospitality. The work of T.Y. is supported in part by the Grant-in-Aid, Priority Area “Supersymmetry and Unified Theory of Elementary Particles” (#707), K.B. is supported in part by Department of Energy Grant No. DE-FG03-98ER41076 and by a grant from the Research Corporation.

References

R-symmetry, Yukawa textures and anomaly mediated supersymmetry breaking

I. Jack *, D.R.T. Jones

Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

Received 15 June 2000; received in revised form 21 August 2000; accepted 6 September 2000

Editor: P.V. Landshoff

Abstract

We explore, in the MSSM context, an extension of the Anomaly Mediated Supersymmetry Breaking solution for the soft scalar masses that is possible if the underlying theory has a gauged R-symmetry. The slepton mass problem characteristic of the scenario is resolved, and a context for the explanation of the fermion mass hierarchy provided. © 2000 Published by Elsevier Science B.V.

Recently there has been interest in a specific and predictive framework for the origin of soft supersymmetry breaking within the MSSM, known as Anomaly Mediated Supersymmetry Breaking (AMSB). The supersymmetry-breaking terms originate in a vacuum expectation value for an $F$-term in the supergravity multiplet, and the gaugino mass $M$, the $\phi^3$ coupling $h^{ijk}$ and the $\phi\phi$-mass $(m^2)^{ij}$ are all given in terms of the gravitino mass, $m_0$, and the $\beta$-functions of the unbroken theory by simple relations that are renormalisation group (RG) invariant [1–20]. Direct application of this idea to the MSSM leads, unfortunately, to negative $m^2$ sleptons: in other words, to a theory without a vacuum preserving the $U_1$ of electromagnetism. Various resolutions of this dilemma have been investigated; here we explore a particularly minimalist one, which requires the introduction of no new fields into the low energy theory. The key lies in a compelling generalisation of the RG invariant solution described above [5]. The basic AMSB solution is given by:

$$M = m_0 \frac{\beta_g}{g},$$

$$h^{ijk} = -m_0 \beta^{ijk}_Y,$$

$$(m^2)^{ij} = \frac{1}{2} [m_0]^2 \mu \frac{\partial \gamma^i_j}{\partial \mu}.$$ 

Now $\beta_m$ is given by [3] (see also [21–25])

$$(\beta_m)^{ij} (m^2, \ldots) = \left( 2 \mathcal{O} \mathcal{O}^* + 2 M M^* g^2 \frac{\partial}{\partial g^2} + \tilde{Y} \frac{\partial}{\partial Y} 
+ \tilde{Y}^* \frac{\partial}{\partial Y^*} + X \frac{\partial}{\partial d} \right) \gamma^i_j, \quad \text{(2)}$$

where

$$\mathcal{O} = \left( M g^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial Y^{lmn}} \right), \quad \text{(3)}$$

$$\tilde{Y}^{ijk} (m^2, Y) = (m^2)^i_j Y^{ijk} + (m^2)^j_i Y^{ilk} + (m^2)^k_i Y^{ijl} \quad \text{(4)}$$
and (in the NSVZ scheme) \[4,26\]^1

\[
X(m^2, M) = \frac{-2 \sqrt{g^3} r^{-1} \text{Tr}[m^2C(R)] - MM^+C(G)}{1 - 2g^2C(G)(16\pi^2)^{-1}}. \tag{5}
\]

(Here \(r\) is the number of generators of the gauge group and \(C(R)\) and \(C(G)\) are the quadratic matter and adjoint Casimirs, respectively.)

It is immediately clear that, given a solution to Eq. (2), \(m^2 = m^2_1\), then \(m^2 = m^2_1 + m^2_2\) is also a solution, where \(m^2_2\) satisfies the equation (linear and homogeneous in \(m^2_1\)):

\[
\mu \frac{d}{d\mu} m^2_2 = \left[ \bar{Y}^*(m^2_1, Y^*) \frac{\partial}{\partial Y^*} + \bar{Y}(m^2_2, Y) \frac{\partial}{\partial Y} \right] \gamma.
\tag{6}
\]

Remarkably, Eq. (6) has a solution of the form \[5,8\]

\[
(m^2_2)^j = \mathcal{S}_0(y'j + \bar{\eta} \delta^j_j),
\tag{7}
\]

where \(\mathcal{S}_0\) and \(\bar{\eta}\) are constants, as long as a set \(\bar{\eta}\) exists that satisfy the following constraints:

\[
(\bar{\eta} + \bar{\eta}' + \bar{\eta}'') \gamma_{ijk} = 0,
\tag{8a}
\]

\[
2 \text{Tr}[\bar{q}C(R)] + Q = 0,
\tag{8b}
\]

where \(Q\) is the one-loop \(\beta_\chi\) coefficient. It is easy to show \[5\] that Eq. (8) corresponds precisely to requiring that the theory have a nonanomalous \(R\)-symmetry (which we will denote \(\mathcal{R}\), to avoid confusion with our notation \(R\) for group representations). Setting

\[
\bar{\eta}' = 1 - \frac{3}{2} r'.
\tag{9}
\]

we see that Eq. (8a) corresponds to \((r' + r' + r'') \times Y_{ijk} = 2Y_{ijk}\), which is the conventional \(\mathcal{R}\)-charge normalisation. Moreover, it is then easy to show (recall that the gaugino has \(\mathcal{R}\)-charge of 1) that Eq. (8b) is simply the anomaly cancellation condition for the \(\mathcal{R}\)-charges.

Our strategy in this paper will be to take the AMSB solution Eq. (1), but with Eq. (1c) generalised to

\[
(m^2_j)^j = \frac{1}{2} [m_0^2 \mu \frac{d}{d\mu} \kappa_j + \mathcal{S}_0(y'j + \bar{\eta} \delta^j_j)].
\tag{10}
\]

For a discussion of a possible origin of \(m_0^2\) as the vacuum expectation value of a \(D_1\) \(D\)-term, see Ref. \[8\].

In a theory with direct product structure there is a relation of the form Eq. (8b) for each gauged subgroup; so in the MSSM case there are three conditions, corresponding to cancellation of the \(\mathcal{R}(SU_3)^2\), \(\mathcal{R}(SU_2)^2\) and \(\mathcal{R}(U_1)^2\) anomalies. We also impose cancellation of the \((\mathcal{R})^2U_1\), \((\mathcal{R})^3\) and \(\mathcal{R}\)-gravitational anomalies, although this is not required to render Eq. (10) \(\mathcal{R}\)-invariant. (In the case of the \((\mathcal{R})^3\) and \(\mathcal{R}\)-gravitational anomalies, we need to invoke the existence of a MSSM-singlet sector (at high energies) to ensure that they cancel.\footnote{Note that the gravitino also contributes to these anomalies \[27, 28\].} Note that we cannot invoke the Green–Schwarz mechanism to cancel these anomalies since in that case the \(\mathcal{R}(SU_3)^2\), \(\mathcal{R}(SU_2)^2\) and \(\mathcal{R}(U_1)^2\) anomalies would no longer be zero.)

Now for the MSSM superpotential

\[
W_{\text{MSSM}} = \mu_s H_1 H_2 + (\lambda_u)_{ab} H_2 Q_a (\bar{u}^c)_b
+ (\lambda_d)_{ab} H_1 Q_a (\bar{d}^c)_b
+ (\lambda_e)_{ab} H_1 L_a (\bar{e}^c)_b,
\tag{11}
\]

there is no possible \(\mathcal{R}\)-symmetry, satisfying the constraints described above, such that all the Yukawa couplings are nonzero.\footnote{Application of the scenario to the MSSM was dismissed in Ref. \[5\], presumably for this reason.} One way out of this dilemma would be to add extra particles \[27\]; here we instead persist with the minimal field content, and are hence forced to distinguish between the generations. Apart from simplicity this also provides a context for explaining the fermion mass hierarchy. We therefore presume an \(\mathcal{R}\)-charge assignment such that only the third generation Yukawa couplings are permitted (we will return later to the origin of the first two generation masses). We will, however, enact the constraint that the first two generations have identical \(\mathcal{R}\)-charges. As we shall see, this will alleviate potential Flavour Changing Neutral Current (FCNC) problems.

Thus for the superpotential to have \(\mathcal{R}\)-charge 2, we require (henceforth we work with the fermionic charges, related to the \(\mathcal{R}\)-charges by \(q_1 = r - 1\)):

\[
q_3 + u_3 + h_2 = q_3 + d_3 + h_1 = l_3 + e_3 + h_1
= -1.
\tag{12a}
\]
\[ h_1 + h_2 = 0, \]  
(12b)

while for cancellation of the mixed anomalies we require:

\[
q_3 + \frac{1}{2}(u_3 + d_3) + 2(q_1 + \frac{1}{2}(u_1 + d_1)) + 3 = 0, \quad (13a)
\]

\[
\frac{1}{2}l_3 + \frac{3}{2}q_3 + 2\left(\frac{1}{2}l_1 + \frac{1}{2}q_1\right) + \frac{1}{2}(h_1 + h_2) + 2 = 0, \quad (13b)
\]

\[
\frac{1}{6}q_3 + \frac{1}{3}d_3 + \frac{2}{3}u_3 + \frac{1}{2}l_3 + e_3
\]

\[+ 2\left(\frac{1}{2}q_1 + \frac{1}{2}l_1 + \frac{1}{2}u_1 + \frac{1}{2}l_1 + e_1\right) + \frac{1}{2}(h_1 + h_2) = 0, \quad (13c)
\]

\[-l_3^2 + e_3^2 + q_3^2 - 2u_3^2 + d_3^2
\]

\[+ 2\left(-l_1^2 + e_1^2 + q_1^2 - 2u_1^2 + d_1^2\right) - h_1^2 + h_2^2 = 0. \quad (13d)
\]

Eqs. (13a)–(13d) correspond to cancellation of the \( \mathcal{R}(SU_3)^2, \mathcal{R}(SU_2)^2, \mathcal{R}(U_1)^2 \) and \( \mathcal{R}^2 U_1 \) anomalies respectively. It is easy to show that even without imposing the quadratic constraint (13d), the system of Eqs. (12) and (13) has no solution if we set \( q_1 = q_3, \ u_1 = u_3, \) etc. Thus, as asserted above, there is no possible generation independent \( \mathcal{R} \)-charge assignment. The above constraints may be solved (for arbitrary values of the leptonic charges) as follows:

\[
q_3 = \frac{4}{9} - \frac{1}{3}l_3 - \frac{1}{9}e_3 - \frac{1}{9}, \quad (14a)
\]

\[
u_3 = -\frac{22}{9} - \frac{2}{9}l_3 - e_3 + \frac{1}{9}, \quad (14b)
\]

\[
d_3 = -\frac{4}{9} + \frac{2}{3}l_3 + e_3 + \frac{1}{9}, \quad (14c)
\]

\[
q_1 = -\frac{1}{15} + \frac{1}{3}\kappa + \frac{1}{15} + \frac{1}{5} \kappa + \frac{1}{30} \kappa.
\]

\[+ \frac{1}{15} \kappa, \quad (14d)
\]

\[
u_1 = \frac{1}{90} - \frac{2}{3} \kappa - \frac{1}{15} + \frac{6}{5} \kappa - \frac{1}{30} \kappa - \frac{1}{18} \kappa, \quad (14e)
\]

\[
d_1 = \frac{1}{90} + \frac{4}{3} \kappa + \frac{1}{15} + \frac{4}{5} \kappa - \frac{1}{30} \kappa - \frac{1}{18} \kappa, \quad (14f)
\]

\[
h_2 = -h_1 = l_3 + e_3 + 1, \quad (14g)
\]

where \( \kappa = l_1 - l_3 + e_1 - e_3 - 3, \) and \( \kappa = -12l_2 - 16e_3 + 10e_1 - 23. \) Thus for any set of rational values for the leptonic charges there exist rational values for all the charges.

We will presently exhibit a set of sum-rules for the sparticle masses that are completely independent of the set of values \( l_3, e_3, \kappa, \kappa. \) Let us first see whether we can gain any insight on the \( \mathcal{R} \)-charge assignments by relating them to a possible origin of the light quark and lepton masses. Suppose [29] there are higher-dimension terms in the effective field theory of the form (for the up-type quarks) \( H_2 Q_i u_i' (\theta / M_U) a_i / \) and \( H_2 Q_i u_i' (\bar{\theta} / M_U) a_i / \) where \( \theta, \bar{\theta} \) is a pair of MSSM singlet fields with \( \mathcal{R} \)-charges \( \pm |r_\theta| \) that get equal vacuum expectation values, and \( M_U \) represents some high energy new physics scale (with similar terms for the light down quarks and leptons). Evidently the \( \mathcal{R} \)-charge assignments will then dictate the texture of the Yukawa couplings, via the relation \( h_2 + q_1 + u_1 + a_{11} r_\theta = -1 \) and similar identities.

We thus obtain Yukawa textures of the general form:

\[
\Delta_u = \begin{pmatrix}
\epsilon_{\kappa} & \epsilon_{\kappa} & \epsilon_{\kappa}
\end{pmatrix},
\]

\[
\Delta_d = \begin{pmatrix}
\epsilon_{\kappa} & \epsilon_{\kappa} & \epsilon_{\kappa}
\end{pmatrix},
\]

\[
\Delta_L = \begin{pmatrix}
\epsilon_{\kappa} & \epsilon_{\kappa} & \epsilon_{\kappa}
\end{pmatrix},
\]

\[
\text{for the up and down quarks,}
\]

\[
\text{for the leptons, where}
\]

\[
\delta_3 = \frac{1}{30} - \frac{1}{30} \kappa + 12l_3 + 6e_3 + 5 \kappa - 10 \kappa + 5 \kappa,
\]

\[
\delta_L = \frac{7}{10} - \frac{7}{10} \kappa + \frac{3}{5} e_3 - \frac{1}{10} \kappa + \kappa,
\]

\[
\epsilon = |\langle \theta \rangle / M_U| \quad \text{and} \quad \sigma = |r_\theta|^{-1} \quad \text{(provided \( r_\theta \) is such that all the exponents in Eqs. (15) and (16) are integers).}
\]

More complex scenarios may be contemplated in which there are more than one pairs of \( \theta, \bar{\theta} \) fields, but we do not consider this further.

In work on Yukawa textures it is common to assume that they are symmetric: this assumption is not dictated by the theoretical structure of our model. Moreover, it is easy to show that to obtain symmetric textures for both up and down quarks requires \( \kappa = \kappa = 0. \) This then implies that the up- and down-quark Yukawa couplings amongst the 1st and 2nd generations are also allowed (and presumably of \( O(1) \), leaving the fermion mass hierarchy unexplained). We therefore
abandon the symmetric paradigm; as an alternative
simplifying assumption, motivated by the similarity of
the hierarchies of the down-quark and lepton masses,
we impose $\Delta d = \Delta L$. This requires
\[ \kappa = -\frac{3}{2}, \quad \kappa^2 = -\frac{21}{4} - \frac{9}{4} = \frac{9}{4}, \quad (18) \]
where $\lambda = 2d_3 + e_3$. We then find $d_4 = \frac{3}{2} + \frac{3}{2}$. The only value of $\lambda$ we have found which leads to nice textures with only one pair of $\theta$, $\bar{C}$ fields is $\lambda = -\frac{1}{2}$; with $r_4 = \frac{3}{2}$, we then obtain texture matrices of the form
\[ \Delta_u = \begin{pmatrix} e^4 & e^6 & e^4 \\ e^6 & e^4 & e^4 \\ e^4 & e^4 & e \end{pmatrix}, \]
\[ \Delta_d = \Delta_L = \begin{pmatrix} e^4 & e^4 & e^4 \\ e^4 & e^4 & e^4 \\ e^4 & e^4 & e \end{pmatrix}. \quad (19) \]

The charges now have the form shown in Table 1.

It is easy to show that as long as $-\frac{1}{2} < e < \frac{1}{2}$ and $m_{0} < 0$, the contribution to each slepton mass term due to the $\tilde{C}$ term in Eq. (10) will be positive, and we may expect to achieve a viable spectrum; however, it turns out that it is still non-trivial to obtain an acceptable minimum because, for example, if $e = 0$ and $m_{0} < 0$, the $m_{0}$ contributions to Eq. (10) from $u_3, q_1$ and $d_1$ are negative. Reverting to the Yukawa texture issue, we see that $\Delta_{u,d,L}$ are not in the class of forms for the texture matrix most frequently considered in the literature, where more attention has focussed on the possibility of texture zeroes. They are of interest, however, in that $\Delta_{u}$ has one zero eigenvalue, and $\Delta_{d,L}$ have two zero eigenvalues. It follows from these properties that mass hierarchies may be produced with matrices of this generic structure. For example, given the following up- and down-quark Yukawa matrices,
\[ \lambda_u \propto \begin{pmatrix} -0.28e^4 & 1.3e^4 & 0.4e^4 \\ -0.32e^4 & 1.45e^4 & 1.36e^4 \\ -0.36e^4 & 1.67e^4 & 1 \end{pmatrix}, \]
\[ \lambda_d \propto \begin{pmatrix} -1.75e^4 & 1.99e^4 & 0.25e^4 \\ -3.01e^4 & 2.53e^4 & 1.18e^4 \\ 0.26e^3 & -0.48e^3 & 0.95 \end{pmatrix}, \quad (20) \]
with $e = 0.25$, we obtain ratios for the quark masses and a CKM matrix within experimental limits. Let us consider the issue of FCNC contributions. The matrices $\lambda_u$ and $\lambda_d$ are both diagonalised by matrices which are approximately of the general form
\[ \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \]
from which it follows, because we chose identical $R$-charge assignments for the first two generations, that if we rotate the squark masses to the basis that diagonalises both the quark masses and the quark–squark–gluino coupling, then all the off-diagonal terms are small, so FCNC contributions mediated by the gluino will be suppressed. Of course even in the absence of squark–flavour mixing there are susy FCNC contributions; consider for example the wino–squark box diagram contribution to $K - \bar{K}$ mixing. Here the up/csm squark contributions will be GIM suppressed and the top-squark contribution suppressed by CKM angles, just as the analogous Standard Model top-quark diagram is. For the charged leptons, we are less constrained given the lack of a (or, if we generalised to the massive neutrino case, our ignorance of the) leptonic CKM matrix.

Naturally because the off-diagonal squark and slepton masses are (though relatively small), not zero, it follows that the whole issue of FCNCs deserves a more detailed analysis.

We cannot entirely claim avoidance of fine-tuning, in as much as the lightest quark masses ($m_{0,d}$) are somewhat sensitive to small changes in the coefficients shown in Eq. (20); for example, if we change 1.3 to 1.4 in $\lambda_u$, then $m_u$ increases by a factor of 4. However, the CKM matrix, $m_s$ and $m_c$ are remarkably stable under such variations.

The mechanism proposed for generating the light fermion masses raises the following issue. As a sym-
metry of the low-energy effective field theory, our $R$-symmetry forbids from the superpotential, Eq. (11), not only the light-fermion Yukawa couplings but also the well-known set of baryon and lepton-number violating terms of the form $QLDc$, $d^c u^c$, $LLe^c$ and $H_2L$. It is clear that a priori the same mechanism we invoke above to generate the light masses might lead to similar contributions to these operators, for example via the operator $d^c d^c u^c (\theta / M_T)^p$. However it is easy to check that, with the charge assignment we make above for the $\theta, \overline{\theta}$ fields, the value of $p$ required to render this operator $R$-invariant is not an integer; and similarly for the other baryon and lepton-number violating operators above. There will in general be higher-dimensional $B$-violating and $L$-violating operators but the effects of these will be strongly suppressed.

The phenomenology of AMSB-models has been discussed at length in the literature. If we compare our model here with the constrained MSSM (where the assumption of soft universality at the unification scale means that the theory is characterised by the usual input parameters, $\tan \beta$, $m_0$, $m_{1/2}$ and $A$), we see that we have the same number of parameters, $\tan \beta$, $m_0$, $m_0^2$ and the $R$-charge $e$. We can try and further constrain the model by demanding that the soft $H_1 H_2$ mass term lies on the same RG trajectory as the other soft terms (see Ref. [3]), but we find it impossible to find a satisfactory vacuum in that case.

A characteristic feature of AMSB models is the near-degenerate light charged and neutral winos; this prediction, depending as it does on Eq. (1a), is preserved in the scenario presented here. A variety of mass spectra for $m_0 = 40$ TeV (corresponding to a gluino mass of around 1 TeV), but with different values of $\tan \beta$, $e$ and $m_0^2$, is presented in Table 2; we were unable to find any values of $e$ and $m_0^2$ corresponding to an acceptable spectrum for $\tan \beta$ significantly larger than 10. The heaviest sparticle masses scale with $m_0$ and are given roughly by $M_{\text{SUSY}} = \frac{1}{16} m_0$. Consequently we take account of leading-log corrections by evaluating the mass spectrum at this scale. In other words, before applying Eq. (10), we evolve the dimensionless couplings (together with $v_1$, $v_2$) from the weak scale up to the scale $M_{\text{SUSY}}$. A dramatic feature of the spectra is the splitting in the slepton masses for different generations. Moreover, unusual [30] is the possibility (exemplified in the first three columns of Table 2) that the $\tilde{\nu}_e$ is the LSP. As is well known, radiative corrections give a sizeable upward contribution to the mass of the light CP-even Higgs, and so we have included the one-loop calculation (in the approximation given by Haber [31]).

A salient feature of the model is the existence of sum rules in which the dependence on the $R$-charge assignment cancels. These sum rules follow from Eq. (14); and thus for the particular solution exhibited in Table 1, they are independent of $e$. We find the following relations for the physical masses (in

### Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\tan \beta$</th>
<th>$\pm$</th>
<th>$+$</th>
<th>$-$</th>
<th>$-$</th>
<th>$-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_s^2$</td>
<td>$+1/9$</td>
<td>$-1/9$</td>
<td>$-1/9$</td>
<td>$-2/9$</td>
<td>$-2/9$</td>
<td></td>
</tr>
<tr>
<td>$m_0^2$</td>
<td>$-0.1$</td>
<td>$-0.1$</td>
<td>$-0.1$</td>
<td>$-0.25$</td>
<td>$-0.2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (in GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{t}_1$</td>
<td>652</td>
</tr>
<tr>
<td>$\tilde{t}_2$</td>
<td>882</td>
</tr>
<tr>
<td>$\tilde{b}_1$</td>
<td>865</td>
</tr>
<tr>
<td>$\tilde{b}_2$</td>
<td>977</td>
</tr>
<tr>
<td>$\tilde{\tau}_1$</td>
<td>94</td>
</tr>
<tr>
<td>$\tilde{\tau}_2$</td>
<td>110</td>
</tr>
<tr>
<td>$\tilde{u}_L$</td>
<td>918</td>
</tr>
<tr>
<td>$\tilde{u}_R$</td>
<td>997</td>
</tr>
<tr>
<td>$\tilde{d}_L$</td>
<td>920</td>
</tr>
<tr>
<td>$\tilde{d}_R$</td>
<td>887</td>
</tr>
<tr>
<td>$\tilde{e}_L$</td>
<td>260</td>
</tr>
<tr>
<td>$\tilde{e}_R$</td>
<td>423</td>
</tr>
<tr>
<td>$\tilde{\nu}_e$</td>
<td>83</td>
</tr>
<tr>
<td>$\tilde{\nu}_2$</td>
<td>251</td>
</tr>
<tr>
<td>$\tilde{\nu}_{1R}$</td>
<td>96</td>
</tr>
<tr>
<td>$\tilde{\nu}_{1L}$</td>
<td>598</td>
</tr>
<tr>
<td>$\tilde{\nu}_{2R}$</td>
<td>251</td>
</tr>
<tr>
<td>$\tilde{\nu}_{2L}$</td>
<td>364</td>
</tr>
<tr>
<td>$\tilde{\nu}_{3R}$</td>
<td>619</td>
</tr>
<tr>
<td>$\tilde{\nu}_{3L}$</td>
<td>637</td>
</tr>
<tr>
<td>$\tilde{\ell}$</td>
<td>1008</td>
</tr>
</tbody>
</table>

Table 2) that the $\tilde{\nu}_e$ is the LSP. As is well known, radiative corrections give a sizeable upward contribution to the mass of the light CP-even Higgs, and so we have included the one-loop calculation (in the approximation given by Haber [31]).
each case independent of e and sign $\mu_s$; in general the numerical results depend on $\tan \beta$, here taken throughout to equal 5, and also on $m_0$, here taken throughout to be 40 TeV, due to the running to $M_{\text{SUSY}}$ (which depends on $m_0$):

$$m^2_{h_1} + m^2_{h_2} + m^2_{h_1} - 2(m^2_{h_1} + m^2_{h_2}) - 2.75m^2_g = 0.92m^2_0 \text{ TeV}^2,$$

(21a)

$$m^2_{h_1} + m^2_{h_2} + m^2_{h_1} - 2(m^2_{h_1} + m^2_{h_2}) - 1.14m^2_g = 0.96m^2_0 \text{ TeV}^2,$$

(21b)

$$m^2_{h_1} + m^2_{h_2} + m^2_{h_1} + m^2_{h_2} - 1.70m^2_g = -3.56m^2_0 \text{ TeV}^2,$$

(21c)

$$m^2_{h_1} + m^2_{h_2} + m^2_{h_1} + m^2_{h_2} - 3.51m^2_g = 0.90m^2_0 \text{ TeV}^2,$$

(21d)

$$m^2_{\chi} = 2\sec 2\beta (m^2_{l_1} + m^2_{l_2} - 2m^2_\tau) - 0.49m^2_g = 1.05m^2_0 \text{ TeV}^2.$$  

(21e)

Eqs. (21c) and (21d) above involve only the first (or second) generations, and so the numerical results here are also independent of $\tan \beta$. Thus these two sum rules hold for every column in Table 2, as is easily verified.

It is interesting to compare these sum rules with the corresponding ones in the Fayet–Iliopoulos scenario described in our previous paper [16]; essentially the distinction lies in the non-zero RHS in Eqs. (21a)–(21e).

In conclusion, we have shown that within the MSSM it is possible to construct a solution to the running equations for $m^2$, $M$ and $h$ that is completely RG invariant, and leads to a phenomenologically acceptable theory, resulting in a distinctive spectrum with sum rules for the sparticle masses. Two sources of supersymmetry-breaking are required, one corresponding to the gravitino mass (at around $m_0 = 40$ TeV) and another, related to a $\mathcal{R}$-symmetry, at around $|m_0| = 500–500$ GeV. The magnitude of the latter suggests the idea of a common origin for it, the $\mu_s$ term and the associated $H_1H_2$ soft term. A convincing demonstration of this would considerably enhance the attractiveness of this model. It would also be interesting to consider variations on the same theme; forbidding the Higgs $\mu_s$ term, or incorporating massive neutrinos, for example.

Acknowledgements

This work was supported in part by a Research Fellowship from the Leverhulme Trust. We thank the referee for helping us to clarify some issues.

References

Higgs mechanism in the Randall–Sundrum model

A. Flachi a, David J. Toms a,*

a Department of Physics, University of Newcastle upon Tyne, Newcastle Upon Tyne, NE1 7RU, UK

Received 7 July 2000; accepted 7 September 2000

Editor: P.V. Landshoff

Abstract

We consider the dimensional reduction of a bulk scalar field in the Randall–Sundrum model. By allowing the scalar field to be non-minimally coupled to the spacetime curvature we show that it is possible to generate spontaneous symmetry breaking on the brane. © 2000 Elsevier Science B.V. All rights reserved.

The idea that spacetime may have some extra dimensions, beyond the usual four of Einstein’s theory, has been shown to provide an interesting solution to the gauge hierarchy problem [1]. An intriguing version of this scenario is the one proposed by Randall and Sundrum [2] — a five dimensional model with one extra spatial dimension having an orbifold compactification. Essentially the model consists of two three-branes with opposite tensions sitting at the two orbifold fixed points. The 5-dimensional line element is

\[ ds^2 = e^{-2kr|\phi|} \eta_{\mu \nu} dx^\mu dx^\nu - r_c^2 d\phi^2, \]

where \( x^\mu \) are the 4-dimensional coordinates, and \( |\phi| \leq \pi \) with the points \((x^\mu, \phi)\) and \((x^\mu, -\phi)\) identified. The factor of \( e^{-2kr|\phi|} \) present in (1) means that the Randall–Sundrum spacetime is not a direct product of 4-dimensional spacetime and the extra fifth dimension. This factor is often referred to as the warp factor. The three-branes sit at \( \phi = 0 \) and \( \phi = \pi \). \( k \) is a constant of order of the Planck scale and \( r_c \) is an arbitrary constant associated with the size of the extra dimension. The interesting feature of this model is the simple way in which it generates a TeV mass scale from higher dimensional Planck scale quantities. A field with mass \( m_0 \) which is confined on the negative tension brane will develop a physical mass \( m = m_0 e^{-kr_\pi} \); therefore, the electroweak scale is naturally realized if one adjusts the length of the extra-dimension to \( kr_\pi \sim 12 \).

In fact, in the original version of the Randall–Sundrum model all the standard model particles are supposed to be confined on the brane with only gravity living in the bulk (5-dimensional) spacetime.

An attractive alternative to confining standard model particles on the brane is to allow all of the fields to live in the bulk spacetime. Several aspects of this model have been studied and the literature is already immense. In [3], the physics of a bulk scalar field is studied, and the authors point out that the warp factor in (1) localises the field on the brane. This situation has been further explored in [4,5], where bulk gauge bosons have been considered. Some aspects of fermion bulk fields have been explored in [6–9], and bulk supersymmetry has been studied in [9]. Both fermions and gauge bosons in the bulk have been studied in [10,11]. In this last case it turns out that the zero modes, interpreted as standard model particles, are localised on...
the brane, explaining why the hierarchy problem is solved in a different setting with respect to the original Randall–Sundrum model in which the standard model was confined on the wall. In [9,11–13] a bulk Higgs field as the origin of spontaneous symmetry breaking was also considered. Unfortunately, phenomenological constraints for gauge boson masses to be of the order of the electroweak scale requires a hierarchically small Higgs mass. This seems to rule out a bulk Higgs field, leaving two alternatives: stick the Higgs on the brane by hand (or by some as yet unknown confinement mechanism), or expect some dynamics to drive a bulk scalar field to a negative mass squared field in four dimensions. Other arguments have been given in [11,12].

In this letter we investigate the possibility of obtaining spontaneous symmetry breaking as a consequence of dimensional reduction within the Randall–Sundrum model with fields living in the bulk. We will show how it is possible to obtain a scalar particle with imaginary mass via a Kaluza–Klein reduction in the Randall–Sundrum spacetime. A scalar field in a higher-dimensional spacetime has been shown to reduce in four-dimensions to the Kaluza–Klein infinite tower of scalar fields whose masses \( m_n \) are quantised [3]. For the Randall–Sundrum metric the masses \( m_n \) are given by solutions of the transcendental equation

\[
0 = y_n(ax_n)j_n(x_n) - j_n(ax_n)y_n(x_n),
\]

where we have defined \( a = e^{-\pi kr_c} \) and \( m_n = kax_n \), with \( x_n \) the \( n \)th positive solution to (2). The functions \( j_n \) and \( y_n \) are given by the following combinations of Bessel functions:

\[
\begin{align*}
  j_n(z) &= 2J_n(z) + zJ_n(z), \\
  y_n(z) &= 2Y_n(z) + zY_n(z).
\end{align*}
\]

The order of the Bessel functions is \( v = \sqrt{4 + \tilde{m}^2/k^2} \) where \( \tilde{m} \) is the mass of the five-dimensional scalar field.

Our model is described by a simple generalisation of that in [3]:

\[
S = \frac{1}{2} \int d^4x \int d\varphi \sqrt{g} \left( g^{AB} \partial_A \Phi \partial_B \Phi - \tilde{m}^2 \Phi^2 - \xi \, R \Phi^2 - \frac{\lambda}{k^4} \Phi^4 \right).
\]

Here \( \lambda \) is dimensionless (for the moment no value for it is specified) and \( \xi \) is also dimensionless and represents a non-minimal coupling of the scalar field to the gravitational background. \( g_{AB} \) represents the metric of (1) and \( \tilde{R} \) is the scalar curvature computed from this metric. We will see that it is the non-minimal coupling of the scalar field which allows the possible generation of Higgs particles in the theory.

Let us now turn to Kaluza–Klein reduction. In the non-minimally coupled case the situation is different from the one presented in [3] due to the presence of the \( \tilde{R} \phi^2 \) term. Decomposing the fields as a sum over modes and setting \( \lambda = 0 \) initially (as in [3]), we write

\[
\Phi(x, y) = \sum_n \psi_n(x) f_n(y),
\]

with

\[
\int \frac{dy}{\pi r_c} e^{-2\sigma} f_n(y) f_n(y) = \delta_{nn}.
\]

The field equation for the modes becomes

\[
-\epsilon^{2\sigma} \partial_y \left( \epsilon^{4\sigma} \partial_y f_n(y) \right) + m^2 \epsilon^{-2\sigma} f_n(y)
- 16 k \epsilon^{2\sigma} \left( \delta(y) - \delta(y - \pi r_c) \right) f_n(y) = m_n^2 f_n(y),
\]

where we have defined \( m^2 = \tilde{m}^2 + 20 \xi k^2 \). It can be seen that the presence of the terms involving Dirac delta distributions in (8) has its origin in the curvature of the Randall–Sundrum spacetime. For \( y \neq 0 \) or \( y \neq \pi r_c \) the linearly independent solutions to (8) are the same Bessel functions as those given in [3]. After applying the boundary conditions appropriate to the orbifold compactification in the model it is easily shown that

\[
f_n(y) = N_n \epsilon^{2\sigma} \left( J_n \left( \frac{m_n}{k} \epsilon^\sigma \right) \\
- \frac{j_n(m_n/k)}{y_n(m_n/k)} \epsilon \left( \frac{m_n}{k} \epsilon^\sigma \right) \right)
\]

with \( v = \sqrt{4 + m^2/k^2} \) and

\[
\begin{align*}
  j_n(z) &= (2 + 8 \xi) J_n(z) + zJ_n(z), \\
  y_n(z) &= (2 + 8 \xi) Y_n(z) + zY_n(z).
\end{align*}
\]

For \( \xi = 0 \) these results reduce exactly to those in [3], as they should. The normalization factor can be evaluated in closed form, but the expression is very lengthy and will not be given explicitly here.
Let us now turn to the mass spectrum. As was said before, it is interesting to extend the Randall–Sundrum model by considering the possibility of having other bulk fields. Since spontaneous symmetry breaking in the bulk appears to be disfavoured for a variety of reasons, the Higgs field is forced to live on the brane. No alternative to generate spontaneous symmetry breaking on the brane starting from an ordinary bulk scalar field has been investigated. We want to show that a non-minimally coupled scalar field offers such a possibility.

The masses of the Kaluza–Klein excitations are given by the zeroes of the function
\[ F_\nu(z, \xi) = y_\nu(az) j_\nu(z) - j_\nu(az)y_\nu(z). \]  
(12)
Clearly, the previous considerations lead us to look for a Higgs field after Kaluza–Klein reduction, which in turn means looking for purely imaginary solutions of (12). After rotating to the complex plane \((z \rightarrow iz)\), \(F_\nu(z, \xi)\) can be rewritten in terms of the modified Bessel functions and the mass spectrum equation reads
\[ 0 = i_\nu(az)k_\nu(z) - k_\nu(az)i_\nu(z). \]  
(13)

Obviously, the previous equation cannot be solved analytically and its numerical study is rather tricky because of the oscillating behaviour of the Bessel functions, the presence of an extremely small exponential factor and of the fact that \(F_\nu(z, \xi)\) becomes very large.

Although it is necessary to resort to a numerical analysis, several points can be addressed analytically. First of all, note that the function (13) does not admit real zeros unless the order of the Bessel function is imaginary. This can be achieved by letting
\[ \xi < -\frac{1}{20}\left(4 + \frac{m^2}{k^2}\right). \]
Secondly, although the modified Bessel functions appearing in the above transcendental equation are complex, the combination appearing in (13) can be shown to be real. Thus it is possible to expedite the numerical procedure by taking the real part.

A negative value of \(\xi\) is necessary if we are to obtain a Higgs type mass for the dimensionally reduced field. The coupling constant \(\xi\) is to be regarded as a free parameter. Although a popular choice is to fix its value to zero or the conformal value \((1/6\text{ for 4-dimensions, and }3/12\text{ for 5-dimensions})\) for computational simplicity, there is no good argument to prefer any specific value. Indeed, there are specific theories in which a prescription for \(\xi\) does exist. For instance, it has been shown in [13] that Higgs scalar fields must have \(\xi \leq 0\) or \(\xi \geq 1/6\) in order to have an absolutely stable ground state. Other examples have been considered in [14–16].

In performing the numerical analysis various features have to be taken into account. The first is related to the bounds on the Higgs mass. The discussion on this issue is quite complicated because in the Randall–Sundrum model, higher dimensional operators need to be included in order to have a reliable upper limit on the Higgs mass [17]. However, the mass spectrum does not depend dramatically on the value of the Higgs mass — this can be easily seen numerically — therefore, we do not expect our analysis to change drastically by altering the Higgs mass within a reasonable range of values. It is interesting to note that from the knowledge of the bounds on the Higgs mass, it is possible to trace back bounds on \(\xi\). In the following we fix \(m_H = 100\text{ GeV}\).

Another parameter, which seems to be quite problematic, is the ratio \(\tilde{m}/k\), or simply \(\tilde{m}\), which needed to be finely tuned when placing the Higgs field in the bulk in previous work [13]. We have chosen \(\tilde{m}/k = 1\), again noting that \(F_\nu(z, \xi)\) is not particularly sensitive to this parameter for values between 0 and 5. The dependence of \(\xi\) on \(\tilde{m}/k\) has been studied and the results are shown in Fig. 1.

The approximate value of \(\xi\) corresponding to a mass of 100 GeV is \(\xi = -0.250020221625119498\). The standard model bounds [18,19] on the Higgs mass correspond to \(\xi\) varying between \(-0.250019\) and \(-0.250061\). It is important to note that for a fixed
value of the Higgs mass $\xi$ can assume different values. In our computation we took the first zero. As we will discuss, this has the virtue that the next mass eigenvalue (if there is one) is extremely large.

When mode expanding the 5-dimensional action we have to consider the self interaction term, which simply needs to be integrated over the extra coordinate. Taking into account only the first low lying mode, after integration, the self interaction term looks like

$$S_{\text{int}} = \lambda_{\text{eff}} \int d^4x \psi_n^4(x),$$

with

$$\lambda_{\text{eff}} = \frac{2\lambda}{k} \int_0^{\pi r_c} dy e^{-4ky} f^4_n(y).$$

Although it does not appear possible to obtain an exact closed form result for this expression, it can be evaluated numerically without difficulties and the result is $\lambda_{\text{eff}} = 9.42553 \times 10^{-18}\lambda$. Even for very large values of $\lambda$ in the 5-dimensional theory, the self-interaction term in the effective 4-dimensional theory is small.

An important feature to consider is the presence of higher Kaluza–Klein modes. It is important to make sure that there are no other low lying modes for at least two reasons: firstly, because their presence would considerably complicate any phenomenological analysis; secondly, because they would give rise in the dimensional reduction of the self-interaction portion to mixed terms. Therefore, we had to extend our numerical investigation not only to the first mode, but to higher modes as well, to check the reliability of the present model. Because of numerical limitations, we have studied the function $F_n(z, \xi)$ only in a relatively large region, $|z| < 740,000,000$ corresponding to modes of masses $m < 10^{11}\text{GeV}$. Our numerical study shows that there is only one root in this region, enabling us to discard in the Kaluza–Klein expansion for the self interaction term the mixed modes safely if we are only interested in the low energy effective theory.

In conclusion, we have discussed the issue of a bulk Higgs field, relevant in any attempt to place the standard model in the bulk. Since this possibility is disfavoured, we have indicated a mechanism which admits a bulk scalar field and spontaneous symmetry breaking on the brane. In other words we have considered a bulk scalar field with a positive mass term and indicated a way of obtaining a scalar field with imaginary mass on the brane. We achieved this by considering a non-minimally coupled theory and letting the scalar coupling be negative. The non-minimal coupling constant is responsible for this. Thus it is possible to generate spontaneous symmetry breaking in a natural way using the geometry.

Acknowledgement

A. Flachi would like to thank the University of Newcastle upon Tyne for the award of a Ridley Studentship.

References

Sparticle spectrum and dark matter in type I string theory with an intermediate scale

D. Bailin \(^a\), G.V. Kraniotis \(^b\), A. Love \(^b\)

\(^a\) Centre for Theoretical Physics, University of Sussex, Brighton BN1 9QJ, UK
\(^b\) Centre for Particle Physics, Royal Holloway and Bedford New College, University of London, Egham, Surrey TW20-0EX, UK

Received 28 July 2000; received in revised form 28 August 2000; accepted 7 September 2000

Editor: P.V. Landshoff

Abstract

The supersymmetric particle spectrum is calculated in type I string theories formulated as orientifold compactifications of type IIB string theory. A string scale at an intermediate value of \(10^{11}–10^{12}\) GeV is assumed and extra vector-like matter states are introduced to allow unification of gauge coupling constants to occur at this scale. The qualitative features of the spectrum are compared with Calabi-Yau compactification of the weakly coupled heterotic string and with the eleven dimensional supergravity limit of \(M\)-theory. Some striking differences are observed. Assuming that the lightest neutralino provides the dark matter in the universe, further constraints on the sparticle spectrum are obtained. Direct detection rates for dark matter are estimated.

In a generic supergravity theory, the soft supersymmetry-breaking terms are free parameters. On the other hand, if the supergravity theory is the low-energy limit of a string theory, these parameters are calculable in principle in terms of the fewer parameters characteristic of string theory. Once the soft supersymmetry-breaking terms have been determined the renormalization group equations may be run from the string scale to the electroweak scale to derive the sparticle spectrum. Such calculations have been performed in the context of the weakly coupled heterotic string in the large modulus limit of Calabi-Yau compactifications [1], of orbifold compactifications of the weakly coupled heterotic string [1,2] and of the eleven dimensional supergravity limit of \(M\)-theory corresponding to the strongly coupled heterotic string [3–8]. Here we extend such calculations to scenarios motivated by type I string theories constructed as orientifold compactifications of type IIB string theory [9,10]. A novel feature of type I theories is that the string scale is a function of the Planck scale and the compactification scale and can, in principle, lie anywhere between about 1 TeV and \(10^{18}\) GeV [11–15]. A rather natural possibility is for the string scale to be at an intermediate scale of order \(10^{11}\) GeV. Type I theories possess an elegant mechanism for this to occur which may be summarized as follows. It is possible to construct type I theories in which the observable gauge group and quark and lepton matter are associated with 9-branes and 5-branes while supersymmetry is broken directly in non-supersymmetric anti-5-brane sectors. The scale of supersymmetry breaking in the anti-5-brane sector is the type I string scale \(M_1\) and the supersymmetry...
breaking will be transmitted gravitationally to the observable sector. We then expect masses for the supersymmetric particles of order $M_2^2/M_P$ where $M_P$ is the Planck mass. For sparticle masses of order 1 TeV we have $M_1 \sim 10^{11}$ GeV. In a string theory, unification of gauge coupling constants at $\sim 10^{12}$ GeV with the unconventional normalization of the standard model $U(1) \ g_1^2/g_2^2 = 3/11$. Scenario I$_b$ is inspired by an explicit $Z_3$ orientifold model [9] with this latter property, though the model does not have all the properties discussed in the next paragraph.

The soft supersymmetry-breaking terms for type I theories are known where the observable sector gauge group and all observable sector matter are associated with 9-branes, 5-branes or open strings linking 9-branes to 5-branes, and all 5-branes sit at the orbifold fixed point at the origin, so that duality transformations can be exploited to the full. We shall consider the case where there are only $S_i$-branes for one value of $i$, say $S_3$-branes, where $i$ labels the complex compact dimension wrapped by the 5-brane, and where there is a single overall modulus $T$. We shall also assume that the observable gauge group is entirely in the 9-brane sector and the cosmological constant $V_0$ is zero, so that $C = 1$ in the notation of Brignole et al. [1], and that the CP violating phases $\alpha_S, \alpha_T$ are zero. Then, the soft supersymmetry-breaking terms are universal and the same as in the large $T$ limit of the Calabi-Yau compactification of the weakly coupled heterotic string:

$$M_{1/2} = \sqrt{3} m_{3/2} \sin \theta,$$
$$m_0^2 = m_{3/2}^2 \sin^2 \theta,$$
$$A = -\sqrt{3} m_{3/2} \sin \theta,$$

where $M_{1/2}, m_0$ and $A$ are the observable sector gaugino mass, scalar mass and trilinear scalar coupling, respectively, and $m_{3/2}$ is the gravitino mass. The Goldstino angle $\theta$ has been introduced by parametrizing the auxiliary fields $F^S$ and $F^T$ for the dilaton $S$ and modulus $T$ in the form

$$F^S = \sqrt{3} m_{3/2}(S + \bar{S}) \sin \theta,$$
$$F^T = \sqrt{3} m_{3/2}(T + \bar{T}) \cos \theta.$$

The effects of twisted sector moduli entering the gauge kinetic function and mixing with the $T$ modulus through a Green–Schwarz term have been neglected.

As mentioned earlier, a novel sparticle spectrum can arise when the renormalization group equations are
run from an intermediate string scale of order \(10^{11}\) or \(10^{12}\) GeV instead of \(10^{16}\) GeV, especially when unification at the intermediate scale is achieved by the introduction of extra matter states, even though the soft supersymmetry-breaking terms at the string scale are not novel. We shall present results for the dilaton dominated case \(\theta = \frac{\pi}{2}\) and for a “typical” case \(\theta = \frac{\pi}{4}\) but not for \(\theta = 0\), in which case the loop corrections become important.

Our parameters are the goldstino angle \(\theta\), sign \(\mu\) (which is not determined by the radiative electroweak symmetry-breaking constraint) [18] where \(\mu\) is the Higgs mixing parameter in the light-energy superpotential and \(\tan \beta\), the ratio of Higgs expectation values \(\langle H^0_u \rangle/\langle H^0_d \rangle\), if we leave \(B\), the coefficient of the soft bilinear term associated with the Higgs mixing term, to be a free parameter to be determined by the minimization of the Higgs potential. Using (1)–(3) as boundary conditions, the renormalization group equations are run from the unification scale to the overall scale which is determined by the gravitino mass. We therefore only present the sparticle spectrum for the two scenarios, type Ia and Ib, described earlier and compare with the eleven-dimensional supergravity limit of M-theory corresponding to the strongly-coupled heterotic string and with the weakly-coupled heterotic string in the large-\(T\) limit of Calabi-Yau compactification.

The results of our calculation of the sparticle spectra arising from the different scenarios are presented in Figs. 1–3. For the Ia scenario the sparticle spectra for \(\theta = \pi/4\) and \(\theta = \pi/2\) differ very little apart from the overall scale which is determined by the gravitino mass. We therefore only present the sparticle spectrum for \(\theta = \pi/4\) in Ia scenario. We find also that the choice of sign of \(\mu\) makes little difference to the spectra. Qualitatively, the principal features are as follows:

- The CP-odd Higgs \(\langle A^0 \rangle\) is much lighter in the Ia scenario than in Ib. For the most part it is lighter than the lightest stop \(\langle st_2 \rangle\) in the Ia scenario, whereas in Ib it is much heavier than \(\langle st_2 \rangle\).
- The lightest stau \(\langle st_2 \rangle\) is also lighter in the Ia scenario than in Ib. In the former it is closer to the lightest chargino \(\langle \chi^{+}_1 \rangle\), whereas in the latter it is much heavier than \(\langle \chi^{+}_1 \rangle\).
- Moreover, in Ia the \(\langle \chi^{+}_1 \rangle\) is much heavier than the lightest neutralino \(\langle \chi^{0}_1 \rangle\), whereas in Ib they are almost degenerate, with \(\langle \chi^{0}_1 \rangle\) being a few GeV lighter. This has important consequences for dark matter due to coannihilation effects.
- In Ib the gluino is almost the heaviest sparticle, whereas in Ia it is in the middle of the spectrum (and lighter than \(A_0\) for \(\tan \beta < 32\)).

It is of interest to compare these spectra with those arising in other string scenarios. To allow this
Fig. 1. $I_3$ scenario sparticle spectrum vs $\tan \beta$ for $m_{3/2} = 140$ GeV, $\mu > 0, \theta = \pi/4$.

Fig. 2. Sparticle spectrum vs $\tan \beta$ $I_3$ scenario with $6L + 3D_R$, $m_{3/2} = 250$ GeV, $\mu > 0, \theta = \pi/2$. 
comparison we present the sparticle spectra for the extreme $M$-theory limit in Figs. 4, 5, for the large-$T$ limit of weakly coupled string theory compactified on a Calabi-Yau space in Fig. 6, and for the case of mirage unification [19], with soft supersymmetry-breaking terms running from $10^{11}$ GeV, in Figs. 7, 8 (in the case of mirage unification there is no extra matter, so the gauge couplings are not unified at the string scale; the “mirage” of unification at $10^{16}$ GeV is given by string loop effects). In the extreme $M$-theory limit case the Goldstino mixing angle $\theta = \frac{\pi}{4}$ is not accessible without the scalar mass squared ($m^2_0$) becoming negative at the unification scale, hence breaking the electroweak gauge symmetry in models with the standard model gauge group.

The noteworthy qualitative features of this comparison are as follows:

- The $I_b$ spectrum is similar in most respects to that of the extreme $M$-theory case with $\theta = \frac{\pi}{4}$.

However, in the $I_b$ case $s_{T_2}$ is heavier than $\chi^+_1$ for $\tan \beta \leq 25$, whereas in the extreme $M$-theory case it is always lighter than $\chi^+_1$. Furthermore, in the $M$-theory case, for $\tan \beta > 23$, ($s_{T_2}$) becomes the lightest supersymmetric particle (LSP).

- Most of the foregoing features are insensitive to the Goldstino angle. However, in the extreme $M$-theory case [3] (see Fig. 5) with $\theta = \frac{7\pi}{20}$ we have $m_0 \ll M_{1/2}$, the common gaugino mass, whereas for $\theta = \frac{\pi}{4}$ we have $m_0 \sim M_{1/2}$ and this difference produces some qualitative changes. For example, $s_{T_2}$ becomes the LSP for $\tan \beta > 9$ when $\theta = \frac{7\pi}{20}$. Also $A^0$ is now heavier than when $\theta = \frac{\pi}{4}$, and $s_{T_2}$ is lighter than $A^0$, as is the case in the $I_b$ scenario.

- The spectra deriving from the extreme $M$-theory limit with $\theta = \pi/4$ (Fig. 4) are similar in most respects to those deriving from the weakly coupled case with $m_{3/2} = 100$ GeV, $\theta = \frac{\pi}{2}$ shown in Fig. 6.
Fig. 4. Sparticle spectrum vs $\tan \beta$ in extreme M-theory limit, i.e. $\alpha(T+\overline{T}) = 2$, $\theta = 7\pi/4$, $m_{3/2} = 200$ GeV, $\mu > 0$.

Fig. 5. Sparticle spectrum vs $\tan \beta$ in extreme M-theory limit, i.e. $\alpha(T+\overline{T}) = 2$, $m_{3/2} = 210$ GeV, $\theta = 7\pi/20$, $\mu > 0$. 
Fig. 6. Sparticle spectrum vs $\tan \beta$, $m_{3/2} = 100$ GeV, $\mu > 0$, in the large-$T$ limit of weakly-coupled CY space, $\theta = \pi/2$.

Fig. 7. Sparticle spectrum vs $\tan \beta$, in mirage unification scenario, $m_{3/2} = 100$ GeV, $\mu > 0$. 
The spectra arising in the mirage unification scenario (with \( \mu > 0 \)) are similar in most respects to those in Ia. However, for mirage unification \( s_{\tau_2} \) is lighter than \( \chi^+_1 \), whereas it is heavier than \( \chi^+_1 \) in Ia for \( \tan\beta < 25 \).

The spectra arising in the two mirage unification scenarios, \( \mu > 0 \) and \( \mu < 0 \), are similar in most respects. However, for \( \mu > 0 \) \( \chi^+_1 \) is always heavier than \( s_{\tau_2} \), whereas for \( \mu < 0 \) \( \chi^+_1 \) is lighter than \( s_{\tau_2} \) for \( \tan\beta < 17 \). Also, for small \( \tan\beta \) the masses of \( A^0 \) and \( s_{\tau_2} \) are very similar for \( \mu > 0 \), but very different for \( \mu < 0 \).

Assuming \( R \)-parity conservation the LSP is stable, and consequently if it is neutral can provide a good dark matter candidate. We assume that the dark matter is in the form of neutralinos. The lightest neutralino is a linear combination of the superpartners of the photon, \( Z^0 \) and neutral-Higgs bosons,

\[
\chi^0_1 = N_{11} \bar{B} + N_{12} \bar{W}^3 + N_{13} \bar{H}_1^0 + N_{14} \bar{H}_2^0.
\]

For both Ia and Ib scenarios the lightest neutralino is the LSP for most of the parameter space and for the mirage unification \( \chi^0_1 \) is not the LSP only for \( \tan\beta > 25 \). For these cases one can calculate the resulting relic abundance.

When the observational data on temperature fluctuations, type Ia supernovae, and gravitational lensing are combined with popular cosmological models, the dark matter relic abundance (\( \Omega_{\text{LSP}} \)) typically satisfies [20]

\[
0.1 \leq \Omega_{\text{LSP}} h^2 \leq 0.4.
\]

We calculated the relic abundance of the lightest neutralino in the scenarios we have considered using standard techniques [21]. When these results are confronted with the (model-dependent) bounds (10) derived from the observational data further constraints on the parameters \( m_{3/2}, \tan\beta, \mu, \theta \) are obtained and these give new constraints on the sparticle spectrum.
Let us start with the $I_a$ scenario. We first present the results of a calculation of the relic abundance for the lightest neutralino as a function of the gravitino mass $m_3/2$ for two representative values of $\tan \beta$, $\tan \beta = 3$ and $\tan \beta = 10$. In Fig. 9 we plot the relic abundance for the lightest neutralino versus the gravitino mass, $m_3/2$, for $\theta = \pi/2$ and $\tan \beta = 3$, $\mu > 0$. The upper and lower limit (10) on the relic abundance constrain $m_3/2$ to lie in the interval $85 \leq m_3/2 \leq 170$ GeV see Table 1.

If instead we take $\tan \beta = 10$, $100 \leq m_3/2 \leq 167$ GeV for $\mu > 0$ whereas for $\mu < 0$, $113 \leq m_3/2 \leq 170$ GeV. In this case one obtains the bounds on the sparticle masses exhibited in Table 1.

The lightest stau for $\mu > 0$ is in the range $114 \leq m_{\tilde{\tau}_2} \leq 185$ GeV. For the same value of $\tan \beta$ the total detection rates for a typical $^{75}$Ge detector are in the range $0.07(2 \times 10^{-2})$–$4.8 \times 10^{-1}(7 \times 10^{-4})$ events/kg/day for $\mu < 0$ ($\mu > 0$), respectively. The lightest supersymmetric particle is almost a Bino for both signs of $\mu$. For $\mu < 0$ the higgsino component is a little bit larger than for $\mu > 0$ but still the LSP is essentially almost a Bino.

In Fig. 10 we plot the relic abundance versus $\tan \beta$ for fixed gravitino mass. From this figure we see that for smaller values for the gravitino mass (i.e. the lighter the spectrum) $\tan \beta$ is restricted to small values.

In the $I_b$ scenario and for $\mu > 0$ the constraints from (10) have dramatic consequences. As noted earlier there is an almost exact degeneracy of the lightest chargino with the lightest neutralino, $M_{\tilde{\chi}^\pm_1} \approx M_{\tilde{\chi}^0_1} \leq 3$ GeV. Because of this coannihilation effects [22] become important and the resulting relic abundance is very small. Therefore, if the lightest neutralino (which is almost wino in this case) makes up most of the nonbaryonic dark matter in the universe this model is excluded.

![Fig. 9. Relic abundance of LSP vs $m_{3/2}$, $I_a$ scenario with $2L + 3E_R$, $\tan \beta = 3$, $\mu > 0$, $\theta = \pi/2$.](image1)

![Fig. 10. Relic abundance of LSP vs $\tan \beta$, $I_a$ scenario with $2L + 3E_R$, $m_{3/2} = 100, 150$ GeV, $\mu > 0$, $\theta = \pi/2$.](image2)

<table>
<thead>
<tr>
<th>Mass</th>
<th>$\tan \beta = 10$</th>
<th>$\mu &lt; 0$</th>
<th>$\mu &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{3/2}$ (GeV)</td>
<td>113–170</td>
<td>100–167</td>
<td>85–170</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}^+_1}$ (GeV)</td>
<td>54–87.5</td>
<td>52.5–89.1</td>
<td>49–93.5</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}^0_1}$ (GeV)</td>
<td>87–150</td>
<td>88–158</td>
<td>93–174</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}^0_1}$ (GeV)</td>
<td>115–123</td>
<td>110–121</td>
<td>89–105</td>
</tr>
</tbody>
</table>
In the mirage unification scenario (without extra matter) cosmological constraints prefer a low \( \tan \beta \) and gravitino mass. For instance for \( m_{3/2} = 90 \text{ GeV}, \mu < 0 \) the relic abundance is in the range \( 0.12 \geq \Omega_{\text{LSP}}^2 h^2 \geq 0.01 \) for \( 3 \leq \tan \beta \leq 8 \). In this case, for \( ^{73}\text{Ge}, ^{208}\text{Pb}, ^{131}\text{Xe} \) detectors, detection rates of the neutralinos are in the range of order \( 10^{-1} - 10^{-5} \) events/kg/day. This illustrates the fact that \( \Omega_{\text{LSP}}^2 h^2 \sim 10^{-37} \text{ cm}^2/\sigma_{\text{nunu}} \) and the neutralino annihilation cross section is roughly proportional to the neutralino scalar cross section. Thus as the LSP abundance decreases, its scattering cross section generally increases. For \( \Omega_{\text{LSP}}^2 h^2 \sim 0.1 \) this results in an increased event rate. Thus in this region of the parameter space even if the neutralino cosmic density is insufficient to close the universe, and other forms of dark matter are needed, the prospects of its direct detection in underground nonbaryonic dark matter experiments could be enhanced. This has also been noted by Gabrielli et al. [24].

This comparatively large direct detection rate is a consequence of the Higgsino component of the lightest neutralino being comparable to or even larger than the gaugino component. As a result the scalar cross section for the scattering of a neutralino with a nucleon through Higgs exchange increases. The scalar nucleon–LSP cross section is given by [23,24]

\[
\sigma_{\text{scalar}}^\text{nucleon} = \frac{8G_F^2}{\pi} M_{\text{h}}^2 m_{\text{red}}^2 \\
\times \left[ \frac{G_1(h_0)I_{h_0} + G_2(H)I_H}{m_{h_0}^2} + \cdots \right]^2, \tag{11}
\]

where

\[
G_1(h_0) = (-N_{11} \tan \theta_W + N_{21}) \\
\times (N_{31} \sin \alpha + N_{41} \cos \alpha),
\]

\[
G_2(H) = (-N_{11} \tan \theta_W + N_{21}) \\
\times (-N_{31} \cos \alpha + N_{41} \sin \alpha), \tag{12}
\]

\[
I_{h_0,H} = \sum_q \frac{\bar{p}_q h_0}{m_q} \frac{N_q |\bar{q} q| N}{}, \tag{13}
\]

\[
p_q \bar{h_0} = \frac{\cos \alpha}{\sin \beta}, \quad p_q \bar{H} = \frac{\sin \alpha}{\sin \beta}, \quad \text{for } q = u, c, t,
\]

\[
p_q \bar{h} = \frac{\sin \alpha}{\cos \beta}, \quad p_q \bar{H} = \frac{\cos \alpha}{\cos \beta}, \quad \text{for } q = d, s, b. \tag{14}
\]

In Eq. (11) \( m_{\text{red}} \) is the neutralino-nucleon reduced mass, \( h_0, H \) denote the lightest Higgs and CP-even heavier Higgs, respectively, and \( \alpha \) is the Higgs mixing angle. We note also the \( \tan \beta \) dependence of the scalar neutralino-nucleon cross section \( \sigma_{\text{scalar}}^\text{nucleon} \). For high values of \( \tan \beta \) the corresponding cross section generically increases. The ellipsis denotes the contribution to the scalar cross-section through squark exchange which we have not written explicitly, although we included it in the calculations [3]. We note that, even if the scalar (spin-independent) interaction is the dominant one, the spin-dependent interaction through \( Z \)-exchange is also appreciable in this case since it is proportional to the difference \( |N_{41}|^2 - |N_{41}|^2 \) [3].

In summary, the three intermediate scale scenarios studied have sparticle spectra with striking qualitative features which distinguish them from each other and from the \( M \)-theory and weakly coupled heterotic string cases. Moreover, the composition of the lightest neutralino differs in the three scenarios. (It is almost Wino for the Ib scenario, with a large Higgsino component for the mirage unification scenario, and almost Bino for the Ia scenario, as well as for the \( M \)-theory and weakly-coupled heterotic string cases.) If we assume that the lightest neutralino provides the dark matter in the universe, constraints on the relic abundance put lower and upper bounds on the sparticle masses in each scenario. Also, the Ib scenario is then excluded because coannihilation effects result in a too small relic abundance. Direct detection rates for the lightest neutralino in the Ib scenario are similar to those for \( M \)-theory and weakly coupled heterotic string models. Interestingly, direct detection rates one or two orders of magnitude larger are obtained in the mirage unification scenario where the lightest neutralino has a large Higgsino component.

Acknowledgements

This research is supported in part by PPARC.

References


[7] T. Li, MADPH-99-1109;
T. Li, hep-ph/9903371.


Supersymmetrizing branes with bulk in five-dimensional supergravity

Adam Falkowski a, Zygmunt Lalak a,b, Stefan Pokorski a,*

a Institute of Theoretical Physics, University of Warsaw, Warsaw, Poland
b Physikalisches Institut, Universität Bonn, Nussallee 12, D-53115 Bonn, Germany

Received 18 May 2000; received in revised form 17 August 2000; accepted 30 August 2000

Abstract

We supersymmetrize a class of moduli dependent potentials living on branes with the help of additional bulk terms in 5d $N = 2$ supergravity. The resulting theories are gauged supergravities with specific relations between bulk and brane cosmological potentials. The space of Poincaré invariant vacuum solutions includes the Randall–Sundrum solution and the M-theoretical solution. After adding gauge sectors to the branes we discuss breakdown of low energy supersymmetry in this setup and hierarchy of physical scales.

The idea of higher-dimensional unification of the fundamental interactions attracts considerable interest and receives more and more concrete realizations. The general setup for such unification consists of hypersurfaces hosting various gauge sectors which are embedded into higher-dimensional bulk space. Bulk interactions are those of higher-dimensional gravity coupled to certain scalar and form fields, as well as to fermions, which are however inert with respect to gauge groups localized on branes. From the low-energy point of view most of the nontrivial features of field theoretical models that are related to spatial separation of gauge sectors should be clearly visible at the level of the simple five-dimensional theory. So far, the most extensively studied [1–4] supersymmetric models of this kind are related to the supergravity model constructed by Horava and Witten as the low energy effective theory of the strongly coupled heterotic $E_8 \times E_8$ superstring [5,6] (see also [7]). In particular, a five-dimensional theory is the simplest nontrivial setup to study spontaneous supersymmetry breakdown and its transmission between the branes [3,4,8]. The agents of that transmission are the bulk fields.

Much attention to five-dimensional gravity has also been drawn by the recent observation that 5d anti-de-Sitter gravity with 3-branes embedded in the bulk allows for solution with localized gravitational field [9]. However, in this scenario it is necessary to add the cosmological terms localized on the boundaries with coefficients determined uniquely by the cosmological term in the bulk. The correlation between bulk and boundary potentials is crucial for obtaining a consistent solution to Einstein equations, as well as vanishing of the cosmological constant in the effective four-dimensional theory. In the original paper [10] no symmetry justifies this apparent fine-tuning.
A considerable effort has been devoted to supersymmetrization of the Randall–Sundrum model. The early papers [11 – 14] are devoted to attempts to generate the Randall–Sundrum 3-brane as a thick brane, i.e., as a smooth solitonic solution of the $N = 2, d = 5$ supergravity coupled to a number of vector multiplets. In that approach the brane tensions and the bulk cosmological term are generated as vacuum condensates of the bosonic fields in the vector multiplet sector. The structure of this sector turns out to be so constrained that smooth solutions of the Randall–Sundrum type do not exist [11,13]. Another approach is to consider singular supersymmetric solutions with delta-type (thin) 3-branes, as in the five-dimensional version of the Horava–Witten model [1,2].

The authors of reference [15] start with 5d $N = 2$ pure supergravity with a cosmological constant $\Lambda$ and demonstrate that inclusion of delta-type 3-branes in a supersymmetric way leads to the Randall–Sundrum action. The Lagrangian is that of gauged supergravity, with $U(1)$ subgroup of the $R$-symmetry group gauged and with a particular choice of the prepotential and $Z_2$ symmetry. However, it turns out that the construction using delta-type 3-branes is not unique as it allows for different ways of imposing $Z_2$ symmetry. In reference [15] the profile of the gravitini mass term in the bulk is chosen to be $Z_2$ symmetric across the wall, while it is possible to choose these terms to have an $Z_2$ antisymmetric, kink-type profile across the wall (thus coupling $Z_2$-odd and $Z_2$-even components of gravitini). In the present paper we give an exhaustive analysis of the second possibility.

In particular, we demonstrate by explicit construction that our approach allows to couple to supergravity in a supersymmetric way hypermultiplets living in the bulk and gauge and matter sectors living on delta-like branes. Neither scalar fields in the bulk nor gauge and matter on the branes are included in [15]. Our construction makes it possible to study low-energy supersymmetry breakdown in the Randall–Sundrum background, which we do in the final part of this paper. The possibility of having $Z_2$ ‘odd’ fermionic mass terms was already considered in Ref. [16], but without studying the full structure of gauged supergravity, in particular without complete corrections to the supersymmetry transformations. These ingredients, as well as bulk hypermultiplets, matter and gauge sectors, are also absent in an interesting attempt of the Ref. [17], where 4-form potential is coupled to 5d supergravity.

To summarize, the purpose of this paper is to study, in a general way, a class of five-dimensional locally supersymmetric theories with delta-type 3-branes. We demonstrate that certain types of potentials introduced on branes, that are not parts of a supersymmetric model on a brane, can be supersymmetrized by modifications of the 5d supergravity in the bulk. The final theory belongs to a class of 5d gauged supergravities. The requirement of supersymmetry yields relations between bulk and brane cosmological potentials, such that in suitable limits we obtain a supersymmetric version of the Randall–Sundrum scenario or the M-theoretical solution. The general case can still preserve the characteristic features of the Randall–Sundrum model and at the same time includes non-trivial potentials for the hypermultiplets of the bulk theory. We find a vacuum solution which preserves one half of the supercharges. This solution can serve as a background for the compactification to the effective 4d theory with $N = 1$ supersymmetry. Next, we include gauge fields on the brane, and discuss the modification of the brane action and supersymmetry transformation laws, necessary to obtain a supersymmetric theory. In this letter we do not give the four-fermi terms in the bulk.

Finally, we discuss supersymmetry breaking and the role played by the warp factor and conclude the paper with some remarks on the consistency of the compactification to 4d.

We start with a five-dimensional $N = 2$ supergravity on the manifold $M_4 \times S_1 / Z_2$ which includes a gravity multiplet $(e^a_\mu, \psi_R, A_\mu)$ coupled to one hypermultiplet $(x^a, V, \sigma, \xi, \bar{\xi})$ forming a $SU(2, 1)/U(2)$ nonlinear sigma model. Two parallel 3-branes are located at $x^3 = 0$ and $x^3 = \pi \rho$. This particular framework is motivated by the Horava–Witten model compactified to 5d [1,3]. The sigma-model metric can be read from the Kähler potential: $K = -\ln(S + \bar{S} - 2\xi \bar{\xi})$, $S = V + \xi \bar{\xi} + i\sigma$. The conventions and normalizations we use are mainly those of reference [2]. The signature of the metric tensor is $(- + + + +)$. The $SU(2)$ spinor indices are raised with antisymmetric tensor $\epsilon^{AB}$, and the $Sp(1)$ indices (those of hyperino) with $\Omega^{ab}$. We choose $\epsilon^{12} = \epsilon_{12} = \Omega^{21} = \Omega_{21} = 1$. The rule for dealing with symplectic spinors is $\frac{\psi_1}{\psi_2} = \frac{\psi_2}{\psi_1}$ (note that $\frac{\psi^1}{\psi^2} = \frac{\psi^2}{\psi^1}$). The $Z_2$ symmetry acts as reflection $x^3 \rightarrow -x^3$ and is represented in such a way...
that bosonic fields \((e^m_{\mu}, e^5_{\mu}, A_5, V, \sigma)\) are even, and \((e^m_{\mu}, e^5_{\mu}, A_\mu, \xi)\) are odd. The indices \(\alpha, \beta, \ldots\) are five-dimensional \((0, \ldots, 3, 5)\), while 4d indices are denoted by \(\mu, v, \ldots\). The action of the \(Z_2\) on fermion fields and on parameter \(\epsilon\) of supersymmetry transformations is defined as:

\[
\begin{align*}
\gamma S \psi_\mu^A (x^5) &= (\sigma^3)^A B \psi_\mu^B (-x^5), \\
\gamma S \psi_5^A (x^5) &= (\sigma^3)^A B \psi_5^B (-x^5), \\
\gamma S \lambda^\alpha (x^5) &= - (\sigma^3)^\beta \lambda_\beta^A (-x^5), \\
\gamma S e^A (x^5) &= (\sigma^3)^B e^B (-x^5),
\end{align*}
\]

(1)

where \(\gamma = (1, 0, 0, 0, \sigma^3 = (0, 0, 1, 0))\). Symplectic Majorana spinors in 5d satisfy \(\bar{\psi}^A = (C \chi^A)^T\) with \(C = -i \gamma^2 \gamma^0\) in 4d chiral representation. The kinetic part of the action and supersymmetry transformation laws up to 3-fermi terms are:

\[
S = -\int d^5x \frac{1}{\sqrt{2}} \left( R + \frac{3}{2} \mathcal{F}_{\alpha \beta} \mathcal{F}^{\alpha \beta} + \frac{1}{\sqrt{2}} \bar{\psi}^{\alpha \gamma \delta \epsilon} \partial_\alpha \mathcal{F}_{\beta \gamma \delta \epsilon} + \frac{1}{2 \sqrt{2}} \bar{\xi} \partial_\alpha \bar{\xi} + \frac{i}{2 \sqrt{2}} (\bar{\xi} \partial_\alpha \bar{\xi} - \bar{\xi} \partial_\alpha \xi)(V + D_\alpha \sigma D^\alpha \sigma) + \frac{2}{\sqrt{2}} \bar{\xi} \partial_\alpha \xi \right) + \frac{i}{\sqrt{2} V} (\bar{\psi}^{\alpha \gamma \delta \epsilon} \partial_\alpha \mathcal{F}_{\beta \gamma \delta \epsilon} + (1 \to 2)),
\]

(2)

\[
\delta e^m_\alpha = -i \frac{\sqrt{2}}{2} \psi^m_\alpha e^1 + (1 \to 2),
\]

\[
\delta \psi^1_\alpha = D_\alpha e^1 - i \frac{4 \sqrt{2}}{4} (\gamma^1_{\alpha \beta} - 4 \delta^1_{\alpha \beta}) \mathcal{F}_{\alpha \beta} \psi^1_\epsilon + \frac{i}{4 \sqrt{2}} \bar{\xi} \partial_\alpha \xi e^1 - \frac{1}{4 \sqrt{2}} \bar{\xi} \partial_\alpha \xi e^2,
\]

\[
\delta \psi^2_\alpha = D_\alpha e^2 - i \frac{4 \sqrt{2}}{4} (\gamma^2_{\alpha \beta} - 4 \delta^2_{\alpha \beta}) \mathcal{F}_{\alpha \beta} \psi^2_\epsilon + \frac{i}{4 \sqrt{2}} \bar{\xi} \partial_\alpha \xi e^2 - \frac{1}{4 \sqrt{2}} \bar{\xi} \partial_\alpha \xi e^2 + \frac{1}{4 \sqrt{2}} \bar{\xi} \partial_\alpha \xi e^1,
\]

\[
\delta \mathcal{A}_\alpha = -i \frac{\sqrt{2}}{2 \sqrt{2}} \psi^m_\alpha e^1 + (1 \to 2),
\]

\[
\delta V = -i \frac{\sqrt{2}}{2} V (\bar{\xi} \lambda_1^1) - (1 \to 2),
\]

\[
\delta \sigma = +i \frac{1}{\sqrt{2}} V (\bar{\xi} \lambda^1_1) + (1 \to 2) + \frac{1}{\sqrt{2}} \bar{\xi} \lambda^2_2, \\
\delta \xi = -i \frac{\sqrt{2}}{2} (\bar{\psi}^1_\alpha e^1) - \frac{i}{\sqrt{2}} (\bar{\psi}^1_\alpha e^1), \\
\delta \lambda^1_1 = -i \frac{\sqrt{2}}{2 \sqrt{2}} V (\bar{\psi}^1_\alpha e^1) - \frac{i}{\sqrt{2}} (\bar{\psi}^1_\alpha e^1) + i \sqrt{2} \epsilon e^1, \\
\delta \lambda^2_2 = -i \frac{\sqrt{2}}{2 \sqrt{2}} V (\bar{\psi}^1_\alpha e^1) - \frac{i}{\sqrt{2}} (\bar{\psi}^1_\alpha e^1) + i \sqrt{2} \epsilon e^1.
\]

(4)

We assume a scalar potential \(\delta (x^5) \bar{\psi}^m_\alpha (\Lambda + \sqrt{2} \alpha / V)\) localized on, say, the first brane (note the delta function), and study the variation of the brane action under supersymmetry transformations. The motivation for the constant \(\Lambda\) part of this expression is that it will finally lead us to the Randall–Sundrum exponential solutions. At the same time we allow for cosmological potentials for hypermultiplet scalars; the above form is motivated by the M-theory example and is a natural extension in the presence of hypermultiplets. The generalizations are possible, but \(\sigma\)-dependent terms in the potential break the translational \(U(1)\) symmetry \(\sigma \to \sigma + \text{const}\) which is useful when we introduce potential in the bulk, while \(\xi\) cannot appear in the boundary potential because of parity assignments. We will be able to supersymmetrize this action by modification of the bulk action only (thus, our construction is alternative to [15]).

For simplicity, we initially put \(\alpha = 0\) and consider a cosmological term of the form:

\[
\mathcal{L}_B = -\frac{\delta (x^5) \epsilon}{k^2} \Lambda,
\]

(5)

where \(\epsilon\) is 4d determinant built from the metric induced on the brane. We wish to supersymmetrize this term. The supersymmetry variation of \(\mathcal{L}_B\) comes from varying \(\epsilon\):

\[
\delta \mathcal{L}_B = -\frac{\delta (x^5) \epsilon}{k^2} \Lambda,
\]
\[ \delta \mathcal{L} = \frac{1}{2} \epsilon (x^5) e \mathcal{A} (\overline{\psi}_1 \gamma^\mu \epsilon^1 + (1 \to 2)). \]  

We observe that, without further modification of the boundary action, we can cancel this variation by modifying gravitino transformation law:

\[ \delta \psi^1_{\alpha} = \frac{4}{12} \epsilon (x^5) \gamma_\mu \epsilon^1, \]
\[ \delta \psi^2_{\alpha} = -\frac{4}{12} \epsilon (x^5) \gamma_\mu \epsilon^2. \]  

(7)

Note that these corrections are compatible with \( Z_2 \) symmetry defined by (1).

If we vary \( \psi \) in the gravitino kinetic term, the fifth derivative acting on the step function produces an expression multiplied by a delta function, which precisely cancels (6). But now the bulk theory is not supersymmetric. It is straightforward to show that the variations of the gravitino kinetic term resulting from (7) and proportional to \( \Lambda \epsilon (x^5) \) can be cancelled by adding a `gravitino mass term':

\[ \mathcal{L}_{\psi^2} = \frac{\epsilon \Lambda}{8x^2} \Lambda \]  

(8)

The gravitino variation \( \delta \psi_{\alpha}^A = D_{\alpha} \psi^A \) in (8) cancels the above mentioned variation, but now (7) applied to the mass term (8) will produce a variation proportional to \( \Lambda \epsilon (x^5) \), which can be cancelled by varying the determinant in a new `cosmological term':

\[ \mathcal{L}_C = \frac{\epsilon \Lambda}{6x^2} \Lambda^2. \]  

(9)

Moreover, in our framework, \( \epsilon (x^5) \) has another discontinuity at \( x^5 = \pi \rho \) so an additional term multiplied by \( \delta (x^5 - \pi \rho) \) appears in the varied bulk Lagrangian. This variation can be cancelled by adding a cosmological term confined to that brane:

\[ \mathcal{L}_{B'} = \delta (x^5 - \pi \rho) \frac{\epsilon \Lambda}{2x^2}. \]  

(10)

(The minus sign relative to (5) appears because \( \epsilon (x^5) \) has a `step down' at \( x^5 = \pi \rho \).)

Note that the cosmological term appeared with a plus sign. The relevant part of the bulk action now reads \( S = -\frac{1}{2} f (R - \frac{1}{2} \Lambda^2) \) which allows for anti-de-Sitter solutions. In fact, the coefficient of (9) is precisely the one we need to obtain the Randall–Sundrum scenario, as we will show soon.

The above mentioned corrections are still not sufficient to supersymmetrize the bulk Lagrangian. We also need the hyperino mass term:

\[ \mathcal{L}_{\Lambda} = \frac{i \epsilon \Lambda}{4\sqrt{2}x^2} \epsilon (x^5) \mathcal{A} (\overline{\psi}_1 A^1 - (1 \to 2)), \]  

(11)

and the coupling of the graviphoton to gravitino:

\[ \mathcal{L}_A = \frac{i \epsilon \Lambda}{\sqrt{2}x^2} \epsilon (x^5) \times \mathcal{A} (\overline{\psi}_1 \gamma^{\alpha \beta} \psi^1_{\beta}), \]  

(12)

In addition a graviphoton dependent correction to gravitino transformation law appears:

\[ \delta \psi_{\alpha}^A = \frac{i \epsilon \Lambda}{2\sqrt{2}x^2} \epsilon (x^5) \mathcal{A} (\overline{\psi}_1 \gamma^{\alpha \beta} \psi^1_{\beta}), \]  

(13)

Further, we need 4-fermi terms in the bulk action to complete the supersymmetrization, but these are not given in this letter.

The corrections (12), (13) should be understood as parts of the covariant derivatives corresponding to gauging of a \( U(1)_R \) subgroup of the \( SU(2)_R \) atomorphism group of the supersymmetry algebra in 5d. In particular:

\[ D_{\mu} \psi_{\alpha}^A = D_{\mu} \psi_{\alpha}^A + A_{\mu} M_{\beta}^{A} \psi_{\beta}^B, \]
\[ D_{\mu} \psi_{\alpha}^A = D_{\mu} \psi_{\alpha}^A + A_{\mu} M_{\beta}^{A} \psi_{\beta}^B, \]  

(14)

where \( D_{\mu} \) are the usual covariant derivatives, \( A_{\mu} \) is the graviphoton and \( M_{\beta}^{A} = \frac{i (x^5)^{A}}{2\sqrt{2}x^2} (\overline{\sigma}^3)_B \). The matrix \( M_{\beta}^{A} \) determines the charges of \( \psi_{\alpha}^A \) and \( \epsilon^A \) under the \( U(1)_R \) gauge transformations \( A_{\mu} \to A_{\mu} + \partial_{\mu} A_R \), where \( A_R \) is \( Z_2 \) odd.

The presence of the mass term (11) is required because of the mixing between gravitini and hyperinui in the kinetic part of the bulk Lagrangian, \( \mathcal{L}_{\phi} = i \lambda_{\alpha} \gamma^{\alpha \beta} \psi_{\alpha} A_{\beta} \mathcal{V}_{\alpha}^{A} D_{\mu} q^A \). In this expression \( q^A \) denotes scalars from the hypermultiplet, and \( \mathcal{V}_{\alpha}^{A} \) is the vielbein which gives the sigma-model metric for the hypermultiplet \( g_{\mu \nu} = \mathcal{V}_{\alpha}^{A} \mathcal{V}_{\beta}^{B} \epsilon_{AB} \Omega_{\mu \nu} \) and can be easily read off from the kinetic Lagrangian for the bosonic components of the hypermultiplet in (2) (given by \( g_{\alpha \beta} \partial q^\alpha \partial q^\beta \), see [2,4]).

Let us now assume \( A = 0 \) and consider the boundary term:
\[ \mathcal{L} = \delta(x^5) \frac{e^{\sqrt{2} \alpha}}{\kappa^2} e. \]  

The variation of the determinant can be canceled by modifying \( \delta \psi \), similarly to the previous case:

\[ \delta \psi^1_a = -\frac{\sqrt{2} \alpha}{12} V^2 (x^5) \gamma_a \epsilon^1, \]

\[ \delta \psi^2_a = \frac{\sqrt{2} \alpha}{12} V^2 (x^5) \gamma_a \epsilon^2. \]  

We must also vary the hyperplet modulus \( V \) in (15):

\[ \delta V = \frac{V}{\sqrt{2}} \left( e^{\alpha^1} \lambda^1 - e^{2 \lambda^2} \right) \]

\[ \delta \mathcal{L} = -i \delta(x^5) \frac{e^{\alpha}}{V} \left( e^{\alpha^1} \lambda^1 - (1 \rightarrow 2) \right). \]  

This variation can be cancelled by modifying supersymmetry transformation law of the hyperino \( \lambda \):

\[ \delta \lambda^1 = \frac{i}{2 V} \alpha \epsilon(x^5) \epsilon^1, \]

\[ \delta \lambda^2 = \frac{i}{2 V} \alpha \epsilon(x^5) \epsilon^2. \]  

A similar mechanism works: in the variation of the hyperino kinetic term the fifth derivative acts on the step function which leads to a term which precisely cancels (17). Note that it is only the potential \( \alpha/V \) which causes the corrections to the hyperino transformation law. As before, we need to supersymmetrize further. Two-fermi terms and, as a consequence, a cosmological potential is necessary:

\[ \mathcal{L} = i \frac{e_s}{2 V \kappa^2} \alpha \epsilon(x^5) \]

\[ \times \left( -\frac{\sqrt{2}}{4} (\gamma^a \gamma^b \psi^1 \psi^1 - (1 \rightarrow 2)) \right. \]

\[ + \left( \frac{\sqrt{2}}{4} \lambda^1 \lambda^1 - (1 \rightarrow 2) \right) \]

\[ \left. + \frac{3 \sqrt{2}}{4} \lambda^1 \right), \]  

\[ \mathcal{L}_\mathcal{C} = -\frac{e_s \alpha^2}{6 V \kappa^2}. \]  

However, this time a minus sign relative to that of (9) appears, and anti-de-Sitter solution is not allowed. Moreover, contrary to the previous case, 2-fermi and cosmological terms are not enough to render the bulk Lagrangian supersymmetric. Closer inspection shows, that terms of the form \( \alpha(\epsilon \psi) \partial_\sigma \sigma \) do not cancel and the bulk Lagrangian must be supplemented with a coupling \( \alpha \partial_\beta \sigma \mathcal{A}^\beta \). In the context of 5d supergravity this means that the translations of the pseudoscalar \( \sigma \) from the hypermultiplet are gauged, with graviphoton being the gauge field. To recapitulate, after starting with the boundary term (15) we are led to 5d gauged supergravity similar to that studied in [1].

One could also imagine other powers of \( V \) occurring in (15), let us say some function \( f(V) \). But then supersymmetrization is possible only if the bulk sigma model quaternionic metric is found. In some simple cases one can appropriately redefine \( \text{Re}(S) \) and end up in the same sigma model, however in general one has to search for new sigma models with quaternionic kinetic metric that allow gauging, which is beyond the scope of this paper.

Interestingly enough, we can join both schemes discussed in this paper and demand a boundary term:

\[ \mathcal{L}_B = \delta(x^5) \frac{e}{\kappa^2} \left( -\frac{\sqrt{2} \alpha}{V} \right). \]  

As explained we need a similar term on the second brane:

\[ \mathcal{L}_{B'} = -\delta(x^5 - \pi \rho) \frac{e}{\kappa^2} \left( -\frac{\sqrt{2} \alpha}{V} \right). \]  

Repeating the same line of arguments, we arrive at the conclusion, that we need the gauged supergravity in the bulk of the kind considered in [1], but with the potential:

\[ \mathcal{L}_\mathcal{C} = \frac{e_s \alpha^2}{6 V \kappa^2} \left( -\frac{\sqrt{2} \alpha}{V} \right)^2 - \frac{e_s \alpha^2}{2 V \kappa^2}. \]  

Additional terms are needed to arrive at completely supersymmetric bulk action and they all fit into the general form of gauged supergravity with local translations of \( \sigma \).

Since we want to compactify this theory down to 4d and demand that the effective theory has \( N = 1 \) supersymmetry, we must search for the background which preserves exactly four supercharges. The supersymmetry transformation laws of fermions, including modifications found in the previous paragraphs are:

\[ \delta \psi^A_a = D_a \epsilon^A \]

\[ -\epsilon(x^5) \frac{1}{12} \left( -\frac{\sqrt{2} \alpha}{V} \right) \gamma_0 (\sigma^3)^A B \epsilon^B, \]

\[ \delta \lambda^a = \frac{i}{2 \sqrt{2} V} \partial_5 V \gamma^5 (\sigma^3)^A B \epsilon^B. \]
+ \alpha \epsilon(x^5) \frac{i}{2V} e^a. \tag{24}

In the above formulas we neglected terms with 4d derivatives \( \partial_\mu \) so as to preserve 4d Poincaré invariance. We also put \( \sigma = \lambda_5 = 0 \) since these fields do not occur in the potential, so this choice is consistent with equations of motion. Finally, we neglected \( \partial_5 \xi \) term since, as we show later in this letter, expectation value of this term generically leads to supersymmetry breaking.

The ansatz for static solutions is: \( ds^2 = a(x^5)dx^0dx^1dx^2dx^3 + b(x^5)(dx^5)^2 \), \( V = V(x^5) \). The relevant supersymmetry transformation laws evaluated for this ansatz are (‘ denotes \( \partial_5 \) and the world indices are with respect to the Minkowski metric \( \eta \)):

\[
\begin{align*}
\delta \psi^A_\mu &= \frac{a'}{a} \gamma_\mu \gamma^\epsilon e^A \\
&- \epsilon(x^5) \frac{\sqrt{a}}{12} \left(-\Lambda + \frac{\sqrt{2a}}{V}\right) \gamma_\mu (\sigma^3)_B e^B, \\
\delta \psi^A_5 &= \partial_5 e^A - \epsilon(x^5) \frac{\sqrt{b}}{12} \left(-\Lambda + \frac{\sqrt{2a}}{V}\right) \gamma^\epsilon (\sigma^3)_B e^B, \\
\delta \lambda^a &= - \frac{i}{2\sqrt{2b} V} V' (\sigma^3)_B^a \gamma^\epsilon e^B + \alpha \epsilon(x_5) \frac{a}{2V} e^a. \tag{25}
\end{align*}
\]

We obtain conditions for unbroken supersymmetry by demanding that the above variations of fermionic fields are vanishing for vacuum configurations

\[
\begin{align*}
a' &= \frac{1}{3} \left(-\Lambda + \frac{\sqrt{2a}}{V}\right) \epsilon(x^5) \sqrt{b}, \\
V' &= \sqrt{2a} \epsilon(x^5) \sqrt{b}, \\
\partial_5 e^A &= \frac{\sqrt{b}}{12} \left(-\Lambda + \frac{\sqrt{2a}}{V}\right) \epsilon(x^5) e^A. \tag{26}
\end{align*}
\]

In addition we need chirality conditions for the supersymmetry generating spinor, which reduce \( N = 2 \) supersymmetry down to \( N = 1 \):

\[
\gamma^1 \epsilon^1 = \epsilon^1, \quad \gamma^2 \epsilon^2 = - \epsilon^2. \tag{27}
\]

It turns out, that if the parameters \( a, b, \ V \) of our ansatz satisfy conditions (26), they automatically satisfy the equations of motion (with delta sources), and give vanishing vacuum energy. We can easily solve the conditions (26). In the coordinate frame where \( b = R_0^2 \) the vacuum solution is:

\[
V = V_0 + \alpha \sqrt{2} R_0 \left( |x^5| - \frac{\pi \rho}{2} \right),
\]

\[
g_{\mu\nu} = \left( 1 + \alpha \sqrt{2} \frac{R_0}{V_0} \left( |x^5| - \frac{\pi \rho}{2} \right) \right)^{1/3}
\times e^{-\frac{R_0 a}{4|x^5|} \eta_{\mu\nu}},
\]

\[
g_{55} = R_0^2, \tag{28}
\]

where a constant coefficient in the solution for \( g_{\mu\nu} \) has been absorbed into a redefinition of the relation between 5d and 4d Planck scales. With the standard procedure we identify the four-dimensional Planck scale as

\[
M_4^2 = 2M_5^2 R_0 \int_0^{x_5} a(x^5)
= 2M_5^2 R_0 \times \int_0^{x_5} \left( 1 + \alpha \sqrt{2} R_0 \left( |x^5| - \frac{\pi \rho}{2} \right) \right)^{1/3}
\times \exp \left( -\frac{R_0}{3} \Lambda x^5 \right). \tag{29}
\]

In particular, for \( \alpha = 0 \) one obtains \( M_4^2 = \frac{6M_5^2}{\Lambda} (1 - e^{-\frac{R_0 a}{4|x^5|}}) \).

For \( \Lambda \to 0 \) we are back in the domain wall solution studied in [1], while in the case \( \alpha \to 0 \) we get the Randall–Sundrum exponential solution (the connection with the normalization of reference [9] is \( \Lambda = 6k \)). If we assume \( a \rho \) to be small (which is the case in the M-theoretical scenario) we are very close to the Randall–Sundrum solution and, in particular, gravity is still localized on the positive tension brane at \( x^5 = 0 \).

For phenomenological applications we need gauge and charged matter fields transforming in representations of the Standard Model. The problem of coupling confined to a boundary gauge and matter fields to 5d supergravity can be studied along the lines of the original Horava–Witten procedure [6] and details of this can be found in [18]. We summarize the results. Let us add a gauge multiplet \( (A'_\mu, \chi^a) \), say, on the first brane \( (a \text{ is the group index}) \) and set the kinetic function to \( -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \). It turns out that supersymmetric coupling is possible and no changes in the bulk La-
defined as an even combination of 5d supersymmetry
expectation value of the hypermultiplet field. This
mechanism arises because \( \xi \), although odd, couples to
gauginos on the boundaries through its fifth derivative.

\[
\mathcal{L}_{YM} = \frac{4}{g^2} \left( -\frac{V}{4} \frac{g_{\mu\nu}}{2} \frac{F_{a\mu} F_{a\nu}}{4} + \frac{1}{4} \sigma \frac{F_{a\mu}}{2} \frac{F_{a\mu}}{2} \right) - \frac{V}{4} \left( \nabla_{\mu} \chi^a + \frac{4}{7} (\nabla_{\nu} \chi^a)^2 \right) F_{5\mu} - \frac{1}{4} \left( \frac{\nabla_{\nu} \chi^a}{2} \right)^2 F_{5\nu} + \frac{1}{8} \frac{\nabla_{\mu} \chi^a}{2} \frac{\nabla_{\nu} \chi^a}{2} \frac{\nabla_{\sigma} \chi^a}{2} \frac{\nabla_{\tau} \chi^a}{2} F_{5\mu} F_{5\nu} F_{5\sigma} F_{5\tau} + (4 \text{-fermi}) \right). 
\]

(30)

In the above the bulk fermions appear in their even (and Majorana in the 4d sense) combinations defined as:

\[
\psi_\mu = \left( \begin{array}{c} i \psi_{L \mu}^1 \\ i \psi_{L \mu}^2 \end{array} \right), \quad \psi_5 = \left( \begin{array}{c} -i \psi_{R 5}^1 \\ i \psi_{R 5}^2 \end{array} \right), \quad \lambda = \sqrt{2} V \left( -\frac{\lambda_{L}}{\lambda_{R}} \right). 
\]

(31)

We also need to modify the supersymmetry transformation laws of the even bulk fermions:

\[
\delta \psi_\mu = \delta (x^5) \frac{\lambda_2}{\lambda_1} \frac{V}{2} \left( g_{\mu \rho} - \frac{1}{2} g_{\frac{\mu}{\rho}} \right) \chi^a \times \chi^a - \frac{V}{4} \left( \chi^a \chi^a \chi^a \right) \chi^a.
\]

\[
\delta \lambda = \delta (x^5) \frac{\lambda_2}{\lambda_1} \frac{V}{2} \left( \chi^a \chi^a \chi^a \right) \chi^a.
\]

(32)

In the above, the supersymmetry parameter \( \epsilon \) is defined as an even combination of 5d supersymmetry parameters:

\[
\epsilon = \left( \begin{array}{c} i e_{L1}^2 \\ i e_{R1}^2 \end{array} \right). 
\]

(33)

Note that no gaugino dependent correction appears in the transformation law of \( \psi_5 \).

In analogy to heterotic models, we have the possibility to break supersymmetry by gaugino condensation on the hidden and/or visible brane. The supersymmetry breaking is transmitted between branes by the expectation value of the hypermultiplet field \( \xi \). This mechanism arises because \( \xi \), although odd, couples to gauginos on the boundaries through its fifth derivative.

The equation of motion for \( \xi \) in the presence of the condensates is:

\[
\frac{1}{\kappa^2} \partial_5 \left( \frac{e_s g_{55}}{V} \partial_5 \xi \right) = \partial_5 \left( -\frac{e_s g_{55} \sqrt{V}}{2g^2 e_5^2} (\delta (x^5) (\mathcal{X}_{L \chi} 1) + \chi^a \xi) \right). 
\]

(34)

We are interested in the solution for \( \partial_5 \xi \) because this expression (and not \( \xi \) alone) appears in the relevant formulae. For \( \alpha = 0 \) the solution is:

\[
\partial_5 \xi = -\frac{\kappa^2}{2g^2} \sqrt{V_0} \partial_{3/2} \left( \frac{\delta(x^5) \chi^2 + \delta(x^5 - \pi \rho) \chi^2}{2} \right) + C \exp \left( \frac{2R_0}{3} \Lambda |y| \right). 
\]

(35)

The nontrivial background effects are due to the exponential factors. The constant \( C \) can be determined from the boundary conditions (in other words, from matching delta singularities in the equation of motion):

\[
C = R_0 A \frac{\kappa^2}{2g^2} \frac{\partial_{3/2} \sqrt{V_0}}{1} \left( \chi_1^2 + \chi_2^2 \right). 
\]

(36)

For \( \Lambda = 0 \) it is customary [1] to go to a different coordinate frame where \( g_{55} = R_2 H^4 \). \( \chi_{\mu \nu} = \frac{1}{R_3} H \tilde{g}_{\mu \nu}, V = V_0 H^3 \) and \( H = 1 + \frac{4 \sqrt{2} \pi \rho R_3 ((x^5)^{3/2} - \frac{\pi}{g^2})} {3} \). In this frame we obtain the solution:

\[
\partial^5 \xi H^{-3} = \frac{\kappa^2}{2g^2} \sqrt{V_0} \partial_{3/2} H^{3/2} \left( \chi_1^2 + \chi_2^2 \right) + C, \quad (37)
\]

\[
C = \frac{\kappa^2}{3g^2} \sqrt{\frac{2}{\pi \rho}} \sqrt{V_0} \partial_{3/2} \left( \chi_1^2 \chi_2^2 \right) + C, \quad (38)
\]

It is worth noting, that in the 5d theory gaugino condensates break supersymmetry (but, if we assume superpotentials on the branes we can cancel their contribution). In the presence of the condensates we have no way to satisfy simultaneously \( \delta \psi_{\mu}^A = 0 \) and neither of the remaining conditions for unbroken supersymmetry. Indeed, \( \partial_5 \xi \) and condensates do not alter the transformation law of \( \psi_{\mu} \), so in particular, the conditions resulting from \( \delta \psi_{\mu}^A = 0 \) include chirality conditions (27). But then, the condensates in \( \delta \lambda^a \) and
\( \delta \psi^A \) multiply the supersymmetry parameter \( \epsilon \), which is of the chirality opposite to other \( \epsilon \)'s occurring in these transformation laws. Thus, conditions \( \delta \psi^A = 0 \) and \( \delta \lambda^a = 0 \) cannot be satisfied.

When we compactify our model to 4d on the background (28), the independent of \( x^5 \) integration constants \( R_0, V_0 \) together with zero modes of \( \sigma \) and \( A_5 \), become the \( \chi^\mu \) dependent moduli of the effective 4d theory. The fluctuations around Minkowski metric in the solution (28) are described by \( \tilde{g}_{\mu \nu} \). To go to the 4d Einstein frame one needs to perform explicit integration over \( x^5 \) and a suitable moduli dependent Weyl rotation. We rescale the metric \( \tilde{g}_{\mu \nu} \rightarrow a_0 \tilde{g}_{\mu \nu} \) with \( a_0 \) chosen (up to a numerical, independent of moduli, factor which can be absorbed into the definition of the 4d gravitational constant) as \( a_0^{-1} = f_0^{2 \pi / \alpha} d x^5 a(x^5) \). The Killing spinors generating an unbroken \( N = 1 \) supersymmetry are:

\[
\begin{align*}
\epsilon^R_k &= e^{-\frac{\lambda A_0 a_0^3}{2}} \left( 1 + \alpha \sqrt{2} \frac{R_0}{V_0} \left| x^5 \right| - \frac{\pi \rho}{2} \right)^{1/12} \times a_0^{-1/4} \eta_R, \\
\epsilon^L_\ell &= e^{-\frac{\lambda A_0 a_0^3}{2}} \left( 1 + \alpha \sqrt{2} \frac{R_0}{V_0} \left| x^5 \right| - \frac{\pi \rho}{2} \right)^{1/12} \times a_0^{-1/4} \eta_L.
\end{align*}
\]  

(39)

Since \( \eta^R = -i \sigma^2 \epsilon^R \) (5d Majorana condition) spinor \( \eta \) is Majorana in the 4d sense. The factor \( a_0 \) in (39) yields canonical form of the reduced 4d supersymmetry transformation law of the gravity multiplet. \( \eta \) depends only on \( x^\mu \) and has an interpretation of a parameter of supersymmetry transformations in the 4d theory.

When \( \Lambda \neq 0 \) it is pretty difficult to compactify the supersymmetric model which we have defined to four dimensions and even more to bring it into the standard 4d supergravity form. Hence we postpone the discussion of this case for a while and discuss first in detail the simpler case with \( \Lambda = 0 \) and \( \alpha \neq 0 \). We can now proceed with the derivation of the 4d effective theory. In that case the Kähler potential is [1]:

\[
K = - \ln(S + \bar{S}) - 3 \ln(T + \bar{T}).
\]  

(40)

The moduli \( S, T \) and their superpartners are defined as:

\[
S = V_0 + i \sigma_0,
\]

\[
T = R_0 + i \sqrt{2} \Lambda_5,
\]

\[
A^S = \left( \frac{H}{V_0} \right)^{1/4} \left( \lambda - \alpha \sqrt{2} \left| x^5 \right| - \pi \rho \right) \psi_S,
\]

\[
T = R_0 + i \sqrt{2} \Lambda_5,
\]

\[
A^T = \left( \frac{H}{V_0} \right)^{1/4} \psi_T,
\]  

(41)

where \( \langle \cdots \rangle = \frac{1}{2 \pi \rho} \int d x^5 \cdots \). The gauge sectors originating from two different branes are described by the gauge functions, which includes corrections linear in \( \alpha \):

\[
\begin{align*}
f_1 &= S - \sqrt{2} \alpha \pi \rho T, \\
f_2 &= S + \sqrt{2} \alpha \pi \rho T.
\end{align*}
\]  

(42)

The physical gauginos can be expressed as \( (\chi)_R = (\chi_1)(H(0)/R_0)^{3/4}, (\chi_2)_R = (\chi_2)(H(\pi \rho)/R_0)^{3/4} \).

The above corrections were extracted from the kinetic terms of the 5d Lagrangian compactified to 4d. Although the functions \( K \) and \( f \) are sufficient to reconstruct the rest of the supergravity Lagrangian, an interesting consistency check would be to obtain explicitly the complete 4d Lagrangian by integrating out the fifth dimension. This is fairly difficult as, e.g., the 4-fermi terms have higher order in \( \alpha \) contributions.

Another approach is to reduce 5d supersymmetry transformation laws to 4d, and check if they are consistent with the results (40), (42). This has the advantage that corrections can be seen at lower order in the expansion in \( \alpha \) and \( \kappa^2 \). As an example we present how to determine the gauge kinetic functions from the transformation laws of moduli superpartners.

We use the definition of \( S \) and \( T \) superpartners (41) and substitute \( \delta \xi \) with the solution of its equation of motion in the relevant part of 5d supersymmetry transformation law of \( \lambda \) and \( \psi_S \). After integrating over fifth dimension the result up to \( \alpha^2 \) corrections is:

\[
\begin{align*}
\delta A^S_L &= \frac{\kappa^2}{2 \alpha^2} V_0^2 (\chi^2_1 + \chi^2_2) \eta_L, \\
\delta A^T_L &= -\frac{\kappa^2}{12 \alpha^2} \left( \frac{H}{V_0} \right)^{1/4} R_0^2 \alpha \sqrt{2} \pi \rho (\chi^2_1 - \chi^2_2) \eta_L.
\end{align*}
\]  

(43)

Noting that in 4d supergravity, scalar gaugino condensates in the transformation law of the fermions \( \Lambda^5 \), \( \Lambda^T \) are multiplied by \( \frac{1}{2} f_S (K^{-1})^5_S \) and \( \frac{1}{2} f_T (K^{-1})^T_T \), respectively, the result indeed agrees with (42). A noteworthy detail in this derivation is that in 5d \( \delta \xi \) appears as a full square: \( \delta \xi = \delta \xi + \frac{\kappa^2}{2 \alpha} \bar{\delta} (x^5) V_0^{1/2} \).
\[ \times ((\mathcal{R} \mathcal{R} L)_1 + (\mathcal{R} \mathcal{R} L)_2) \text{ in } \delta \lambda \text{ but not in } \delta \psi_S. \]

Thus, when we calculate \( \delta A^T \) the linear part of the solution for \( \delta g \xi \) cancels to zeroth order in \( \alpha \) with delta functions occurring in this solution, leading to the correct form of \( f_T \). Note also, that the admixture of \( \psi_S \) in the definition of \( A^5 \) is crucial to obtain the correct form of \( f_S \). These conclusions confirm fully the results of [3,4,8].

From the transformation laws (43) it can be read off that presence of gaugino condensates breaks supersymmetry also in the 4d effective theory. Although one can adjust \( \chi^2_1 = -\chi^2_2 \) so that the condensates cancel in the regular part of the solution (37) for \( \delta g \xi \) and in consequence lacking of the ‘full square’ structure of \( \delta \psi_S \). However, if we allow for boundary scalar fields, by appropriate adjusting of their superpotentials we have the possibility to cancel the contribution of the condensates.

To be precise, we note that the above considerations for the case \( \Lambda = 0 \) are valid only to linear order in \( \alpha \). In \( \alpha \) order, with \( n > 1 \), further corrections appear, but they are difficult to calculate. One needs to solve the equations of motion for \( K K \) modes of the bulk fields (to linear order in \( \alpha \) it suffices to know the expectation value of the bulk fields on the branes).

While neglecting the higher order corrections in \( \alpha \) can be justified by the expected smallness the expansion parameter \( \alpha \pi \rho \), this is not the case for \( \alpha \pi \rho \), since \( \Lambda \) is expected to be set by the string scale. Thus, finding the effective theory with a nonzero \( \Lambda \), even for the case \( \alpha = 0 \), requires more elaborate tools. However, although we do not know the complete effective Lagrangian, we can still try to extract some information about the low energy 4d theory by computing physically important 4d operators, and using experience gained in the study of the simpler model. Let us put \( \alpha = 0 \) in what follows. We know already that the parameter controlling the breakdown of low energy supersymmetry is the regular part of \( \delta g \xi \) given in formula (37). This equals in the present case

\[ \delta g \xi (x^5) = \frac{1}{e^{\frac{1}{2}R_0 \Lambda |x|^5} - 1} \frac{\kappa^2 R_0 \Lambda}{6g^2} V_0^{1/2} \left( \chi_1^2 + \chi_2^2 \right) \]

We note that in the presence of gaugino condensates the vacuum expectation value of \( \delta g \xi \) is nonzero, and is modulated by an exponential, \( x^5 \)-dependent factor. The above expression can be inserted into the bulk kinetic term of \( \delta g \xi \) (the singular part of the solution drops out due to the ‘full square’ structure), to read off physical gaugino masses on each wall. These are the important parameters as they can tell us directly the physical magnitudes of induced global supersymmetry breaking terms in the boundary gauge sectors. In the limiting case \( \Lambda \pi \rho \ll 1 \), to the lowest order (and keeping \( \alpha = 0 \)), we recover this way soft gaugino masses which we have obtained in the compactification of the pure M-theoretical model. To perform the task in a general case, we need to expand metric around the vacuum solution. At the same time, to obtain canonical normalization of kinetic terms of fields living on the branes we need to rescale them by exponential factors. This also gives the standard form of the supersymmetry transformation law of the gauge field \( A_i \) reduced in our background, \( \delta A_{\mu} = \partial_{[\mu} \chi_{\nu]} \).

The needed rescaling is \( \chi_1 = a_i^{1/3} (\chi_1)_p \), i.e., \( (\chi_1)_5 = a_0^{1/3} (\chi_1)_p \) and \( (\chi_2)_5 = a_0^{-3/4} e^{\frac{1}{4}R_0 \Lambda \pi \rho} (\chi_2)_p \). If we interpret quartic gaugino terms as leading to gaugino masses after condensation the result for the masses is (in the limit \( \Lambda \pi \rho \gg 1 \)):

\[ M_1 = \frac{V_0 \kappa^2}{2g^4} e^{\frac{1}{2}R_0 \Lambda \pi \rho} \left( (\chi_1^2)_p + (\chi_2^2)_p \right) e^{\frac{1}{2}R_0 \Lambda \pi \rho}, \]

\[ M_2 = \frac{V_0 \kappa^2}{2g^4} e^{\frac{1}{2}R_0 \Lambda \pi \rho} \left( (\chi_1^2)_p + (\chi_2^2)_p \right) e^{\frac{1}{4}R_0 \Lambda \pi \rho}. \]

For definiteness of the discussion, let us consider one after another two possible visible-hidden sector configurations. First we put the visible sector on the negative tension brane at \( x^5 = \pi \rho \) and switch on the hidden condensate on the positive tension brane. First of all, we see that \( M_2 \approx e^{-\frac{1}{6}R_0 \Lambda \pi \rho} (\chi_1^2)_p \) becomes exponentially suppressed as a function of the distance between walls, and it scales exactly as expected on the basis of the argument given in [10]. If we now revert the roles of the branes, and assume that the positive tension

\[ 2 \text{ It is useful to express the 5d gravitational coupling } \kappa^2 \text{ through the 4d coupling } \kappa^2_4 \text{. In the two cases of interest the relation is a) } \kappa^2_4 = \kappa^2 / (2 \pi \rho) \text{ when } \Lambda \pi \rho \ll 1 \text{ and b) } \kappa^2_4 = \kappa^2 / 6 \text{ when } \Lambda \pi \rho \gg 1. \]
braner contains observable fields, the situation is similar, i.e., $M_i \approx e^{-\frac{1}{2} R_0 \lambda_{1i}} (\lambda_i^2)$ vanishes with growing distance between the walls. The thing to be noted in this case, which can be called standard hidden sector scenario, is the presence of the additional exponential factor in front of the usual dynamically generated mass scale $a^{1/2}(\pi\rho A_{1i})^3 M_{1i}^2$, which is the (not necessarily welcome) source of additional hierarchy. In conclusion we stress that in the limit of large warp factors the transmission of supersymmetry breaking from the hidden brane is exponentially suppressed. Thus, in this scenario walls indeed decouple with the growing distance between them.

A different role of the warp factor can be seen when one considers a condensate forming on the same wall where the observable sector lives. If this happens on the negative tension wall the induced gaugino mass seems to be exponentially enhanced, like $e^{-\frac{1}{2} R_0 \lambda_{1i}}$. On the other hand the visible brane is the positive tension wall and the presence of the additional exponential factor in front of the usual dynamically generated mass scale, is the presence of the additional exponential factor in front of the usual dynamically generated mass scale $a^{1/2}(\pi\rho A_{1i})^3 M_{1i}^2$, which is the (not necessarily welcome) source of additional hierarchy. In conclusion we stress that in the limit of large warp factors the transmission of supersymmetry breaking from the hidden brane is exponentially suppressed. Thus, in this scenario walls indeed decouple with the growing distance between them.

A different role of the warp factor can be seen when one considers a condensate forming on the same wall where the observable sector lives. If this happens on the negative tension wall the induced gaugino mass seems to be exponentially enhanced, like $e^{-\frac{1}{2} R_0 \lambda_{1i}}$. On the other hand the visible brane is the positive tension one, the supersymmetry breaking mass is suppressed by the factor $e^{-\frac{1}{2} R_0 \lambda_{1i}}$. One obvious comment on this is that the walls are not equivalent, in the sense of being interchangeable, as was already the case in the nonsupersymmetric Randall–Sundrum scenario. Second, let us note that expecting large, say well above $1 \text{ TeV}^3$, condensates on the negative tension brane where all physical scales are scaled down to say $1 \text{ TeV}$ may be inconsistent in the present framework.

At this point we should consider matter fields on the boundaries, interacting by means of a trilinear superpotential $W$. New terms in the Lagrangian which should be taken into account are

$$S_{\text{scalar}} = \int d^5 x \frac{e^4}{g^2} \bar{\psi}(x^5)$$

$$\times \left( -D_{\mu} \Phi D^{\mu} \Phi - \frac{2}{V} \frac{\partial W}{\partial \Phi} \frac{\partial \bar{W}}{\partial \Phi} \right)$$

$$- \frac{4g^2}{V} \bar{W} W + \frac{2}{V e^2} W \partial_5 \xi + \text{h.c.} \right)$$ (45)

(same for the second wall). The results of the coupling between $W$ and $\partial_5 \xi$ are twofold. First, in the previous formulae for the vacuum solution for $\partial_5 \xi$ one should substitute

$$\chi; \bar{\psi}; \rightarrow \chi; \bar{\psi}; - \frac{4W_i}{V^{3/2}}.$$ (46)

This means that expectation value of $W$ contributes to the supersymmetry breaking, and can in principle cancel the contribution of condensates. Let us note, that the canonical normalization of kinetic terms of scalars $\Phi_i$ living on the $i$th wall leads to rescaling $\Phi_1 \rightarrow \Phi_1 a_0^{-1/2}$, $\Phi_2 \rightarrow \Phi_2 a_0^{-1/2} e^{\frac{1}{2} R_0 \Delta \pi_{12}}$. This implies that superpotential from the $i$th wall scales like a condensate from the same wall, hence the earlier discussion of decoupling applies here without modifications.

The second result of new couplings is the appearance of softly breaking global supersymmetry trilinear scalar terms when condensates are switched on. These terms, say on the second brane, are proportional to

$$a_0^{1/2} \frac{1}{V e_5^2} W_2 \frac{1}{V^{3/2}} (\chi^0_i \chi^0_i - \frac{1}{4} W_1 \rho)$$ (47)

and one can easily work out their scaling properties. These terms are the physical soft terms assuming that the effective 4d vacuum energy, after switching on vevs for boundary scalars, vanishes.

After presenting this preliminary and somewhat speculative interpretation of the supersymmetry breaking pattern, we want to stress that definite conclusions can be made only when one constructs the complete effective 4d theory.

To put this discussion into a wider framework, let us remind ourselves that what we have done here so far is the traditional compactification of the fifth dimension, where one tries to combine both, in principle different, gauge sectors at the ends of the five-dimensional world into a single effective theory. However, a different approach is possible, see [19]. One can imagine that the model living on the negative tension brane located at $x^5 = \pi \rho$ is a holographic image of the same model living on the Planck brane located at $x^5 = 0$. Flow of the second brane along the $x^5$ axis accompanied by rescaling of all the mass scales on that brane by a factor $a^{1/2}(x^5)$ might be considered to be equivalent to renormalization group flow along momentum scale towards the IR limit. In this context we want to notice, that in the $N = 1$ theory which we study here the mass scales scale exactly in the way required by holographic principle, but the gauge coupling does not scale with the changing warp factor. This is easy to see, since the warp factor cancels out from the expression $e^{\frac{1}{2} \Delta_{12} g_{\mu\nu} g^\rho_{\mu} F_{\mu\nu}}$. The intriguing observation is that if one would try to improve for that, and scale also the gauge coupling according to one-loop scaling anomaly, see [20],
\[
\frac{1}{g^2(x^5)} = \frac{1}{g^2(M_5)} + b_0 \log \left( \frac{M_5}{m(x^5)} \right)^2 \\
= \frac{1}{g^2(M_5)} - b_0 \log a(x^5),
\]
then assuming \( \frac{1}{g^2(x^5)} = \frac{1}{g_{\text{GUT}}(x^3)} \), this flow would compensate the relative enhancement factor between gaugino condensate from the second and first wall,
\[
A_{\text{cond}}(\pi\rho) = M_5 e^{-\frac{1}{\alpha_0} \left( \frac{1}{\pi} - b_0 \log a(\pi\rho) \right)}
\approx a^{1/2}(\pi\rho) A_{\text{cond}}(0).
\]

At the end we would like to comment on two aspects of the models we discuss in this paper. Firstly, it is interesting to note that the additional terms which we have put on the boundary, \( \delta L = e f(V) \), are not parts of a globally supersymmetric sigma model living on a brane. When one integrates over the fifth dimension these terms cancel against the bulk potential and drop out completely from the effective four-dimensional model. Hence, the presence of the additional dimension offers the possibility of supersymmetrizing certain boundary terms along the direction transverse to the branes, with partner terms living in the bulk. Secondly, the vanishing vacuum energy in the pure bulk moduli sector which we observe does not solve automatically the cosmological constant problem. When we allow matter chiral superfields on the branes to follow their local dynamics given by nontrivial superpotential and gauge interactions, the new vacuum approach is not guaranteed to give automatically a vanishing contribution to the 4d vacuum energy, and in general next instance of tuning is necessary.

To summarize, we have presented a class of five-dimensional supergravities with gauge sectors living on 4d boundaries, which admit exponential warp factors analogous to that of the Randall--Sundrum model. The required fine-tuning between bulk and boundary cosmological potentials was explained by supersymmetry. These models can be considered to be deformations of the M-theoretical model constructed in [1]. We have discussed hidden sector supersymmetry breaking and its transmission between branes in the present models. The setup and results are likely to be relevant for the discussion of the holographic projection of \( N = 1 \) supersymmetric gauge models.

Acknowledgements

This work has been supported by TMR programs ERBFMRX--CT96--0045 and CT96--0090. Z.L. and S.P. are supported by the Polish Committee for Scientific Research grant 2 P03B 05216(99-2000) and by M. Curie-Skłodowska Foundation grant MEN/DOE-96-279.

References

Inflation and supersymmetry breaking

Wilfried Buchmüller, Laura Covi *, David Delépine

Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

Received 16 June 2000; received in revised form 24 August 2000; accepted 6 September 2000

Editor: P.V. Landshoff

Abstract

We study the connection between inflation and supersymmetry breaking in the context of an O’Raifeartaigh model which can account for both hybrid inflation and a true vacuum where supersymmetry is spontaneously broken. For a weakly coupled inflaton field, the dynamics during the inflationary phase can be determined by the supersymmetry breaking scale $\mathcal{M}_S \sim 10^{10} \text{ GeV}$, even if $H_I \gg m_{3/2}$. The spectrum of density fluctuations is then almost scale invariant, with a spectral index $n - 1 = \mathcal{O}(\mathcal{M}_G^2 / \mathcal{M}_P^2)$. The mass parameter $\mathcal{M}_G$ of the O’Raifeartaigh model is determined by the COBE normalization for the cosmic microwave background to be the grand unification scale, $\mathcal{M}_G \sim 10^{16} \text{ GeV}$. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

It is well known that an inflationary phase in the early history of the universe can explain its present flatness, isotropy and homogeneity [1]. From the COBE measurement of the cosmic microwave background (CMB) anisotropy it has soon been realized that the scale of inflation has to be lower than the Planck scale, but much larger than the electroweak scale. It is therefore clear that supersymmetry may play an important role for inflation. Many models have been proposed describing an inflationary phase in the context of globally supersymmetric theories [2]. But since supersymmetry is not exact in nature, we know that globally supersymmetric models can give us only an approximate description of the real world. Supergavity corrections can strongly affect the inflationary phase [3], and a variety of models have been constructed in a general supergravity framework [2]. However, it is often thought that, when the scale of inflation is much larger than the supersymmetry breaking scale, a globally supersymmetric model is sufficiently accurate to describe the inflationary phase as well as the reheating process.

As we shall see, this is not the case. The goal of this paper is to study explicitly a model containing both, supersymmetry breaking in the true vacuum and during the inflationary phase. On the one hand, in such a case the supersymmetry breaking sector is influenced by the inflaton dynamics, and the true vacuum is reached only at the end of inflation. On the other hand, also the inflationary potential is modified by the presence of the supersymmetry breaking sector. This modification turns out to be very important in the case of a weakly coupled inflaton field.

* Corresponding author.
E-mail address: covil@mail.desy.de (L. Covi).

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.
PJE: S0370-2693(00)01005-4
In the following we first describe the model, which combines a Fayet term [5] for global symmetry breaking with a Polonyi term [6] for supersymmetry breaking, leading to a particular O’Raifeartaigh model [7]. We then analyze the model without and with supersymmetry breaking effective potential where the constant and the linear term dominate in an expansion in powers of $S$

Combining the two superpotentials (1) and (2) we arrive at
\[ W = W_G + W_S = \lambda T (M_G^2 - \Sigma^2) + M_S^2 (\beta + S). \] (4)

Further, we choose the canonical Kähler potential for the fields $T$, $\Sigma$ and $S$. Note, that the superpotential (4) is a particular O’Raifeartaigh model. This becomes apparent after a change of variables. Defining
\[
\Phi = \frac{\xi S}{\sqrt{1 + \xi^2}} + \frac{T}{\sqrt{1 + \xi^2}}, \\
\Psi = \frac{S}{\sqrt{1 + \xi^2}} - \frac{\xi T}{\sqrt{1 + \xi^2}},
\] (5)
with $\xi = M_S^2/(\lambda M_G^2)$, and

\[ \lambda_1 = \frac{\lambda}{\sqrt{1 + \xi^2}}, \quad \lambda_2 = \frac{\lambda^2}{\sqrt{1 + \xi^2}}, \]

one obtains
\[ W = \lambda_1 \Phi (M^2 - \Sigma^2) + \lambda_2 \Psi \Sigma^2 + M_S^2 \beta. \] (7)

This is the more familiar form of an O’Raifeartaigh model [7]. The two superpotentials (4) and (7) are equivalent, and in the following we will use one or the other according to our convenience. As we shall see, successful inflation requires $\xi$ to be very small, so that effectively $T \simeq \Phi$, $S \simeq \Psi$ and $M \simeq M_G$.

3. Hybrid inflation

For global supersymmetry the scalar potential reads
\[ V_G + V_S = \lambda^2 |M_G^2 - \Sigma^2|^2 + 4\lambda^2 |T|^2 |\Sigma|^2 + M_S^4, \] (8)
and the corresponding ground state is given by
\[ \langle T \rangle = 0, \quad \langle \Sigma \rangle = M_G. \] (9)

while $\langle S \rangle$ is undetermined. Hence, the supersymmetry breaking sector decouples, and an inflationary phase can take place as in ordinary hybrid inflation, starting
with a large value of $T$. The field $\Sigma$ is then pushed to the origin by a large mass term and the potential is perfectly flat along $T$. A small curvature needed for the "slow roll" is generated by the quantum corrections due to the loops of the $\Sigma$ particles [9], which are non-vanishing since supersymmetry is broken by $F_T = \partial W_G / \partial T \neq 0$. The corresponding one-loop correction to the scalar potential reads,

$$\Delta V_G = \frac{\lambda^4 M_G^4}{8 \pi^2} \left( \ln \left( 2 \lambda^2 \phi^2 / \mu^2 \right) + O(M_G^4 / \phi^4) \right), \quad (10)$$

where $\phi$ is the real part of the complex scalar field $T$ and $\mu$ is a renormalization scale.

Inflation ends at $\phi_\ell \simeq M_G$, where the mass of $\Sigma$ becomes negative and the field acquires a non-vanishing expectation value. For $M_S \ll \lambda_{1/2} M_G$, the potential (10) satisfies the slow-roll conditions [2] down to $\phi_c$,

$$\epsilon = \frac{M_G^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{\lambda^4}{32 \pi^4} \frac{M_G^2}{\phi^2} \ll 1, \quad (11)$$

$$\eta = \frac{M_G^2}{V} \frac{V''}{V} = -\frac{\lambda^2}{4 \pi^2} \frac{M_G^2}{\phi^2}, \quad |\eta| \ll 1, \quad (12)$$

as long as $\lambda$ is of order $M_G / M_p$.

The number of e-folds between the inflaton field value $\phi$ and the end of inflation at $\phi_c$ is given by

$$N(\phi) = \left[ \int_{t(\phi)}^{t(\phi_c)} H_I dt \right] \frac{\phi}{\sqrt{V'}} = \frac{2 \pi^2 \phi^2 - \phi^2_c}{\lambda^2 M_p^2}, \quad (13)$$

where $t$ denotes time and $H_I \simeq \sqrt{V_G / (3 M_G^2)} \simeq \lambda M_G^2 / (\sqrt{3} M_p)$ is the slow-roll approximation.

Adiabatic density perturbations originate as vacuum fluctuations during inflation. The COBE normalization [14] then gives

$$\delta_H \equiv \frac{1}{\sqrt{75 \pi M_p^2} |\nu_4|} \left[ \frac{V_G}{3} \right]^{3/2} = \frac{4 \pi}{\sqrt{75 \lambda M_p^2}} \phi^*_c = 1.94 \times 10^{-5}, \quad (14)$$

where the * indicates that the potential and its derivative are evaluated at the epoch of horizon exit for the comoving scale $k_\ast \simeq 10 H_0$ [14]. Defining

$$N_* \text{ as the number of e-folds at that epoch, the corresponding inflaton value is given by } \phi_* / M_p \simeq \sqrt[1/2]{\frac{\lambda^2 N_* / (2 \pi^2)}{\phi_c^2 / M_p^2}}. \quad \text{We thus obtain the relation}$$

$$\frac{M_G^2}{M_p^2} \left( N_* + \frac{2 \pi^2 \phi_c^2}{\lambda^2 M_p^2} \right)^{1/2} \simeq 5.9 \times 10^{-5}, \quad (15)$$

where the number of e-folds is $N_* \simeq 50$.

From Eq. (15) one reads off that a consistent picture is obtained for $M_G / M_p \sim \lambda \sim 10^{-3}$, with $H_I \sim 10^{-8} M_p$. The exact relation between $\lambda$ and $M_G$ imposed by the COBE constraint is shown in Fig. 1. It is interesting that $M_G$ is naturally of order the grand unification scale. Moreover, due to the smallness of $\lambda$, the observable number of e-folds corresponds to inflaton fields $\phi$ close to the critical value $\phi_c \simeq M_G \ll M_p$.

**4. Supergravity corrections**

Let us now consider the effect of the supersymmetry breaking Polonyi potential and of corrections suppressed by powers of $1 / M_p$. The supergravity scalar potential reads

$$V = e^{K / M_p^2} \left[ \frac{\partial W}{\partial \phi} + \frac{z^*_W W}{M_p^2} \right] - \frac{3 |W|^2}{M_p^2}, \quad (16)$$
where the sum extends over all fields $z_i$, and $K$ is chosen to be the canonical Kähler potential, $K = \sum_i |z_i|^2$.

The additional non-renormalizable terms modify slightly the vacuum expectation values of the fields $T$ and $\Sigma$ and give a large expectation value to the Polonyi field $S$. Since the corrections to the derivatives of the superpotential are always proportional to $(W)/M_p^2 \ll 1$, it is possible to expand the potential in powers of $M_S/M_p$. This yields for the first corrections to the vacuum expectation values,

$$\langle S \rangle = M_G \left[ 1 + \frac{2 - \sqrt{3}}{4} \frac{M_S^2}{M_p^2} + O \left( \frac{M_S^4}{M_p^4} \right) \right],$$

(17)

$$\langle T \rangle = \frac{1}{2\lambda} \frac{M_p^2}{M_S} + O \left( \frac{M_S^4}{M_p^4} \right).$$

(18)

Also the value of $\beta$, which is adjusted to have vanishing cosmological constant, and the vacuum expectation value of $S$ acquire corrections,

$$\langle S \rangle = (\sqrt{3} - 1) M_p - \frac{9 - 4\sqrt{3}}{24} \frac{M_S^2}{\lambda^2 M_p^2},$$

(19)

$$\beta = (2 - \sqrt{3}) M_p - \frac{\sqrt{3} M_G^2}{6 M_p}.$$  

(20)

Clearly, all corrections $O(1/M_p^4)$ to the vacuum expectation values are very small. One may therefore be tempted to think that also during the inflationary expectation values are very small. One may therefore be tempted to think that also during the inflationary phase supergravity corrections are practically negligible. This, however, is not the case.

During the inflationary phase $\Sigma$ is driven to zero by the large value of $T$. The potential is then most easily computed in the basis $\Phi, \Psi$. Neglecting the one-loop correction, one has

$$V = \lambda^2 M_G^2 \left[ 1 + \xi^2 - \frac{2\sqrt{2} \xi \beta \bar{\psi} \bar{\eta}}{M_p^2} \frac{\xi^2 \beta^2 (\bar{\psi}^2 + \bar{\eta}^2)}{M_p^2} \right]$$

$$- \frac{\xi \bar{\psi} \bar{\eta}^3}{\sqrt{2} M_p^4} \frac{\sqrt{2} \xi \beta \bar{\psi} \bar{\eta} \bar{x}^2}{M_p^4} + \frac{(\bar{\psi}^2 + \bar{\eta}^2)^2}{8 M_p^2}$$

$$+ \frac{|\Psi|^4}{M_p^2} \left[ 1 + \frac{\bar{\psi}^2 + \bar{\eta}^2}{2 M_p^2} \right] + \cdots,$$  

(21)

where $\xi = M_S^2/(\lambda M_G^2) \ll 1$ and $\Phi = (\psi + i \chi)/\sqrt{2}$. Here we have neglected terms which are small for values of $\Phi$ in the range $1 > |\Phi| > \xi$. Note, that the potential for $\Phi = 0$ is just the Polonyi potential, but with the “wrong” constant $\xi \beta$, i.e., while the supersymmetry breaking scale during inflation is given by $\lambda^{1/2} M_G$, the constant is still related to the supersymmetry breaking scale $M_S$ in the true vacuum.

The hierarchy between the two scales is exactly what makes the potential flat enough, contrary to the simple expectation for the Polonyi potential with only one scale of supersymmetry breaking. This hierarchy also implies that in the potential (21) the term linear in $\psi$ is larger than the $\psi$ mass term, which is suppressed by an additional power of $\xi$.

We remark that in the case of a charged inflaton field, or in general when the superpotential contains only second and higher powers of the inflaton field, no linear term is generated by the supersymmetry breaking sector. Then the first supergravity correction is a mass term and inflation can be realized even with $H \approx m_3/2$, if the inflaton mass is sufficiently suppressed either by cancelations or by the running mass mechanism [15].

The minimum of the potential (21) with respect to $\Psi$ and $\chi$ lies at the origin, but it is very flat. However, for initial values $\varphi = O(M_p)$ one can have $m_\varphi, m_\chi > H$, which may be sufficient to drive $\Psi$ and $\chi$ to the origin before the beginning of inflation. In the following we shall assume $\Psi \simeq \chi \approx 0$ as initial conditions.

Comparing (21) with (10) it is clear that the standard hybrid inflation scenario may be significantly modified depending on the values of $\lambda$ and $M_G$. The one-loop radiative corrections dominate over the linear term in (21) for $\lambda^2/(2\pi)^2 \gg \xi \beta \varphi/(\sqrt{2} M_p^2)$. Substituting $\beta/M_p = 2 - \sqrt{3}$ and using $\varphi > \psi_c \gg M_G$, one obtains the lower bound on $\lambda$,

$$\lambda > 3 \left( \frac{M_S^2}{M_G M_p} \right)^{1/3}.$$  

(22)

For the hybrid inflation value $M_G \simeq 3 \times 10^{-3} M_p$, this yields $\lambda > 0.7 \times 10^{-4}$. As discussed above, hybrid inflation takes place in the vicinity of $\psi_c$.

For couplings $\lambda$ above the lower bound the one-loop radiative corrections also dominate over the supergravity induced quartic term in (21). Hence, for $M_G \simeq 3 \times 10^{-3} M_p$ and couplings in the range

$$0.7 \times 10^{-4} < \lambda < 6 \times 10^{-3}$$  

(23)

the standard hybrid inflation scenario is only weakly affected by supergravity corrections.
5. Scale invariant inflation

Consider now the case of small couplings,
\[ \lambda < 3 \left( \frac{M_S^2}{M_G M_P} \right)^{1/3}, \tag{24} \]
for which the linear term in (21) dominates over the one-loop radiative corrections (cf. Fig. 1). From the COBE normalization (14) one then obtains, independently of \( N_* \),
\[ H_D \left( \frac{V_{\phi}}{V_0} \right) = \left( \frac{2}{2\sqrt{2\pi} \xi^2 \beta M_P} \right)^{1/2} \approx 1.9 \times 10^{-5}. \tag{25} \]
Fixing \( M_S \approx 1.4 \times 10^{10} \) GeV, this implies \( \xi = M_S^2/(\lambda M_G^2) \approx 5 \times 10^{-7} \). Note, that \( \xi \) is the ratio of the gravitino masses in the true vacuum and in the inflationary phase. Since \( \xi \ll 1 \), a huge number of e-folds is generated near \( M_G \),
\[ N(\phi) = -\frac{M_G}{2\sqrt{2\xi\beta}} \varphi - \varphi_c, \tag{26} \]
which include the cosmologically relevant scales with \( N \approx 50 \). The linear term dominates over the quartic term in (21) if \( |\varphi|^3/(2M_P^2) < 2\sqrt{2\xi\beta} \). Together with (25), this yields an upper bound on \( M_G \). Similarly, a lower bound on \( M_G \) follows from (24). Inserting numerical values for \( \xi \) and \( M_S \) one finds that the linear term dominates in the range
\[ 2 \times 10^{15} \text{ GeV} < M_G < 2 \times 10^{16} \text{ GeV}. \tag{27} \]

The slow-roll conditions are clearly satisfied for \( \varphi \approx \varphi_c \),
\[ \epsilon = \frac{4\xi^2 \beta^2}{M_P^2} \left( 1 - \frac{\varphi^3}{2\sqrt{2\xi\beta} M_P^2} + \cdots \right) \ll 1, \tag{28} \]
\[ \eta = \frac{3}{2} \frac{\varphi^2}{M_P^2} + \cdots \ll 1. \tag{29} \]

Here we have kept the quartic supergravity correction to the linear term in (21), which affects the spectral index,
\[ n - 1 \approx 3 \frac{\varphi^2}{M_P^2} \leq 2.4 \times 10^{-4}. \tag{30} \]

An inflationary phase dominated by a linear term is very interesting, since it gives a scale invariant spectrum to high accuracy. For standard hybrid inflation, on the contrary, one has \( n \approx 0.98 \) [9]. Future satellite experiments may eventually be able to distinguish between these two versions of hybrid inflation.

The linear term in the potential breaks the symmetry \( \varphi \rightarrow -\varphi \) (cf. Fig. 2). For negative \( \varphi \) hybrid inflation can take place, as discussed above. The potential has a Polonyi-type minimum at \( \varphi_{\text{min}} \approx (4\sqrt{2}\xi\beta/M_P)^{1/3} \approx 0.9 \times 10^{-3} M_P \). For positive initial condition hybrid inflation can take place as long as \( M_G > \varphi_{\text{min}} \). Otherwise, the inflaton field is trapped at \( \varphi_{\text{min}} \). Inflation then continues in this metastable state and has to terminate in a different way.

Let us finally turn to the case where the quartic term dominates during inflation, a possibility already considered in [16]. This occurs for \( M_G > 2 \times 10^{16} \) GeV. The slow-roll conditions,
\[ \epsilon = \frac{\varphi^6}{4 M_P^6} \left( 1 - \frac{2\sqrt{2\xi\beta} M_P^2}{\varphi^3} + \cdots \right) \ll 1, \tag{31} \]
\[ \eta = \frac{3}{2} \frac{\varphi^2}{M_p^2} + \cdots \ll 1, \]  
(32)

are satisfied for field values small compared to \( M_p \). The number of e-folds is given by

\[ N(\varphi) = \int_{\varphi_c}^{\varphi} \frac{2M_p^2}{\varphi^3} \frac{M_p^2}{\varphi_c^3} \frac{M_p^2}{\varphi^2} = \frac{M_p^2}{\varphi_c^3} \frac{M_p^2}{\varphi^2}. \]

(33)

For \( \varphi_c < 10^{-3} M_p \) the cosmologically relevant scales again correspond to \( \varphi_c \simeq \varphi_c \).

The COBE normalization determines \( \lambda \) as function of \( M_G \) (cf. Fig. 1), with \( \lambda > 3 \times 10^{-6} \). For the spectral index one obtains

\[ n - 1 \simeq 3 \frac{\varphi_c^2}{M_p^2} \geq 2.4 \times 10^{-4}. \]

(34)

The three regimes of hybrid inflation, the loop regime, the linear regime and the quartic regime, are summarized in Fig. 1. The COBE normalization defines a curve in the \( \lambda - M_G \) plane. The dashed lines are obtained by assuming that a single term dominates the derivative of the supergravity potential. The full line is based on the full potential. Increasing (decreasing) the scale of supersymmetry breaking \( M_S \) shifts the curve in the linear regime, as well as the boundaries, to larger (smaller) values of \( \lambda \). The inflaton potentials in the three regimes are compared in Fig. 2. Note, that the flattest potential corresponds to the linear regime.

6. Moduli problem and reheating

At the end of the inflationary period the field \( \Sigma \) has to change from 0 to \( M_G \), the field \( T \) from \( M_G \) to \( M_T^2/(2\lambda M_P) \) and the field \( S \) from 0 to \((\sqrt{3} - 1)M_P \). As in the usual Polonyi model \( S \) acquires a small mass \( m_S \sim m_3/2 \) in the true vacuum, like the standard model fields which have only gravitational interactions with \( S \). The late decays of \( S \) are then incompatible with nucleosynthesis, which is the so-called cosmological moduli problem [17].

Several ways have been proposed to circumvent the moduli problem. For instance, it does not occur if the amplitude of the moduli field is reduced via an effective mass term during the evolution to the true vacuum [18]. This can be implemented in the present model by adding the following non-renormalizable term to the superpotential,

\[ W_M = \frac{\alpha}{M_p} \Sigma^2 T^2. \]

(35)

This interaction is negligible during inflation, where \( S \ll M_p \), and modifies only slightly the expectation values in the true vacuum.

However, at the end of inflation, \( T \sim M_G \) and \( S \) acquires the mass \( m_S = 2\alpha M_G^2/M_P \). The amplitude of the \( S \) field oscillations is then sufficiently damped for \( m_S \gg H_1 = \lambda M_G^2/(\sqrt{3} M_P) \) [18]. This is the case for \( \alpha \gg \lambda \), which can be easily satisfied.

The interaction (35) also induces a large mass for the \( T \) field, \( m_T \sim \alpha M_P \gg \lambda M_G \), when \( S \) approaches its minimum. \( T \) then decays rapidly to other particles. The computation of the corresponding reheating temperature is not straightforward, since it depends on the dynamics of the field \( S \). A detailed analysis of this process, including the thermal and non-thermal production of gravitinos, is in progress. A rough estimate, providing a lower bound on the reheating temperature, can be obtained by considering the decay of \( \Sigma \) into quarks. Supergravity always induces the non-renormalizable couplings

\[ L = Y_q Q H q \frac{\Sigma^2}{M_p^2}. \]

(36)

where \( Y_q \) is the quark Yukawa coupling, \( Q \) (\( q \)) the quark doublet (singlet) and \( H \) the corresponding Higgs doublet. From the top-quark contribution alone, one obtains a reheating temperature \( T_R \sim 10^6 \) GeV [1]. Clearly, this simple picture may be strongly modified by additional interactions.

Finally, let us comment on the possible production of topological defects at the end of inflation. The potential (4) has a \( Z_2 \) symmetry with respect to the field \( \Sigma \), that would give rise to domain walls at the end of inflation [19]. In order to avoid them, it is sufficient, either to consider higher order non-renormalizable terms breaking the \( Z_2 \) symmetry or, like in many formulations of hybrid inflation, give to the \( \Sigma \) field a charge under a gauge group and substitute \( \Sigma^2 \) by \( \Sigma \Sigma \) or \( \text{Tr}(\Sigma^2) \), depending on the gauge group representation. In the last case, at the end of inflation other topological defects could arise, e.g., strings, and could give a non-negligible contribution to the density perturbations [20].
7. Conclusions

We have studied in detail the connection between inflation and supersymmetry breaking in the context of an O’Raifeartaigh model, which can account for both a hybrid inflationary phase and a true vacuum where supersymmetry is spontaneously broken. Crucial ingredients of the model are two contributions to the superpotential: a term linear in the inflaton field and a constant which is required by the nearly vanishing cosmological constant in the true vacuum. This constant generates a linear and higher order terms in the inflaton field. This does not spoil the flatness of the inflaton potential since the energy scale during inflation turns out to be large compared to the scale of supersymmetry breaking. For the same reason the linear term in the inflaton potential dominates over the mass term.

The dynamics during the inflationary phase depends on the size of the Yukawa coupling in the O’Raifeartaigh model. For \( \lambda > 10^{-4} \), the usual picture of hybrid inflation driven by loop corrections applies. However, for smaller coupling \( \lambda < 10^{-4} \), and \( M_G < 2 \times 10^{16} \) GeV, the linear term in the effective potential dominates the evolution of the inflaton field. As a consequence, the spectrum of fluctuations is almost scale invariant. The deviation of the spectral index from one is determined by the mass parameter \( M_G \) of the O’Raifeartaigh model, \( n - 1 = \mathcal{O}(M_G^2/M_P^2) \). It is remarkable that for \( M_S \sim 10^{10} \) GeV the COBE normalization for the cosmic microwave background determines \( M_G \) to be the unification scale, \( M_G \sim 10^{16} \) GeV.

Acknowledgement

We would like to thank D.H. Lyth for clarifying comments.

References

Entropy generation and inflation in wave collision induced pre-big-bang cosmology

A. Feinstein a,*, K.E. Kunze b, M.A. Vázquez-Mozo c,d

a Departamento de Física Teórica, Universidad del País Vasco, Apdo. 644, 48080 Bilbao, Spain
b Département de Physique Théorique, Université de Genève, 24 Quai Ernest Ansermet, 1211 Genève 4, Switzerland
c Instituut voor Theoretische Fysica, Universiteit van Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands
d Spinoza Instituut, Universiteit Utrecht, Leuvenlaan 4, 3584 Utrecht, The Netherlands

Received 14 April 2000; received in revised form 27 June 2000; accepted 24 August 2000

Abstract

We study inflation and entropy generation in a recently proposed pre-big-bang model universe produced in a collision of gravitational and dilaton waves. It is shown that enough inflation occurs provided the incoming waves are sufficiently weak. We also find that entropy in this model is dynamically generated as the result of the nonlinear interaction of the incoming waves, before the universe enters the phase of dilaton driven inflation. In particular, we give the scaling of the entropy produced in the collision in terms of the focusing lengths of the incoming waves.

1. Introduction

Pre-big-bang cosmology (PBB) [1] has evolved in recent years into one of the main trends in string cosmology. In spite of its many nice features there is a current discussion in the literature as to what extent the resolution of the flatness and homogeneity problems within the PBB picture requires a certain amount of fine tuning [2].

In a recent paper [3] we have proposed a cosmological scenario in which the universe starts in a trivial state characterised by a bath of plane gravitational and dilatonic waves which, upon collision, generate PBB bubble universes. These bubbles act as possible seeds for a Friedman–Robertson–Walker (FRW) universe in the spirit of PBB cosmology. The proposed scenario [3] should be seen as a realisation of the picture of Bounanno, Damour and Veneziano (BDV) [4].

The aim of this letter is to study some relevant phenomenological implications of the wave collision induced PBB cosmology. To this end we will be focusing our attention on two issues: the conditions for successful inflation and the production of entropy in the scenario.

Though it was shown in [3] that there exists a dense region in parameter space, for which the PBB inflation takes place, it is important to clarify further as to whether such favourable initial conditions would lead to sufficient inflation. In particular, we will see that inflationary requirements leading to consider large black holes as the seeds of PBB bubbles in the original BDV picture here imply the weakness of the incoming waves. Therefore, these waves can be regarded as emerging from perturbations of flat spacetime. With
respect to entropy production, unlike in the original BDV scenario where the origin of entropy remains rather vague, we will see that entropy in our model is generated dynamically as a result of the wave collision.

In the wave collision induced model universe, one starts in the remote past with non-interacting plane gravi-dilatonic waves. It was argued in [3] that such a state should be identified as one with a minimum entropy, for both gravitational and matter sectors. This assumption is based on the following reasons:

(i) Simplicity: plane waves have no nonvanishing curvature invariants, and thus are, apart from Minkowski spacetime, the simplest geometries one may think of.

(ii) Symmetry requirements: with the advent of the string theory and its alphabetic generalisations (M-theory and F-theory) our view on the initial state of the universe has changed considerably. In particular one is led to think that the primordial universe should be described by some exact string background which at later time evolves into a FRW-like universe. In this light, Penrose’s proposal [5] of assigning zero gravitational entropy to backgrounds with vanishing Weyl curvature (a particular case of which is the FRW metric) seems to be too restrictive. This is especially evident in the context of PBB cosmology where the universe originates in a highly perturbative state and only evolves into a FRW phase after a graceful exit phase. Plane gravitational waves, being exact string backgrounds [6], are ideal candidates to represent this PBB primordial state [7].

(iii) Absence of particle creation: the fact that in the vicinity of plane waves there is no quantum creation of particles enforces the idea that plane waves are zero gravitational entropy states. Even using Penrose’s original suggestion [5,8] where the “number of gravitons” contained in the gravitational field could be adopted as a measure of entropy, plane waves are likely to be associated with minimum gravitational entropy.

We now briefly summarise some features of the model universe described in [3] (the reader is referred for details to the original paper). The universe starts in a distant past with weak strictly plane waves propagating on a flat background. Eventually these waves collide breaking the plane symmetry in the interaction region (Fig. 1). Just after the collision takes place at some \( t_i < 0 \), physics is dominated by the superposition of two noninteracting null fluids so the geometry is described by a Doroshkevich–Zeldovich–Novikov (DZN) line element [9]. This phase is followed by an intermediate region where the nonlinearities of gravity become important and the evolution then becomes nonadiabatic. Finally, as we approach the caustic singularity the universe enters a Kasner phase (for \( t_K < t < 0 \) ) where matter is effectively described in terms of a stiff fluid and the evolution is adiabatic. In this region the geometry is given by a generalised Kasner line element whose exponents depend on one spatial coordinate alone. In Einstein frame this metric can be written as

\[
\begin{align*}
\frac{ds^2}{dt} & = -dt^2 + \left( \frac{-t}{-t_K} \right)^{2a_1(z)} dx^2 + \left( \frac{-t}{-t_K} \right)^{2a_2(z)} dy^2 + \left( \frac{-t}{-t_K} \right)^{2a_3(z)} dz^2,
\end{align*}
\]

where we have normalized the scale factors to their values at the beginning of the Kasner epoch, \( t = t_K \).

The generalised Kasner exponents \( a_i(z) \) are determined by the initial data on the null boundaries of the interaction region. In terms of the gravitational and scalar source functions \( \epsilon(z) \) and \( \phi(z) \) given, in turn, by quite complicated line integrals of incoming data [3], the Kasner exponents can be written as

\[
\begin{align*}
a_1(z) & = \frac{1 + \epsilon(z)}{a(z) + 2}, \\
a_2(z) & = \frac{1 - \epsilon(z)}{a(z) + 2}.
\end{align*}
\]
\[ \alpha_3(z) = \frac{a(z)}{a(z) + 2}, \tag{2} \]

where \( a(z) \equiv \frac{1}{2} [\epsilon(z)^2 + \varphi(z)^2 - 1] \). They satisfy the following relations
\[ \sum_{i=1}^{3} \alpha_i(z) = 1, \quad \sum_{i=1}^{3} \alpha_i(z)^2 = 1 - \frac{2\varphi(z)^2}{[a(z) + 2]^2}. \tag{3} \]

On the other hand the dilaton field near the singularity behaves like
\[ \phi(t, z) = \beta(z) \log \left( \frac{-t}{-t_K} \right) + \phi_0 \tag{4} \]
with
\[ \beta(z) = \frac{2\varphi(z)}{a(z) + 2} \tag{5} \]
and \( \phi_0 \) being some constant zero mode that determines the string coupling constant at the beginning of dilaton driven inflation (DDI), \( g_s = e^{2\phi_0} \). Since we have normalized the metric and the dilaton at the beginning of DDI the natural scale in Einstein frame in this regime will be the Planck scale at \( t_K \), \( \ell_{Pl} = e^{\phi_0} \ell_{st} \). We will assume in the following that the dilaton field does not vary very much from the initial boundaries to the onset of the Kasner regime, so \( \phi_0 \sim \phi(t \to -\infty) \). In this case asymptotic past triviality implies that \( \phi_0 \ll -1 \).

2. Successful inflation

In [3] it was shown that there exists a dense set of points in the \( \epsilon(z) - \varphi(z) \) plane for which the models undergo PBB inflation in string frame. These inflationary models correspond in Einstein frame to a contracting universe where all Kasner exponents \( \alpha_i(z) \) are positive.

The question however remains of whether successful inflation, i.e., enough number of e-foldings, can be achieved in the collision induced PBB scenario and if so how the amount of inflation relates with the initial conditions in the collision. In this section we will analyse how the requirement on the number of e-foldings translates into the strengths of the incoming waves. Since the Kasner exponents are analytically related to the initial conditions, we can perform a fully general analysis of the PBB inflationary era by just looking at the Kasner regime. Different particular solutions in the interaction region will be encoded into different source functions \( \epsilon(z) \) and \( \varphi(z) \), which in turn will translate into different set of Kasner exponents through Eq. (2).

In the picture of colliding waves one starts with the collision of two gravi-dilatonic waves with some focal lengths \( L_1 \) and \( L_2 \). As a result of their interaction, the waves will focus each other to form a caustic singularity in the interaction region after a time \( |t_1| \sim \sqrt{L_1L_2} \) has elapsed, as measured in the Einstein frame [10]. Physically speaking, the focusing lengths of the incoming waves give the measure of their strength; long focal lengths correspond to weak waves and thus it takes very long time till a singularity forms, whereas strong waves are characterized by short focal lengths.

The amount of inflation occurring during some time period in the string frame in each direction can be expressed as the ratio of the initial \((i = l_1)\) and final \((i = l_2)\) commoving Hubble radii given by the three functions
\[ Z_i = \frac{\bar{a}_i(l_2)\bar{H}_i(l_2)}{\bar{a}_i(l_1)\bar{H}_i(l_1)} \quad (i = 1, 2, 3), \tag{6} \]
where the tilde indicates quantities in the string frame. Evaluating explicitly \( Z_i \) and using Einstein frame variables in the Kasner regime where both the expressions of \( a_i(t) \) and \( \phi(t) \) are given by Eqs. (1) and (4), we find
\[ Z_i = \left( \frac{-t_2}{-t_1} \right)^{\alpha_i^{-1}}. \tag{7} \]
Notice that the largest \( Z_i \) corresponds (for \( t_1 < t_2 \)) to the direction with the smallest Einstein frame Kasner exponent.

In order to analyse the total amount of inflation during DDI we still need to determine when the inflationary regime comes to an end. We consider that DDI begins at \( t_l = t_K \) when the universe enters the Kasner regime. On the other hand, to determine the moment at which inflation is over we should look at the instant of time when the low energy approximation breaks down. Thus, we will take \( t_f = t_f \) as the time at which either curvature reaches the string scale or the effective string coupling becomes of order 1.

Let us work out the first possibility. By imposing \( \bar{H}_i^2(l) \sim \ell_{st}^{-2} \) at least one of the three directions, we find for \( t_2 \) the following expression in the Einstein frame
\[ |t_f|_{\alpha'} \sim |p_{\min}|^{\frac{2}{\beta+2}} \left( \frac{\ell_{st}}{-t_K} \right)^{\frac{2}{\beta+2}} t_K^{\frac{2}{\beta+2}} \]

where \( p_{\min} \) and \( \alpha_{\min} \) are the smallest Kasner exponents in string and Einstein frame, respectively.

The reason for this is that in the inflationary region \( 2/(\beta + 2) > 1 \) [3], therefore we conclude that in order to define \( t_f \), we should take the largest \(|p_j|\). Since in this region \( p_i \) are negative, the direction with the largest value of \(|p_i|\) corresponds to the one with the smallest Kasner exponent (in both the Einstein and the string frame).

Yet a different constraint on \( t_f \) can be obtained by looking at the time when the string theory coupling becomes of order 1. Given the form of the dilaton field in the Kasner regime (4) we have that the effective string coupling is given by

\[ g_{\text{eff}} = \left( -\frac{t}{t_K} \right)^{\frac{\beta}{2}} e^{\frac{1}{2} \phi_0}. \quad (9) \]

In particular, it can be seen that, since \( \beta < 0 \) for the solutions leading to PBB inflation, the effective coupling blows up when \( t \sim 0 \). If we take \( t_f \) as the time of the onset of strong coupling effects \((g_{\text{eff}} \sim 1)\) where our approximation breaks down, we find

\[ |t_f|_{\alpha'} \sim e^{-\frac{\phi_0}{2} t_K}. \]

By looking at Eqs. (8) and (9) we see that long periods of inflation \((t_{\text{inf}} = |t_K - t_f|)\) can be achieved by either taking \(|t_K| \gg \ell_{st}\) or \(e^{\phi_0} \ll 1\). The first case will correspond, generically, to the case of long focusing times (i.e., very weak waves), whereas the second condition is just equivalent to asymptotic past triviality [4].

To quantify the amount of inflation we can now proceed to evaluate \( Z_i \) using the definition (6) to find (cf. [11])

\[ Z_i = \min \left[ |p_{\min}|^{\frac{2(\alpha_i - 1)}{\beta + 2}} \left( -\frac{t_K}{\ell_{st}} \right)^{\frac{2(1 - \alpha_i)}{\beta + 2}} e^{\frac{1}{2} \phi_0} \right], \quad (10) \]

and in order to solve the flatness/horizon problem we have to require

\[ Z_i \gtrsim e^{60}. \quad (11) \]

Now, the condition for getting a sufficiently large value for \( Z_i \) translates into a condition on two parameters, namely \( t_K \) (which depends on the initial focal lengths \( L_1, L_2 \)) and the zero mode of the dilaton \( \phi_0 \).

In any case, it is possible to find which kind of corrections (\( \alpha' \) or \( g_{\alpha} \)) will end inflation by looking at Eqs. (8) and (9). From there we can get a critical value of \( t_K \) such that when

\[ t_K > t_{\text{crit}} \equiv -|p_{\min}| e^{\frac{\phi_0}{2}}. \]

DDI will be terminated by the onset of \( \alpha' \) corrections. On the other hand if \( t_K < t_{\text{crit}} \) string loop corrections will become relevant first and will signal the end of the inflationary period. Notice that the requirement of asymptotic past triviality, together with the fact that for inflationary solutions \( \beta < 0 \), implies for generic models that \(|t_{\text{crit}}| \gg \ell_{\text{st}} \gg \ell_{\text{pl}}\) so the crossover between the two regimes occurs well inside the region of applicability of the low energy approximation.

We can now translate the condition (11) into constraints on the parameters of the initial waves (their focal lengths \( L_1 \) and \( L_2 \)) and the zero-mode of the dilaton field \( \phi_0 \). In the situation when the duration of inflation is determined by the excitations of massive string states \((t_K > t_{\text{crit}})\) \( Z_i \) is given by the first entry on the right-hand side of Eq. (10). Thus, the requirement (11) implies for a model with reasonably small deviations from isotropy, i.e., \(|p_{\min}|^{\frac{2(\alpha - 1)}{\beta + 2}} \sim 1 , \)

\[ t_i < t_K < -\frac{60(\beta + 2)}{\alpha_{\min}} \ell_{st}. \]

Since \(|t_i| \sim \sqrt{L_1 L_2}\) we have a bound on the strength of the incoming gravi-dilaton waves

\[ L_1 L_2 \gtrsim e^{\frac{60(\beta + 2)}{1 - \alpha_{\min}} \ell_{st}^2}. \quad (12) \]

Therefore, assuming for simplicity that both incoming waves have similar focal lengths, we arrive at the conclusion that successful inflation will be achieved for extremely weak waves. In the cases with \( t_K < t_{\text{crit}} \) the end of inflation is triggered by string loops corrections we get a bound on the zero mode of the dilaton, namely

\[ \phi_0 \lesssim \frac{60\beta}{1 - \alpha_{\min}} \sim -10^2 \quad (13) \]

which corresponds to a very weakly coupled string theory at the beginning of DDI. Actually, this bound
of the dilaton field leads to the same constraint on the focal lengths of the incoming waves found in Eq. (12). This can be realized by combining Eq. (13) with the condition that whenever inflation is terminated by string loop corrections $|f_K| > |c_{\text{crit}}|$. It is remarkable that, independently of the physical mechanism responsible for the end of the inflationary regime, one obtains the same bound on the focal lengths of the colliding waves.

Consequently, we conclude that successful inflation occurs for very weak incoming waves and/or very weak initial string coupling. The condition of having extremely weak initial waves is very satisfactory, since these may be considered as emerging from fluctuations in flat spacetime. Besides, since the focusing time for these waves will be very long we can argue that, on general grounds, for sufficiently weak waves $t_K < t_{\text{crit}}$ and inflation therefore will be terminated by the onset of string loop corrections. If we compare with the original BDV proposal we see that our condition on the strength of the incoming waves corresponds to their condition of starting with a large black hole. It is worth noticing, however, that in the wave collision induced version of the BDV scenario the “fine tuning” of the size of the black hole is automatically achieved if the incoming waves are viewed as small fluctuations on an otherwise trivial background.

Incidentally, we find as a bonus that particle production in the interaction region for weak incoming waves is negligible [12,13]. Thus, as in [4], emerging homogeneity is not expected to be spoiled by quantum corrections.

### 3. Entropy production

A different phenomenological issue which will be addressed here is the entropy production. We start with the assumption that before the collision the waves take place, both matter and gravitational entropy are zero [3]. As the result of the collision, entropy will be generated from its zero value close to the null boundaries to a nontrivial entropy content as the universe approaches the singularity. Since the rate at which gravitational entropy grows is difficult to estimate we will focus our attention on the production of matter entropy.

As explained in introduction, the interaction region of the two incoming waves can be divided into three regions (Fig. 1). Just after the collision takes place at $t_i$ the matter content of the universe can be satisfactorily described in terms of a superposition of the two incoming non-interacting null fluids and the metric is given by a DZN line element. In this regime the evolution is very approximately adiabatic and almost no entropy is produced. This almost linear regime comes to an end as soon as the gravitational nonlinearities take over and we enter an intermediate phase where the dynamics of the universe is dominated by both velocities and spatial gradients. Now the evolution is no longer adiabatic and matter entropy is generated. In this regime the matter content of the universe cannot be described in terms of a perfect fluid equation of state and some effective macroscopic description of the fluid, such as an anisotropic fluid or some other phenomenological stress-energy tensor, should be invoked [14].

The production of entropy will stop at the moment the velocities begin to dominate over spatial gradients and the evolution becomes adiabatic again, the matter now being described by a perfect fluid with stiff equation of state $p = \rho$. The transition to an adiabatic phase will happen either before or at $t_K$ when the universe enters the Kasner phase. From that moment on no further entropy is produced up to the end of DDI. The relative duration of these three regimes crucially depends on the strength of the incoming waves and on the initial data. It is straightforward to show that for both gravitational and scalar waves the ratio of spatial gradients versus time derivatives dies off as $1/L_1/L_2$ and therefore it takes the universe a time of order $\sqrt{L_1L_2}$ before the Kasner-like regime is reached, fixing the duration of the nonadiabatic phase.

As discussed above, all the entropy is generated in the intermediate phase between the adiabatic DZN and Kasner regimes. On the other hand, in the Kasner regime the total entropy generated in this intermediate region is carried adiabatically by the perfect stiff fluid represented by the dilaton field $\phi(t)$. If we consider a generic bariotropic equation of state, $p = (y - 1)\rho$, the first law of thermodynamics implies that the energy density $\rho$ and entropy density $s \equiv S/V$ evolve with the temperature $T$ as

$$\rho = \sigma T^{1/y - 1},$$

(14)
\[ s = \gamma \sigma T^{-\frac{1}{2}}, \quad (15) \]

where the temperature is given by the usual relation \( T^{-1} = (\partial s / \partial \rho)_T \). Here \( \sigma \) is a dimensionful constant which in the particular case of radiation (\( \gamma = 4/3 \)) is just the Stefan–Boltzmann constant. Using Eqs. (14) and (15) we easily find that for a stiff perfect fluid (\( \gamma = 2 \)) the entropy density \( s \) can be expressed in terms of the energy density as

\[ s = 2\sqrt{\sigma \rho}. \quad (16) \]

It is important to stress here that \( \sigma \) is a parameter that gives the entropy content of, and thus the number of degrees of freedom that we associate with, the effective perfect fluid. In principle it could be computed provided a microscopical description of the fluid is available. However in the case at hand, the stiff perfect fluid is just an effective description of the classical dilaton field in the Kasner regime. Since the evolution in the DDI phase is adiabatic, \( \sigma \) can be seen as a phenomenological parameter that measures the amount of entropy generated during the intermediate region where the dilaton field should be described by an imperfect effective fluid. This effective description of the dilaton condensate, and the entropy generated, would then depend on a number of phenomenological parameters.

We can now give an expression for the total entropy inside a Hubble volume at the beginning of DDI in terms of the initial data. The energy density carried by the dilaton field in the Kasner epoch is given by

\[ \rho(t) = \frac{\phi^2}{4\ell^2_{\text{Pl}}} = \frac{\beta^2}{4\ell^2_{\text{Pl}} t^2}, \]

where \( \beta \) is the source function for the dilaton defined in Eq. (5). From here we get the entropy density by applying Eq. (14). Thus, the total entropy inside a Hubble volume at the beginning of the Kasner regime can be written in terms of the Kasner exponents and the source function for the dilaton as

\[ S_H(t_K) = \frac{t_K^2}{\ell^2_{\text{Pl}}} \sqrt{\sigma \ell^2_{\text{Pl}}} |\beta| |\alpha_1\alpha_2\alpha_3|. \]

Now we can write \( t_K = \eta \sqrt{L_1 L_2} \) with \( 0 < \eta \lesssim 1 \), so finally we arrive at

\[ S_H(t_K) = \left[ \frac{\eta^2}{2} \sqrt{\sigma \ell^2_{\text{Pl}}} \frac{|\beta|}{\alpha_1\alpha_2\alpha_3} \right] \frac{L_1 L_2}{\ell^2_{\text{Pl}}} \equiv \kappa \frac{L_1 L_2}{\ell^2_{\text{Pl}}}. \]

It is interesting to notice that this scaling for the entropy in the Hubble volume at the beginning of DDI can be also retrieved by considering a particular example when, as a result of the wave collision, a spacetime locally isometric to that of a Schwarzschild black hole is produced in the interaction region. In that case the focal lengths of the incoming waves are related with the mass of the black hole by \( M = \sqrt{L_1 L_2}/\ell^2_{\text{Pl}} \) (see for example [12]). Now we can write the Bekenstein–Hawking entropy of the black hole \( S = 4\pi \ell^2_{\text{Pl}} M^2 \) in terms of the focal lengths of the incoming waves as \( S = 4\pi L_1 L_2/\ell^2_{\text{Pl}} \). This analogy is further supported by the fact that the temperature of the created quantum particles in both the black hole, and the colliding wave spacetime scales like \( T \sim 1/M \) and \( T \sim 1/\sqrt{L_1 L_2} \), respectively, [12,13], as well as by the similarities between the thermodynamics of black holes and stiff fluids [15]. It is important to notice that this scaling of the temperature of the created particles with the focal lengths implies that, whenever enough inflation occurs, the contribution of these particles to the total entropy is negligible.

One may have thought, in principle, that it is possible to avoid the entropy production before the DDI by just taking a solution in the interaction region for which the evolution is globally adiabatic. The simplest possibility for such a solution would be a Bianchi I metric for which the Kasner regime extends all the way back to the null boundaries. This, however, should be immediately discarded due to the constraints posed by the boundary conditions in the colliding wave problem [16]. Or, put in terms of the null data, it is not possible to choose the initial data on the null boundaries in such a way that the metric is globally of Bianchi I type in the whole interaction region and at the same time \( c^1 \) and piecewise \( c^2 \) across the boundary.

Before closing this section we briefly discuss whether the entropy produced before the DDI phase complies with different cosmological entropy bounds. To study Bekenstein’s entropy bound [17] it is convenient to define the function [18] \( b = c = 1 \)

\[ f \equiv \frac{1}{2\pi} \frac{s}{\rho R} \lesssim 1, \quad (17) \]

where \( R \), the effective radius of the system, can be defined following Refs. [18,19] as the maximal extension of the particle horizon in the three spatial
directions. If we concentrate our attention on the region near the end of inflation $t \sim t_f$ where we are in the Kasner regime it is easy to see that the particle horizon is largest in the direction with the smallest Kasner exponent $\alpha_{\text{min}}$. Then, the function $f^{(P)}(t)$ scales with time as (the superscript indicates that we are using the particle horizon to define the size of the system)

$$f^{(P)}(t) \sim \left( \frac{-t}{-t_K} \right)^{1-\alpha_{\text{min}}}. \quad (18)$$

This is of course expressed in the Einstein frame. As it turns out, evaluating Bekenstein bound in the string frame just comes down to substituting Einstein frame quantities by string frame quantities in the above equation.

Looking at Eq. (18) we see that, since $1 - \alpha_{\text{min}} > 0$ the function $f^{(P)}(t)$ goes to zero as $t$ approaches the singularity. Actually, the value of this function at the end of DDI scales with the number of $e$-foldings in the direction of $\alpha_{\text{min}}$ as $f^{(P)}(t_f) \sim 2^{1/\alpha_{\text{min}}}$. Therefore Bekenstein bound is well satisfied near the curvature singularity. On the other hand, if we were to evaluate the function $f^{(P)}$ at earlier times a detailed analysis of the intermediate region where entropy is produced would be needed.

In the cosmological version of Bekenstein entropy bound the size of the system is determined by the particle horizon. However, one could use instead the event horizon which is proportional to the Hubble radius. In the Kasner regime the Hubble radius in the $j$th direction is given by

$$|H_j(t)|^{-1} \equiv \left| \frac{a_j(t)}{\dot{a}_j(t)} \right| \equiv -\frac{t}{\alpha_j}$$

so by taking $R$ in (17) to be $R = \text{Max} \left[ |H_1|^{-1}, |H_2|^{-1}, |H_3|^{-1} \right]$ we are led again to select the direction with the smallest $\alpha_j$ (recall that for inflationary models $0 < \alpha_j < 1$). Evaluating the function $f^{(H)}(t)$ we find that it is constant for the whole Kasner epoch and solely determined by the initial conditions

$$f^{(H)}(t) \simeq \frac{2\sqrt{\sigma_{\text{Pl}}^2}}{\pi |\beta|} \alpha_{\text{min}}, \quad (19)$$

where by $\simeq$ we mean that we are dropping the multiplicative constant of order one relating the event horizon with the Hubble radius. The condition $f^{(H)} \leq 1$ corresponds then to the entropy bound proposed by Veneziano (see [20] and references therein). This bound will be satisfied depending on the value of the dimensionless quantity $\sigma_{\text{Pl}}^2$ that, as discussed above, measures the total amount of entropy generated during the intermediate regime. The corresponding function in string frame, $f^{(H)}$, is also constant and can be obtained by substituting the constants on the right-hand side of (19) by their string frame counterparts.

We therefore see from (19) that the Hubble entropy bound will be satisfied depending on initial conditions and the thermodynamics of the intermediate region.

It is interesting to notice that, since the Hubble radius is a “local” quantity which only depends on the expansion/contraction rate at a given instant, the function $f^{(H)}(t)$ contains no information about the history of the universe beyond the one encoded in the value of $\sigma_{\text{Pl}}^2$. In particular, there is no dependence on what happened before the universe entered the Kasner regime between the times $t_i$ and the time $t_K$. This contrasts with $f^{(P)}(t)$ which, due to the nonlocal character of the particle horizon, encodes even in the Kasner regime information about the pre-Kasner epoch. Furthermore, one should not be surprised that Veneziano’s bound is time-independent in this example. By using the event horizon one is basically assigning a black hole (maximum) entropy to the solution which in the case of the stiff fluid coincides with that of the black hole anyway (cf. [15]).

4. Discussion

In this letter we have investigated some phenomenological aspects of the picture for PBB cosmology proposed in Ref. [3]. In particular, we have focused our attention on two special aspects of the model: the conditions for successful inflation and the entropy generation.

As to the requirement of successful inflation we have found that in order to meet the necessary number of $e$-foldings to solve the homogeneity and flatness problems the focal lengths of the incoming waves have to be exponentially large with respect to the string scale. Actually it is interesting that the bound on the product $L_1 L_2$ is independent of the physical mechanism terminating inflation, $\alpha'$ corrections or strong coupling effects. This provides quite a natural picture
where the waves might be seen as small perturbations on a flat spacetime producing upon collision the PBB bubbles. One of the alleged "fine-tunings" of the BDV picture was the necessity of starting with very large black holes in order to produce sufficient inflation during the PBB phase. In our picture, this requirement is translated into a rather "natural" condition of having extremely weak incoming waves.

With respect to the production of entropy we have seen, that entropy is generated dynamically as the result of the nonlinear interaction of the incoming waves. Thus, the following picture emerges. As argued in [3] the universe starts as a bath of gravitational and dilatonic plane waves with zero total (matter + gravitational) entropy. The waves eventually interact and, before producing a singularity, the universe passes through a stage of nonadiabatic evolution where entropy is created (Fig. 1). The process of entropy generation ends when the universe enters the regime of adiabatic (perfect fluid) evolution. This happens at the onset of the Kasner phase or earlier, depending whether velocities dominate over spatial gradients at or before entering the Kasner regime. We have also seen that the total entropy inside a Hubble volume at the beginning of DDI is related to the focal lengths of the incoming waves by

\[ S = \kappa L_1 L_2 / \ell_{Pl}^2, \]

with \( \kappa \) a numerical constant depending on the initial conditions of the collision and the thermodynamics of the effective imperfect fluid describing the dilaton condensate in the nonadiabatic intermediate regime.

An immediate question to be asked is whether the analytical relation between the initial data and the structure of the singularity [3] in this scenario allows a quantitative evaluation of the entropy produced. This, unfortunately, cannot be done since any analysis of this type would require some input about the thermodynamical properties of the effective fluid in the whole interaction region. This is easy to understand on physical grounds, since the initial data only determines the dynamics of the background fields (the metric and the dilaton in this case), but not the thermodynamics of the dilaton condensate. In order to specify from first principles the phenomenological parameters required for the description of the fluid in the nonadiabatic regime (which would determine the amount of entropy generated and therefore the value of \( \sigma \ell_{Pl}^2 \)) one would need to invoke a microscopical description of the matter content. This situation is analog to the case of standard FRW cosmology, where the gravitational dynamics of a radiation dominated universe does not determine the value of the Stefan–Boltzmann constant.

We have checked that the Bekenstein entropy bound is satisfied close to the singularity and can only be violated at earlier times. However, the conditions favourable for a long lasting inflation seem to comply with the Bekenstein entropy bound. On the other hand we have seen that the Hubble entropy bound will be satisfied depending on the value of the Kasner exponents (and as a consequence on the initial data on the null boundaries) as well as on the amount of entropy generated during the intermediate regime.

To close, the results of our analysis suggest a picture in which some collisions occur within a primordial "cold" bath of weak gravit-dilatonic waves. Before entering into the DDI stage the universe undergoes a nonadiabatic transient epoch where entropy is produced. If the waves are weak enough (as one would expect if they result from small fluctuations of the background) the resulting PBB bubble can become the seed of a FRW universe of the kind we see at present. In order to study the scenario analytically we have confined the initial data to strictly plane incoming waves. Nonetheless, it would be great to understand the problem in terms of more general \( \text{pp} \) waves which have finite transverse size. Note, however, that unlike plane waves, \( \text{pp} \) waves do polarise the vacuum, and assigning initial zero entropy to those would be somehow less natural.

Acknowledgements

It is a pleasure to thank Jacob Bekenstein, Enric Verdaguer and especially Gabriele Veneziano for useful discussions and correspondence. A.F. has been supported by University of the Basque Country Grant UPV 122.310-EB150/98, General University Research Grant UPV172. 310-G02/99 and Spanish Science Ministry Grant PB96-0250. K.E.K. acknowledges the support of the Swiss National Science Foundation. The work of M.A.V.-M. has been supported by FOM (Fundamenteel Onderzoek van der Materie) Foundation and by University of the Basque Country Grants UPV 063.310-EB187/98 and UPV 172.310-G02/99, and Spanish Science Ministry Grant AEN99-0315. K.E.K. and M.A.V.-M. wish to thank the Department
of Theoretical Physics of The University of the Basque Country for hospitality. M.A.V.-M. also thanks CERN Theory Division for hospitality during the final stages of this work.

References

G. Veneziano, Inflating, warming up, and probing the pre-bangian universe, hep-th/9902097;  
J.E. Lidsey, D. Wands, E.J. Copeland, Superstring cosmology, hep-th/9909061;  
For an updated collection of papers on string cosmology, see http://www.to.infn.it/~gasperin.

G. Veneziano, String cosmology: the pre-big-bang scenario, hep-th/0002094.


K.E. Kunze, Class. Quantum Grav. 16 (1999) 3795.


(3 + 1)-dimensional Yang–Mills theory as a local theory of evolution of metrics on 3-manifolds

Pushan Majumdar a,*, H.S. Sharatchandra a

a Institute of Mathematical Sciences, C.I.T. campus Taramani, Madras 600-113, India

Received 9 August 2000; accepted 2 September 2000

Abstract

An explicit canonical transformation is constructed to relate the physical subspace of Yang–Mills theory to the phase space of the ADM variables of general relativity. This maps (3 + 1)-dimensional Yang–Mills theory to local evolution of metrics on 3-manifolds.

© 2000 Published by Elsevier Science B.V.

PACS: 11.15.-q; 11.15.Tk

The question of whether the dynamics of Yang–Mills theory can be completely captured in terms of gauge invariant quantities has been raised many times. This is important especially because of confinement in QCD. One approach has been to rewrite the theory as dynamics of the loop variables. These Wilson loops are non-local variables and they also form an over-complete set. The possibility of using the gauge invariant combination $E^i \cdot E^j$ of the non-Abelian electric field has also been explored [1–3]. In another approach, the gauge invariant variables $\bar{B}_i[A] \cdot \bar{B}_j[A]$ have also been considered [4,5]. Analogy to gravity yields a nice geometric interpretation for (2 + 1)-dimensional Yang–Mills theory [6]. Such an approach for 3 + 1 dimensions has been attempted in [7].

In this article we use certain techniques motivated by the Ashtekar formulation of gravity [8]. We map the physical phase space of Yang–Mills theory to the phase space of the ADM variables of general relativity by an explicit canonical transformation. To do this we augment the ADM variables $(g_{ij}, \pi^{ij})$ by a set of auxiliary variables $(\theta, \chi^a)$ to match in number, the variables $(A^a_i, E^{ij})$ of the extended phase space of Yang–Mills theory. It turns out that the non-Abelian Gauss law simply becomes the constraint $\chi^a = 0$. Therefore the physical subspace of the phase space of the Yang–Mills theory is exactly mapped to the phase space of the ADM variables and the dynamics can be rewritten as a local theory of evolution of metrics on 3-manifolds.

We use the language of functional integrals, but every step below may be interpreted in terms of dynamics of the classical theory. We begin with the Euclidean partition function

$$Z = \int \mathcal{D}A^a_\mu \exp \left\{ - \frac{1}{4g^2} \right\} \times \left\{ \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu + \tilde{A}_\mu \times \tilde{A}_\nu \right\}^2 \right\}. \quad (1)$$

* Corresponding author.
E-mail addresses: pushan@imsc.ernet.in (P. Majumdar), sharat@imsc.ernet.in (H.S. Sharatchandra).
Introducing an auxiliary field $E^{ia}$, and integrating over $A_{0}^{a}$, we get

$$Z = \int D\bar{A}_{0}^{a} D E^{ia} \delta(D_{1}[A]|E^{i})$$

$$\times \exp \left\{ \int (-\mathcal{H} + i \bar{E}^{i} \cdot \partial_{0} \bar{A}_{i}) \right\} ,$$

where

$$\mathcal{H} = \frac{1}{2} \left( g^{2} E^{2} + \frac{1}{g^2} B^{2} \right) ,$$

is the Hamiltonian density, and

$$\bar{B}^{i}[A] = \frac{1}{2} e^{ijk} \left( \partial_{j} \bar{A}_{k} - \partial_{k} \bar{A}_{j} + \bar{A}_{j} \times \bar{A}_{k} \right)$$

is the non-Abelian magnetic field. Using the Feynman time slicing procedure, it is also clear that $\bar{A}_{i}$, $E^{i}$ are the conjugate variables of the phase space. There are also three first class constraints, the non-Abelian Gauss law:

$$D_{i}[A] \bar{E}^{i} = 0 .$$

(5)

Motivated by the Ashtekar variables, we define a driebein $e$ by

$$\bar{E}^{i} = \frac{1}{2} e^{ijk} \bar{e}_{j} \times \bar{e}_{k} .$$

(6)

Assuming $\| E \| = \| e \|^{2}$ is nonzero, we can invert (6) to get $e^{a}_{i} = \| E \|^{1/2}(E^{-1})^{a}_{i}$. Define $\bar{A}[E]$ as the connection one form which is torsion-free with respect to the driebein $\bar{e}_{i}$. We have

$$\epsilon_{ijk} \left( \partial_{j} \bar{e}_{k} + \bar{A}_{j}(E) \times \bar{e}_{k} \right) = 0 .$$

(7)

Therefore,

$$D_{i}[\bar{A}(e)] \bar{E}^{i} = 0$$

(8)

is identically valid. Hence we may replace Gauss law (5) by

$$\bar{a}_{i} \times \bar{E}^{i} = 0 ,$$

where $a_{i} = A_{i} - \bar{A}_{i}(E)$ transforms homogeneously under gauge transformation.

We now observe that the change of variables from $[A_{i}, E^{i}]$ to $[a_{i}, E^{i}]$ is a canonical transformation. Consider the generating function

$$\mathcal{S}[a^{a}_{i}, E^{ia}] = \int d^{3}x \ a^{a}_{i} E^{ia} + \tilde{S}[e] ,$$

(10)

where

$$\tilde{S}[e] = \frac{1}{2} \int d^{3}x \ e^{ijk} \bar{e}_{i} \cdot \partial_{j} \bar{e}_{k} .$$

(11)

The momentum conjugate to the new coordinate $a^{a}_{i}$ is $\delta S/\delta a^{a}_{i} = E^{ia}$, same as for $A^{a}_{i}$. The relation between the old and the new coordinates is $A^{a}_{i} = \delta S/\delta E^{ia}$ so that $A^{a}_{i} - a^{a}_{i} = \delta S/\delta E^{ia}$. Now

$$\frac{\delta \tilde{S}[e]}{\delta e^{a}_{i}} = e^{ijk} \partial_{j} e^{a}_{m} = -e^{ijk} e^{def} \bar{A}_{j}^{e}[E] e^{f}_{m} .$$

(12)

Therefore we have,

$$\frac{\delta \tilde{S}[e]}{\delta E^{ia}} = \frac{\delta e^{a}_{i}}{\delta E^{ia}} = \bar{A}_{j}^{e}[E] ,$$

(13)

since

$$\frac{\delta e^{a}_{i}}{\delta E^{ia}} = \frac{1}{\| e \|} \left( \frac{1}{2} e^{a}_{i} e^{e}_{i} - e^{a}_{i} e^{e}_{i} \right) .$$

(14)

We now show that there exists a canonical transformation from the phase space $(a, E)$ with the constraint (9) to the phase space of the ADM variables $(\pi^{ij}, g_{ij})$. (The first class constraints involving the ADM variables, related to space–time translations is not relevant for the present context of Yang–Mills theory.) As the variables $(a, E)$ are more in number than $(\pi, g)$, we first augment the latter set by a canonically conjugate set $[\chi^{a}, \theta_{a}]$. Here

$$g_{ij} = \bar{e}_{i} \cdot \bar{e}_{j} ,$$

(15)

and to define $\theta_{a}$ we consider the Polar decomposition of $e^{a}_{i}$:

$$e^{a}_{i} = e_{ij} O^{ia}$$

(16)

into a symmetric matrix $e_{ij}$ and an orthogonal matrix $O^{ia}$. $\theta^{a}$ is the Lie algebra element corresponding to $O^{ia}$.

$$O = \exp \left( i \theta_{a} T^{a}_{2} / 2 \right) .$$

(17)

The Eqs. (15)–(17) give us the relation between the old momenta and the new coordinates. We relate the momenta via a generator of the canonical transformation

$$S(\pi, \chi, E) = \int \left( (\bar{e}_{i} \cdot \bar{e}_{j}) \pi^{ij} + \theta_{a}(e) \chi^{a} \right) .$$

(18)

This gives us $g_{ij} \equiv \delta S/\delta \pi^{ij} = \bar{e}_{i} \cdot \bar{e}_{j}$ as we want. Among the other quantities, $\theta_{a}$ is given by
In this case the conjugate variables \(g_{ij}\) invariants. The new Hamiltonian is defined in Appendix A. Now note that

\[
\frac{\delta S}{\delta e^i_j} = 2\pi I^{ij} e^i_j + \frac{1}{2} \epsilon_{ijk} M^{-1} \theta^b_k \times ((e - 1 \text{Tr} e)^{-1})_{ij} O^{ja} O^{kc},
\]

(21)

where \(M\) is defined in Appendix A. Now note that

\[
\frac{\delta S}{\delta e^i_j} = \frac{\delta E^{bj}}{\delta e^i} a^{bj}.
\]

(22)

This explicitly relates \(a\) to the other variables. We now show that \(\chi\) is related to the Gauss law. Contracting \(\delta S/\delta e^i_j\) by \(e^i_j\), we get from (22)

\[
e^i_a \frac{\delta S}{\delta e^i_j} = -E^{ia} a^i_j + E^{ib} a_{ib} a_{ac}.
\]

(23)

For the part antisymmetric in \((a, c)\), we get from (21),

\[
(a_i \times E_i)^a = \frac{1}{2} (M^{-1})^b_k [\theta^a \theta^b].
\]

(24)

Thus the canonical momentum \(\pi^{ij}\) drops out in the Gauss law equation (5). With the new variables, the Gauss law is implemented by simply setting \(\chi = 0\).

The partition function (2) in the new variables is

\[
Z = \int Dg_{ij} D\pi^{ij} D\theta^a D\chi^a \delta((M^{-1})^a_k [\theta^a \theta^b] \chi^b)
\times \exp \left( -\mathcal{H}' + i \pi^{ij} \partial_0 g_{ij} + i \chi^a \partial_0 \theta^a \right).
\]

(25)

\(\theta^a\) represents the gauge degrees of freedom. We may adopt the Faddeev–Popov procedure to choose \(\theta^a = 0\). In this case \(M^a_k [\theta] = \delta^a_k\), and

\[
Z = \int Dg_{ij} D\pi^{ij} \exp \left( -\mathcal{H}' [g, \pi] + i \pi^{ij} \partial_0 g_{ij} \right).
\]

(26)

Thus the functional integral is rewritten in terms of the conjugate variables \((g_{ij}, \pi^{ij})\) which are gauge invariant. The new Hamiltonian \(\mathcal{H}'\) is obtained from (3), by the replacements

\[
E^{ia} \rightarrow \frac{1}{2} \epsilon^{ijk} \epsilon^{abc} e^{j_b} e^{k_c},
\]

(27)

\[
A^a_i \rightarrow A^a_i [E] + a^a_i,
\]

(28)

\[
a^a_i \rightarrow \frac{1}{\|e\|} (\pi^{jk} g_{jk} e^i_a - 2 \pi^{jk} g_{ik} e^j_a),
\]

(29)

where \(e^{i_a}\) is regarded as the symmetric square root of \(g_{ij}\). Thus the \(\langle \tilde{E}' \rangle^2\) term in the Hamiltonian becomes \(g^{ij}/\|g\|^2\), while

\[
B_i [A] = B_i [\tilde{A} [E]] + \epsilon^{ijk} D_j [\tilde{A} [E]] a_k
\]

(30)

\[
+ \frac{1}{2} \epsilon^{ijk} (a_j \times a_k)
\]

\[
= \tilde{a}_i \cdot \tilde{a}_j e^i_a \cdot e^j_a - (\tilde{a}_i \cdot \tilde{a}_j)^2.
\]

(32)

Similarly

\[
(D_i [\tilde{A} [E]])^a_j = \Gamma^{b}_{ij} [g] e^a_k.
\]

(33)

where \(\Gamma^{b}_{ij}\) is the affine connection corresponding to the metric \(g_{ij}\). \(D_k [\tilde{A} [E]]\) can be replaced by the covariant derivative corresponding to the affine connection \(\Gamma^{b}_{ij} [g]\) when acting on \(g_{ij}\) or \(\pi^{ij}\). This way, \((B_i [A])^2\) can be written as a local expression in \(g_{ij}\) and \(\pi^{ij}\).

We have thus mapped the physical phase space of Yang–Mills theory onto the phase space of the ADM variables and the dynamics is now a local evolution of the metrics on 3-manifolds. This completes the program envisaged in [1].

The canonical transformation constructed here can be used to map all the ADM constraints of general relativity to certain constraints on the Yang–Mills phase space without requiring complexification of the gauge field. In fact Barbero’s constraints [9] are reproduced.

Here we have related the gauge invariant combination \(\tilde{E}' \cdot \tilde{E}'\) to a metric on a 3-manifold. It is also possible to construct a metric from the vector potential \(A^a_i\) and use it to rewrite the Yang–Mills dynamics. These two approaches are dual of each other. This will be addressed elsewhere.
Appendix A

To evaluate $\delta \theta^b [e] / \delta e^a_i$, note that

$$e_i^a + \delta e_i^a = (e_{ij} + \delta e_{ij})$$

$$\times \exp \left( i \theta [e] + \delta \theta [e] \right) \frac{T^b}{2}. \quad (A.1)$$

This gives

$$\delta e_i^a O^k a = \delta e_{ik} + \epsilon^{dca} e_{ij} O^{jd} O^k a \delta \theta^c$$ \quad (A.2)

where $\delta \theta^c = M_c^f [e] \delta \theta^e$ and the matrix $M_c^f$ is given by

$$O^T (\theta) O (\theta + \delta \theta) \approx 1 + T^a M_b^c (\theta) \delta \theta^b.$$ 

Taking the antisymmetric part of (A.2) we can solve for $\delta \theta^b [e] / \delta e^a_i$. We get

$$\frac{\delta \theta^b [e]}{\delta e^a_i} = -\epsilon_{ijk} M^{-1} [\theta]^b_c \left( (e - \text{tr} e)^{-1} \right)_{kl} O^{ja} O^{lc}.$$ 

When two or all eigenvalues of the symmetric matrix $e_{ij}$ are degenerate, so are those of $(e - \text{tr} e)$. In this case, the variables $\theta [e]$ Eq. (A.1) are ill-defined and more care is required to define the new variables.

References

Heat kernel expansions for distributional backgrounds

Ian G. Moss

Department of Physics, University of Newcastle upon Tyne, Newcastle upon Tyne NE1 7RU, UK

Received 26 June 2000; accepted 21 August 2000

Editor: P.V. Landshoff

Abstract

Heat kernel expansion coefficients are calculated for vacuum fluctuations with distributional background potentials and field strengths. Terms up to and including $t^{5/2}$ are presented. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 03.70.+k; 98.80.Cq

The heat kernel is often used to investigate the effects of vacuum polarisation in quantum field theory. An important property of the heat kernel is that coefficients in the expansion of the heat kernel in powers of proper time $t$ give information about renormalisation and anomalies. In many cases it is possible to evaluate these coefficients simply from the information provided in the operator, and there is an extensive literature on the subject [1,2].

Most of the results for the heat kernel coefficients have concentrated on non-singular potentials. However, singular potentials often arise in idealised models. Cosmic strings, for example, have been regarded as distributional line sources in calculations of vacuum energy [4]. More recently, in the brane–world models inspired by superstring theory [5–8], the branes are regarded as distributional sources.

Bordag and Vassilevich [3] have obtained the most complete results so far on the heat kernel coefficients for delta function potentials, obtaining terms up to $t^{5/2}$. In this letter, similar methods are used to obtain results for gauge fields with distributional field strengths. Such fields appear to be present in the reduction of the Horava–Witten model [5,6,9], making the results of topical interest.

The results given here will be for the integrated heat kernel in flat space of $d + 1$ dimensions with a delta function potential concentrated on a smoothly embedded surface $\Sigma$ which bounds a region $\Omega$. The heat kernel satisfies

$$ (\Delta - \partial_t)K(x, x', t) = -\delta(t)\delta(x - x'), $$

where the operator takes the form

$$ \Delta = -\left(\partial + iA(x)\right)^2 + V(x), $$

with gauge field $A$ and potential $V$,

$$ A = a\theta_{\Omega} + A, $$

$$ V = v\delta_{\Sigma} + V. $$

The function $\theta_{\Omega} = 1$ for $x \in \Omega$ and zero otherwise. Its derivative is a delta function, $\partial\theta_{\Omega} = -\delta_{\Sigma} n$, where $n$ is the unit normal to $\Sigma$. The field strength tensor $F_{\mu\nu} = A_{\mu\nu} - A_{\nu\mu}$ therefore contains a distributional part,

$$ F_{\mu\nu} = f_{\mu\nu} \delta_{\Sigma} + F_{\mu\nu}, $$

E-mail address: ian.moss@ncl.ac.uk (I.G. Moss).
where $f_{\mu \nu} = a_i n_{\mu} - n_{i} a_{\mu}$.

The integrated heat kernel $K(t)$ is obtained by integrating $K(x, x, t) - K_0(x, x, t)$ over space, where $K_0$ is the free heat kernel. It has an asymptotic expansion of the form

$$K(t) \sim (4\pi t)^{-(d+1)/2} \sum_{n=0}^{\infty} C_n t^{n/2},$$

where, as we shall see, the coefficients can be expressed in terms of integrals of local invariants,

$$C_n = \int dx b_n(x) + \int dx c_n(x).$$

The method for obtaining the heat kernel coefficients is based on perturbation theory [3]. Suppose that $A = -\hbar^2 + V(x)$, then

$$K(x, x', t) = K_0(x, x', t) - \int_0^t dt_1 \int dx_1$$

$$\times K_0(x, x_1, t - t_1) V(x_1) K(x_1, x', t).$$

The iterative solution to this equation is the Born series with terms $K^{(n)}$. For the integrated heat kernel we have,

$$K^{(n)}(t) = (-1)^n t \int_0^t dt_1 \cdots \int_0^{t_{n-1}} \int dx_n \cdots \int dx_1$$

$$\times V(x_1) K_0(x_1, x_2, t - t_1) \cdots V(x_n)$$

$$\times K_0(x_n, x_1, t_{n-1}).$$

If $\mathcal{A} = 0$, the first term in the series becomes

$$K^{(1)}(t) = -(4\pi t)^{-(d+1)/2} \int dx V(x).$$

The second term in the Born series can be simplified by integrating out the intermediate time variable $t_1,

$$K^{(2)}(t) = \frac{t^{1-d}}{2^{d+3/2} \pi^{d+1/2}}$$

$$\times \int dx dx' e^{-\zeta} U\left(\frac{1}{\zeta}, \frac{d+1}{2}, \zeta\right) V(x) V(x'),$$

where $\zeta = (x - x')^2/t$ and $U(a, b, z)$ is a confluent hypergeometric function.

If the gauge fields are nonzero then it is advantageous to collect together terms which are quadratic in the field strength tensor. Denoting this combination by $K^{FF}$, one finds

$$K^{FF}(t) = \frac{t^{1-d}}{2^{d+3/2} \pi^{d+1/2}}$$

$$\times \int dx dx' e^{-\zeta} U\left(\frac{1}{\zeta}, \frac{d+1}{2}, \zeta\right)$$

$$\times F_{\mu \nu}(x) F^{\mu \nu}(x'),$$

where $U'$ is the derivative of $U$ with respect to $z$.

Let us consider the delta function potential $\delta_\Sigma$. First of all, the second Born approximation reduces to a surface integral

$$K^{(2)} = \frac{t^{1-d}}{2^{d+3/2} \pi^{d+1/2}} \int dx$$

$$\times \int dx' e^{-\zeta} U\left(\frac{1}{\zeta}, \frac{d+1}{2}, \zeta\right) v(x) v(x').$$

For small $t$ the integral is dominated by $x' \approx x$. Let $\xi$ be the unit tangent vector at $x$ to the geodesic joining $x$ to $x'$ in the surface with length $\sigma$. The integrand can be expanded in powers of $\sigma$. Denoting the extrinsic curvature by $k_{ab}$ and the intrinsic ricci curvature by $r_{ab}$, the Euclidean distance becomes

$$(x - x')^2 = \sigma^2 - \frac{1}{12} (k_{ab} e^a e^b)^2 \sigma^4 + \cdots.$$  

The surface area element is

$$dx' = (1 - \frac{1}{12} r_{ab} e^a e^b) \sigma^d d\sigma d\xi,$$

where $\xi$ parameterises a unit sphere in $d$ dimensions. These can be substituted into (13) to obtain an asymptotic expansion in $t$,

$$K^{(2)} \sim (4\pi t)^{-(d+1)/2}$$

$$\times \int_\Sigma \left\{ \frac{\sqrt{\pi}}{4} v^2 t^{3/2} + \frac{\sqrt{\pi}}{32} v^2 u t^{5/2}$$

$$- \frac{\sqrt{\pi}}{128} v^2 (k^2 - 2 k_{ab} e^n e^b)^{5/2} + \cdots \right\}.$$  

The operator $\nabla^2$ is the Laplacian on $\Sigma$. The coefficients appearing here, namely $c_3$ and $c_5$, are in agreement with Bordag and Vassilevich [3].

The same calculation for the gauge fields, using (12), produces
\[ K^{FF} \sim (4\pi t)^{-(d+1)/2} \int \left\{ -\frac{\sqrt{\pi}}{16} f^2 t^{3/2} \right. \\
- \frac{1}{6} f_{\mu\nu} F^{\mu\nu} f^2 - \frac{\sqrt{\pi}}{256} f^4 t^{5/2} \\
+ \frac{\sqrt{\pi}}{1024} f^2 (k^2 - 2 k_{ab} k^{ab}) t^{5/2} + \cdots \left\} \right. \]

where \( f^2 \) denotes \( f_{\mu\nu} f^{\mu\nu} = 2a^2 - 2(a \cdot n)^2 \).

Further terms can be evaluated by returning to the Born series (9) and taking constant values of the gauge field \( a \) and potential \( v \). The integrals can be performed for a plane surface \( \Sigma \), regarding this as the leading term in the curvature expansion (15). The terms in the Born series reduce to expressions of the form

\[ K^{(n)} = (4\pi t)^{-(d+1)/2} \int b^{(n)} + (4\pi t)^{-(d+1)/2} \int c^{(n)}, \]

where, after dropping nonasymptotic terms,

\[ c^{(1)} \sim -vt, \]
\[ c^{(2)} \sim -\frac{\sqrt{\pi}}{4} v^3 t^{3/2} - \frac{\sqrt{\pi}}{8} a^2 t^{3/2} + \frac{1}{16} v a^2 t^2, \]
\[ c^{(3)} \sim -\frac{1}{6} v^3 t^2 + \frac{3\sqrt{\pi}}{16} a^4 t^{5/2} - \frac{1}{8} v a^2 t^2 - \frac{\sqrt{\pi}}{8} v^2 a^2 t^{5/2}, \]
\[ c^{(4)} \sim \frac{\sqrt{\pi}}{32} v^4 t^{5/2} - \frac{29\sqrt{\pi}}{256} a^4 t^{5/2} + \frac{5\sqrt{\pi}}{64} v^2 a^2 t^{5/2}. \]

The volume integrals depend only on the gauge field \( a \) and cancel amongst themselves. Since these terms would violate gauge invariance, the cancellation provides a useful consistency check.

From the preceding analysis it has emerged that the distributional fields only contribute to surface terms in the heat kernel expansion (6). These surface terms for the operator (2) can now be listed by examination of (16), (17) and (19),

\[ c_2 = -v, \]
\[ c_3 = \frac{\sqrt{\pi}}{4} v^2 - \frac{\sqrt{\pi}}{16} f^2, \]
\[ c_4 = -\frac{1}{6} v^3 + Vv - \frac{1}{6} f_{\mu\nu} F^{\mu\nu} + \frac{1}{12} v f^2, \]
\[ c_5 = \frac{\sqrt{\pi}}{32} v^4 + \frac{\sqrt{\pi}}{32} v \nabla^2 v - \frac{\sqrt{\pi}}{128} v^2 (k^2 - 2 k_{ab} k^{ab}) - \frac{\sqrt{\pi}}{4} V v^2 - \frac{\sqrt{\pi}}{256} f \nabla^2 f + \frac{\sqrt{\pi}}{1024} f^2 (k^2 - 2 k_{ab} k^{ab}) + \frac{3\sqrt{\pi}}{1024} (f^2)^2 + \frac{\sqrt{\pi}}{16} V f^2 - \frac{3\sqrt{\pi}}{128} v^2 f^2 + \frac{\sqrt{\pi}}{16} v f_{\mu\nu} F^{\mu\nu}. \]

(Terms involving \( V \) have been recovered by multiplying the series by \( e^{-V t} \).) For comparison, \( V, v \) and \( k_{ab} \) correspond to \(-E, -V\) and \( L_{ab} \) in Ref. [3]. Terms involving \( f \) are new, although some of these terms are implicit in other work (e.g., [11]).

The heat kernel coefficients can be used for regularisation in calculations of vacuum fluctuations on distributional backgrounds. If distributional sources are present at a fundamental level, it might also be argued that any divergent terms appearing in the heat kernel expansion must be taken into account in the renormalisation of the theory.

Distributional potentials can also be used to model imperfectly reflecting boundaries, for example as in [12]. This means that they are often a more physically realistic choice for modeling the vacuum energy in the Casimir effect than Dirichlet or Robin boundary conditions.

The results can be extended to higher orders in the proper time expansion if that is required. It is also possible to construct alternative expansions of the heat kernel, for example in powers of derivatives as suggested in Ref. [10]. Resummation of the potential terms to obtain the first terms in the derivative expansion for \( v \) is relatively simple. For constant values of \( v \) the results can be checked against the full Green function for a delta function potential in one dimension, which is known [13]. The corresponding calculation for the gauge fields appears to be more complicated.
References

Abstract

New non-abelian supersymmetric generalizations of the four-dimensional Born–Infeld action are constructed in \( N = 1 \) and \( N = 2 \) superspace, to all orders in \( \alpha' \). The proposed actions are dictated by simple (manifestly supersymmetric and gauge-covariant) non-linear constraints.

1. Introduction

The abelian Born–Infeld (BI) action is known to be the low-energy part of any (gauge-fixed, world-volume) D-brane effective action. The BI action is merely dependent upon the abelian vector field strength, being independent upon spacetime derivatives of the field strength by definition. The supersymmetric BI actions with linearly realized \( N = 1 \) or \( N = 2 \) supersymmetry in four dimensions describe the low-energy (world-volume) dynamics of a single D3-brane propagating in four or six dimensions, respectively.\(^2\) The D3-brane action is supposed to have the Goldstone–Maxwell interpretation associated with partial \((1/2)\) spontaneous supersymmetry breaking, with the Goldstone fields in a (Maxwell) vector supermultiplet with respect to unbroken (linearly realized) supersymmetry. Hence, a supersymmetric BI action should be the low-energy part of the corresponding Goldstone–Maxwell action. The unbroken \( N = 1 \) and \( N = 2 \) supersymmetry can be made manifest in superspace. The \( N = 1 \) manifestly supersymmetric BI action was first formulated in Ref. \[2\], while its Goldstone–Maxwell interpretation was later established in Ref. \[3\]. The \( N = 2 \) manifestly supersymmetric BI action was first formulated in Ref. \[4\], whereas its relation to \( \text{(yet to be fully determined)} \) \( N = 2 \) Goldstone–Maxwell action is briefly discussed in Section 2, see also Ref. \[5\] for more.

As was pointed out by Witten \[6\], there is a non-abelian gauge symmetry enhancement when \( N \) parallel D3-branes coincide. A supersymmetric abelian BI action is then supposed to be replaced by a non-abelian Born–Infeld (NBI) action where the world-volume fields are valued in the Lie algebra of \( U(N) \). Both abelian and non-abelian BI actions are the effective actions, defined modulo local field redefinitions.
Nevertheless, the bosonic abelian BI action (with tension $T_3$)
\[ S_{\text{BI}} = -T_3 \int d^4x \sqrt{-\det(\eta_{\mu\nu} + 2\pi a' F_{\mu\nu})} \]  
(1)

is unambiguous, being only dependent of the abelian field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ but not of space-time derivatives of it ($\partial F$). In contrast, a bosonic NBI action is not well-defined, while there are two principal sources for ambiguities [1]. The first type of non-abelian ambiguities is related to the obvious fact that the terms dependent of the gauge-covariant derivatives of the non-abelian field strength cannot be unambiguously separated from the $F$-dependent commutators since $[D_\mu, D_\nu]F_{\lambda\rho} = [F_{\mu\nu}, F_{\lambda\rho}]$. Any concrete proposal for an NBI action has to specify an order of the $F$-matrices and, hence, it may effectively include some of the $DF$-dependent terms, even if they do not explicitly appear in the action. Though the full (abelian or non-abelian) D3-brane effective action certainly includes the derivative-dependent terms, it does not make much sense to keep some of them while ignoring other possible terms. Perhaps, the best one can do with a bosonic NBI action is to define it for almost covariantly constant gauge fields with almost commuting field strengths, which does not seem to be very illuminating. The second (related) type of ambiguities is connected to the trace operation over the gauge group. For example, when using the abelian identity \[ \det(\eta_{\mu\nu} + b F_{\mu\nu}) = 1 + \frac{b^2}{2} F^2 - \frac{b^4}{16} (F \bar{F})^2, \] 
(2)

one gets two natural candidates for the bosonic NBI action,
\[ S_{(a)} = -T_3 \int d^4x \sqrt{1 + \frac{b^2}{2} \text{tr}(F^2) - \frac{b^4}{16} \text{tr}(F \bar{F})^2} \]  
(3a)

and
\[ S_{(b)} = -T_3 \int d^4x \text{Str}\sqrt{-\det(\eta_{\mu\nu} + b F_{\mu\nu})}, \]  
(3b)

where $F_{\mu\nu} = F_{\mu\nu}^{ab}t_a$, $\{t_a\}$ are the Hermitian generators of the gauge group, $\{t_a, t_b\} = i f_{abc}^{ab} t_c$, $\text{tr}(t_a t_b) = \delta_{ab}$, and $\text{Str}$ is the symmetrized trace, \[ \text{Str}(t_{a_1} \cdots t_{a_k}) = \frac{1}{k!} \sum_{\pi \text{ permutations}} \text{tr}(t_{\pi(a_1)} \cdots t_{\pi(a_k)}). \]  
(4)

The $F$-matrices effectively commute under the symmetrized trace, so that the formal definition (2) of the determinant still applies in Eq. (3b). It is not difficult to verify that the equations of motion in the NBI theory (3b) on self-dual (Euclidean) configurations $(F_{\mu\nu} = \bar{F}_{\mu\nu})$ coincide with the ordinary Yang–Mills equations, so that they have the same BPS solutions [8], though the existence of a BPS bound is not obvious in the non-abelian case. Away from self-dual configurations the action (3a) is much simpler than (3b), while it is also known to admit solitonic (glueball) solutions [9].

The gauge-invariant actions (3a) and (3b) are obviously different, so that further resolution requirements are needed. Some extra conditions are provided by string theory, because the BI action is well-known to represent the effective action of slowly varying gauge fields in open string theory. The most basic requirement of string theory is the overall single trace of the non-abelian gauge field strength products [1]. The overall symmetrized trace advocated by Tseytlin [1] is a stronger condition based on the observation that it reproduces the $F^4$-terms in the non-abelian effective action of open superstrings in ten dimensions [7]. In this Letter I show that adding supersymmetry unexpectedly gives rise to some more constraints on supersymmetric NBI actions in four dimensions, which are not apparent in the bosonic case.

At first sight, it seems to be straightforward to supersymmetrize any NBI action, so that supersymmetry would not add anything new towards its intrinsic definition. However, in fact, supersymmetry does tell us something more about the BI actions. For example, linearly realized supersymmetry apparently prefers the parametrization of the abelian BI actions in terms of the (anti)self-dual combinations, $F^\pm = \frac{1}{2}(F \pm \bar{F})$, rather than in terms of the naively expected tensors $F$ and $\bar{F}$. More importantly, it is the spontaneously broken (non-linearly realized) supersymmetry on top of the unbroken (linearly realized) supersymmetry that is responsible for the complicated non-linear structure of the D3-brane action to be considered as the Goldstone–Maxwell action. Hence, the non-linear structure of the supersymmetric abelian BI
actions [1,2,4] should also be dictated by similar features. Though a Goldstone interpretation of the supersymmetric NBI actions is far from being obvious, if any, the well-established Goldstone form of the \( N = 1 \) supersymmetric abelian BI action in superspace gives us the natural starting point for a construction of its generalizations, either non-abelian or with extended (linearly realized) supersymmetry.

The paper is organized as follows. In Section 2 the \( N = 1 \) and \( N = 2 \) supersymmetric abelian BI actions are reviewed by emphasizing their relation to the Goldstone–Maxwell actions. In Section 3 the non-abelian generalizations of the BI actions are proposed in \( N = 1 \) and \( N = 2 \) superspace. Section 4 is my conclusion.

2. \( N = 1 \) and \( N = 2 \) abelian BI actions

The abelian bosonic BI Lagrangian \( \mathcal{L}_{\text{BI}}(F) \) can be thought of as the unique non-linear generalization of the Maxwell Lagrangian, \(-\frac{1}{2} F^2\), under the conditions of preservation of causality, positivity of energy, and electric-magnetic duality. In particular, the duality invariance of an abelian Lagrangian \( \mathcal{L}(F) \) amounts to the constraint \[ G_{\mu
u} \tilde{G}^{\mu\nu} + F_{\mu\nu} \tilde{F}^{\mu\nu} = 0, \]
where \( \tilde{G}^{\mu\nu} = \frac{1}{2} e^{\mu\nu\lambda\rho} G_{\lambda\rho} = 2 e^{\mu\nu} \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}}. \) (5)

Supersymmetry is known to be consistent with all these physical properties, so that the supersymmetric abelian BI actions enjoy similar features. The \( N = 1 \) supersymmetric abelian BI action reads \[ S_{\text{BI}} = \frac{1}{2} \left( \int d^4 x \ d^2 \theta \ W^2 + \text{h.c.} \right) + \int d^4 x \ d^4 \theta \ Y(K, \bar{K}) W^2 \bar{W}^2, \] where the structure function \( Y \) is given by

\[
Y(K, \bar{K}) = \left[ 1 - \frac{1}{2} (K + \bar{K}) \right].
\]

### Notes
3 The deformation parameter \( b \) is set to be one. The dependence upon \( b \) can be easily restored for dimensional reasons. I also ignore for dimensional reasons. I also ignore \( T_j \) for simplicity.
Similarly, the $N = 2$ supersymmetric Abelian BI action in $N = 2$ superspace reads \cite{4}

$$S_{2\text{BI}} = \frac{1}{2} \int d^4 x \, d^4 \theta \, \mathcal{W}^2 + \frac{1}{4} \int d^4 x \, d^8 \theta \, \mathcal{Y}(\mathcal{K}, \mathcal{K}) \mathcal{W}^2 \mathcal{\bar{W}}^2, \quad (14)$$

with the same structure function

$$\mathcal{Y}(\mathcal{K}, \mathcal{K}) = \frac{1}{1 - \frac{1}{2}(\mathcal{K} + \mathcal{K}) + \sqrt{1 - (\mathcal{K} + \mathcal{K})^2 + \frac{1}{4}(K - \mathcal{K})^2}}. \quad (15)$$

but

$$\mathcal{K} = D^4 \mathcal{W}^2, \quad D^4 = \prod_{i, \alpha} D^4_{i, \alpha} = \frac{1}{12} D_{ij} D^{ij}.$$ \quad (16)

in terms of the $N = 2$ restricted chiral gauge superfield strength $\mathcal{W}$ satisfying the off-shell constraints ($N = 2$ Bianchi identities)

$$\nabla^4 \mathcal{W} = 0, \quad D^4 \mathcal{W} = \Box \mathcal{W}. \quad (17)$$

The action (14) can be rewritten (modulo $\partial \mathcal{W}$-dependent terms) in the “non-linear sigma-model” form \cite{5}

$$S_{2\text{BI}} = \frac{1}{4} \int d^4 x \, d^4 \theta \, \mathcal{X} + \text{h.c.}, \quad (18)$$

whose $N = 2$ chiral Lagrangian $\mathcal{X}$ satisfies the non-linear $N = 2$ superfield constraint

$$\mathcal{X} = \frac{1}{4} \mathcal{\bar{X}} \mathcal{\bar{X}} + \mathcal{W}^2. \quad (19)$$

Similarly to the $N = 1$ abelian BI action, the non-linear constraint (19) gives us the convenient way of handling the complicated $N = 2$ BI abelian action (14). For example, as was demonstrated in Ref. \cite{11}, electric-magnetic self-duality of an $N = 2$ action $S(\mathcal{W}, \mathcal{\bar{W}})$ amounts to the following $N = 2$ supersymmetric extension of the $N = 1$ non-local constraint (13):

$$\int d^4 x \, d^4 \theta \, (\mathcal{W}^2 + \mathcal{M}^2) = \int d^4 x \, d^4 \theta \, (\mathcal{\bar{W}}^2 + \mathcal{\bar{M}}^2),$$

$$\frac{i}{4} \mathcal{M} = \frac{\delta S}{\delta \mathcal{W}}. \quad (20)$$

while it appears to be satisfied in the case of $S_{2\text{BI}}$ defined by Eqs. (18) and (19).

Unlike its $N = 1$ BI counterpart, the $N = 2$ BI action (14) or (18) does not give the full $N = 2$ Goldstone–Maxwell action, but rather represents its low-energy part, i.e., $S_{2\text{GM}} = S_{2\text{BI}} + \mathcal{O}(\partial \mathcal{W}, \partial \mathcal{\bar{W}})$. The infinitesimal parameters of the spontaneously broken (non-linearly realised) rigid symmetries can be naturally unified into a single (spacetime-independent) superfield $\Lambda = \lambda + \theta^a \lambda^a + \theta \lambda^U$, where $\lambda$ is the complex parameter of the Peccei–Quinn-type symmetry associated with two spontaneously broken supersymmetries, whereas $\lambda^U$ are the parameters of spontaneously broken R-symmetry $SU(2)$. The natural ansatz for the transformation laws of the non-linearly realised symmetries is given by an $N = 2$ analogue of Eq. (12) as follows [5]:

$$\delta_2 \mathcal{X} = 2 \mathcal{W},$$

$$\delta_2 \mathcal{W} = \Lambda \left(1 - \frac{1}{4} \mathcal{\bar{X}} \mathcal{\bar{X}} \right) + \cdots, \quad (21)$$

where $\mathcal{X}$ is the perturbative solution to the non-linear constraint (19) by iterations, to all orders in $\mathcal{W}$ and $\mathcal{\bar{W}}$, while the dots stand for some $\partial \mathcal{W}$-dependent terms needed for consistency with the second Bianchi identity (17). A variation of the $N = 2$ BI action (18) under the non-linear transformations (21) does not vanish, but it appears to be only dependent upon higher spacetime derivatives of $\mathcal{W}$ and $\mathcal{\bar{W}}$. This may not be surprising since the BI action and its supersymmetric extensions were defined modulo such terms. However, it also means that the $N = 2$ BI action has to be modified, order by order in $\partial \mathcal{W}$ and $\partial \mathcal{\bar{W}}$, in order to get the full $N = 2$ Goldstone–Maxwell action. A derivation of the derivative-dependent terms is beyond the scope of this Letter, and we do not need them for our main purpose formulated in the title.

A manifestly $N = 4$ supersymmetric abelian BI action is not known (see, however, Ref. \cite{1} and references therein for some partial results).
3. \( N = 1 \) and \( N = 2 \) supersymmetric NBI actions

Having understood the fact that the simple non-linear constraints (11) and (19) fully determine the structure of the highly complicated abelian BI actions (6) and (14), respectively, it is natural to define the \( N = 1 \) and \( N = 2 \) supersymmetric NBI actions by non-abelian generalizations of Eqs. (11) and (19).

The non-abelian (Yang–Mills) \( N = 1 \) chiral superfield strength is given by the well-known formula (see, e.g., Ref. [13] for a review or an introduction) \(^4\)

\[
W_\alpha = \frac{1}{8} \bar{D}^2 (e^{-2V} D_\alpha e^{2V}) , \quad \bar{D}_\alpha W_\alpha = 0 ,
\]

where the real scalar gauge superfield potential \( V \) transforms under gauge transformations with the chiral parameter \( \Lambda(x, \theta, \bar{\theta}) \) in the standard way [13]:

\[
e^{2V} \rightarrow e^{-2iA} e^{2V} e^{2i\Lambda}, \quad \bar{D}_\alpha \Lambda = 0 ,
\]

so that \( W_\alpha \) and \( \bar{W}_\alpha \) transform covariantly, viz.

\[
W_\alpha \rightarrow e^{-2iA} W_\alpha e^{2i\Lambda}, \quad \bar{W}_\alpha \rightarrow e^{-2iA} \bar{W}_\alpha e^{2i\Lambda} .
\]

The non-abelian gauge-covariant generalization of Eq. (11) is given by

\[
\Phi = \frac{1}{2} \phi \bar{D}^2 (e^{-2V} \bar{\phi} e^{2V}) + \frac{1}{2} W^2 ,
\]

where \( \Phi \) is the \( N = 1 \) chiral superfield Lagrangian that transforms like \( W_\alpha \) under the gauge transformations. The invariant action reads

\[
S_{1NBI} = \int d^4 x \ d^2 \theta \ \mathrm{tr} \, \Phi + \text{h.c.} ,
\]

or

\[
S_{1NBI}^{(x)} = \int d^4 x \ d^2 \theta \ \text{Str} \, \Phi + \text{h.c.}.
\]

The NBI actions (26a) and (26b) are supersymmetric and gauge-invariant, while they both have the single overall trace. The symmetrized trace in Eq. (26b) is supposed to be applied to the gauge-covariant operators only, by definition.

It is instructive to take a look at the structure of the quartic (\( F^4 \)) terms in the actions (26), which arise from the standard "adjoint chiral matter" term,

\[
\int d^4 x \ d^4 \theta (\text{tr} \, \phi e^{-2V} \bar{\phi} e^{2V}) , \quad \phi = W^2 .
\]

It is straightforward to verify that taking the trace as in Eq. (26a) results in the non-abelian generalization of the Euler–Heisenberg Lagrangian, in the bosonic sector of the component expansion of the action (26a),

\[
\frac{1}{4} \left[ \text{tr} F^2 + \text{tr} (F \bar{F})^2 \right] .
\]

In contrast, taking the symmetrized trace, as in Eq. (26b), exactly yields the \( F^4 \)-terms appearing in the expansion of the bosonic NBI Lagrangian (3b) [14]. Hence, if one insists on the choice (3b) of the bosonic NBI action, its supersymmetric extension in compact form is provided by Eq. (26b) — cf. Ref. [14]. Supersymmetry alone does not provide a resolution between the two different actions (26a) and (26b), so that more physical input is apparently needed. One natural option is to restrict the gauge group \( G \) to its (abelian) Cartan subgroup and then impose the condition of the generalized (non-abelian) self-duality for the resulting supersymmetric field theory of rank \( G \) (abelian) gauge fields along the lines of Ref. [12]. As was argued in Ref. [12], this may ultimately support the symmetrized trace. Still, in the absence of more physical reasons, the ordinary trace in Eq. (26a) is much simpler, being dependent of only two matrix building blocks, \( W^2 \) and \( \bar{W}^2 \) (or \( F^2 \) and \( F \bar{F} \)), and their covariant derivatives (see below). The action (3a) does not seem to have a nice supersymmetric generalization.

It is possible to rewrite the action (26) into the manifestly gauge-invariant and \( N = 1 \) supersymmetric form, by using the \( N = 1 \) supersymmetric gauge-covariant derivatives in superspace, which satisfy the standard \( N = 1 \) super-Yang–Mills constraints [13]

\[
\{ \nabla_\alpha , \nabla_\beta \} = \{ \bar{\nabla}_\alpha , \bar{\nabla}_\beta \} = 0 ,
\]

\[
\{ \nabla_\alpha , \bar{\nabla}_\beta \} = -2i 6_{\alpha \beta} \bar{\nabla}_\beta , \quad \left[ \nabla_\alpha , \bar{\nabla}_\beta \right] = 2i 6_{\alpha \beta} \bar{\nabla}_\beta ,
\]

where \( \bar{\nabla}_\alpha \) is the \( N = 1 \) covariantly-chiral gauge superfield strength,

\[
\bar{\nabla}_\alpha \bar{\nabla}_\alpha = 0 \Rightarrow \bar{\nabla}_\alpha \bar{\nabla}_\beta = \bar{\nabla}_\alpha \bar{\nabla}_\beta .
\]

Eq. (26) then takes the form

\[
\int d^4 x \ d^4 \theta (\text{tr} \, \phi e^{-2V} \bar{\phi} e^{2V}) , \quad \phi = W^2 .
\]
\[ S_{\text{NBI}} = \int d^4x \, d^2\theta (S) \text{tr} \, \tilde{\Phi} + \text{h.c.}, \]  
(31)

where the \( N = 1 \) covariantly-chiral Lagrangian \( \tilde{\Phi} \) is the perturbative (iterative) solution to the manifestly gauge-covariant and supersymmetric nonlinear superfield constraint
\[ \tilde{\Phi} = \frac{1}{2} \tilde{\Phi} \tilde{\nabla}^2 \tilde{\Phi} + \frac{1}{2} \tilde{\nabla}^2. \]  
(32)

It is not difficult to generalize Eqs. (31) and (32) further to the case of \( N = 2 \) supersymmetry, by doing a similar construction in \( N = 2 \) superspace. The standard \( N = 2 \) superspace constraints, defining the off-shell \( N = 2 \) supersymmetric Yang–Mills theories, are given by [15]
\[ \left\{ \nabla_{a\sigma}, \nabla_{\frac{1}{2}} \right\} = -2 \varepsilon_{a\sigma}^{\epsilon \eta} \nabla_{i\epsilon j\eta}, \]
\[ \left\{ \nabla_{a\rho}, \nabla_{\frac{1}{2}} \right\} = -2 \varepsilon_{a\rho}^{\epsilon \eta} \nabla_{i\epsilon j\eta}, \]
\[ \left[ \nabla_{a\sigma}, \nabla_{\frac{1}{2}} \right] = i \varepsilon_{a\sigma}^{\epsilon \eta} \nabla_{i\epsilon j\eta}, \]
\[ \left[ \nabla_{a\rho}, \nabla_{\frac{1}{2}} \right] = i \varepsilon_{a\rho}^{\epsilon \eta} \nabla_{i\epsilon j\eta}, \]
\[ \left\{ \nabla_{a\sigma}, \nabla_{\frac{1}{2}} \right\} = -2 i \delta_{a\sigma}^{i\epsilon j\eta} \nabla_{i\epsilon j\eta}, \]  
(33)

where the non-abelian \( N = 2 \) gauge superfield strength \( \nabla \) obeys the off-shell constraints (\( N = 2 \) Bianchi identities)
\[ \nabla_{a\sigma}, \nabla_{\frac{1}{2}} = 0 \quad \text{and} \quad \nabla_{i\epsilon j\eta}, \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \nabla_{i\epsilon j\eta}. \]  
(34)

I use the following book-keeping notation:
\[ \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \quad \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \quad \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \]
\[ \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \quad \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \quad \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \]
\[ \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \quad \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \quad \nabla_{i\epsilon j\eta} = \nabla_{i\epsilon j\eta}, \]  
(35)

where all symmetrizations have unit weight. The non-abelian generalization of the \( D^3 \) operator, which converts the (covariantly) anti-chiral \( N = 2 \) superfields into (covariantly) chiral \( N = 2 \) superfields, is most easily (and unambiguously) identified in the \( SL(4, \mathbb{C}) \) notation of Ref. [16], by combining fundamental \( SL(2, \mathbb{C}) \) and \( SU(2) \) indices into a single (fundamental) \( SL(4, \mathbb{C}) \) index \( a = (i, i) = 1, 2, 3, 4 \). The \( \nabla \) of Eq. (33) in the \( SL(4, \mathbb{C}) \) notation takes the familiar Dirac-type-form
\[ \nabla_{A\alpha} = 2 C_{A\alpha}^{\beta} \nabla_{\beta}, \quad \nabla_{A\alpha} \nabla_{\beta} = 0, \]  
(36)

with the constant metric \( C, \quad C^2 = 1 \) and \( C^T = C \). The desired gauge-covariant operator is just given by the “\( \gamma \)-type” top product
\[ \nabla^4 = \frac{1}{4!} \epsilon^{abcd} \nabla_{a\sigma} \nabla_{b\epsilon} \nabla_{c\eta} \nabla_{d\rho}. \]  
(37)

In the notation (35) it reads
\[ \nabla^4 = \frac{1}{24} (\nabla_{i\epsilon j\eta} - \nabla_{i\epsilon j\eta}, \nabla_{a\rho}^{* \beta}) - \frac{2}{3} \nabla^2. \]  
(38)

The \( N = 2 \) supersymmetric non-abelian Born–Infeld action is given by
\[ S_{\text{NBI}} = \frac{1}{4} \int d^4x \, d^4\theta (S) \text{tr} \, \hat{X} + \text{h.c.}, \]  
(39)

whose \( N = 2 \) covariantly chiral Lagrangian \( \hat{X} \) is the perturbative (iterative) solution to the \( N = 2 \) superfield constraint
\[ \hat{X} = \frac{1}{4} \hat{X} \nabla^4 \hat{X} + \hat{Y} \nabla^2. \]  
(40)

4. Conclusion

The proposed \( N = 1 \) and \( N = 2 \) supersymmetric NBI actions in components contain only even powers of \( F \), while they reduce to the known super-Born–Infeld actions in the abelian case. Both actions enjoy “auxiliary freedom” by keeping the auxiliary fields \( D \) (in the Wess–Zumino gauge) away from propagation, with \( D = 0 \) being a solution to their equations of motion.

Unlike the supersymmetric abelian BI actions (Section 2), their supersymmetric non-abelian counterparts (Section 3) are dependent of the gauge superfields not only via their gauge superfield strengths but also directly (via the gauge-covariant derivatives). This does not allow us to extend the notion of abelian electromagnetic duality to the supersymmetric NBI actions. Similarly, it is unclear to us whether our supersymmetric non-abelian BI actions admit any Goldstone–Yang–Mills interpretation.

It would be also interesting to investigate the structure of BPS solutions to the new supersymmetric NBI actions and find a precise relation between these actions and the non-abelian Dirac–Born–Infeld actions describing clusters of D3-branes with “deformed”
(non-linear) supersymmetry. A connection to noncommutative geometry seems to exist along the lines of Ref. [17] too.

Acknowledgement

I thank S.J. Gates Jr. and E.A. Ivanov for discussions. I also acknowledge kind hospitality extended to me at the University of Maryland in College Park during preparation of this paper.

References

Transversality of the logarithmic divergences in the classical finite temperature $SU(N)$ self-energy

A. Arrizabalaga*, B.J. Nauta, Ch.G. van Weert

Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

Received 28 July 2000; accepted 6 September 2000

Editor: P.V. Landshoff

Abstract

We show that the logarithmic divergences that appear in the classical approximation of the finite temperature $SU(N)$ self-energy are transverse. We use the Ward identities in linear gauges and the fact that the superficial degree of divergence $d$ of a classical diagram only depends on the number of loops $\ell$ via $d = 2 - \ell$. We comment on the relevance of this result to the construction of a low-energy effective theory beyond hard thermal loops.

© 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

As it is well known, low-momentum excitations in a high-temperature plasma behave classically. Nevertheless, to study these excitations one cannot simply replace the quantum thermal field theory that describes them by a classical thermal field theory, because of the appearance of ultraviolet Rayleigh–Jeans type divergences. One should think of a classical thermal field theory as an effective theory at large scales (larger than the typical interparticle distance $\sim h/T$). This involves the introduction of a cut-off $\Lambda$ in the momentum of the order of the temperature $\Lambda \sim T/h$. The resulting cut-off dependences reflect directly the divergences of the classical theory and indicate its different high-momentum behavior with respect to the quantum theory. One might hope that for low-momentum correlation functions this different behavior does not play a role so that the physics involved at the cut-off scale $\sim T/h$ is unimportant. However, this is not so as the high-momentum modes affect, through interactions, also the low-momentum sector of the theory in an essential way [1].

A general strategy to improve the classical theory is to include the dominant quantum contributions from the high-momentum (hard) modes. In this context it is important to understand the classical divergences, since they correspond to the dominant hard-mode contributions in the quantum theory. For instance, the linear divergences in classical non-abelian gauge theories have a one-to-one correspondence to the well-known hard thermal loops (HTLs) [2,3]. The fact that hard thermal loops have to be included in an effective theory for the soft (low-momentum) modes has been known since the work of Braaten and Pisarski [4]. These hard thermal loops have the following remarkable properties, namely, they are gauge invariant, they satisfy abelian-like Ward identities [5] and they allow a kinetic formulation in terms of a Vlasov equation [6].
Besides linear divergences, classical theories contain also logarithmic divergences starting at two-loop. (The explanation of how loops arise in a classical theory can be found in [7].) Higher-loop diagrams are superficially finite, although they may contain linear or logarithmic divergences in the form of one- or two-loop subdiagrams, respectively. Thus, we have two essential kinds of classical divergences, linear and logarithmic.

The relevance and the physical significance of the linear divergences (the HTLs) has been extensively studied, in particular, an effective action which incorporates them has been developed (see [8] or [9], for example).

A further step towards an effective theory beyond hard thermal loops would then be to include the logarithmic divergences (“log-divergences”) into the effective action. At this point some natural questions arise, namely: do the log-divergences have the same or similar properties as the linear divergences, namely the hard thermal loops? What are the quantum contributions corresponding to the log divergences?

In any case, an extra term in the effective action containing the physics of the logarithmic divergences would enter the equations of motion of the fields as a current, which must be conserved for the effective theory to be consistent. In this letter we will use the Ward identities for the self-energy at finite temperature in the equations of motion. For that we will use the current conservation at the level of two-point functions.

Gauge invariance provides us with identities among Green functions, the so-called Ward identities at one-loop. In different Green functions, the so-called Ward identities are then valid for the two cases $T = 0$ and $T 
eq 0$ [11,13–15].

The identities can be written for the self energy $\Pi_{\mu \nu}$ using the relation $(D^{\text{full}})^{-1} = (D^0)^{-1} - \Pi$. Now, a difference arises for the two cases $T = 0$ and $T 
eq 0$. This difference can be better seen in the covariant gauge. At zero temperature, $\Pi_{\mu \nu}$ must be a linear combination of the two available tensors $g_{\mu \nu}$ and $K_{\mu}K_{\nu}$, which immediately leads to the result that the self-energy is transverse, namely $K^\mu \Pi_{\mu \nu} = 0$. However, at finite temperature the difference arises due to the presence of a heat bath with four-velocity $U^\mu$. From (2) and the presence of $U^\mu$ the Lorenz structure of $D^{\text{full}}_{\mu \nu}$ allows different independent tensors combination of both $K_{\mu}$ and $U_{\mu}$, for instance $g_{\mu \nu}$, $K_{\mu}K_{\nu}$, $U_{\mu}U_{\nu}$ and $K_{\mu}U_{\nu} + U_{\mu}K_{\nu}$. More convenient are the dimensionless tensors $A_{\mu \nu}$, $B_{\mu \nu}$, $C_{\mu \nu}$ and $D_{\mu \nu}$ detailed in [10–13,15]. Here we will use the fact that both $A_{\mu \nu}$ and $B_{\mu \nu}$ are transverse to the four-momentum $K^\mu$, i.e. $K^\mu A_{\mu \nu} = 0$ and $K^\mu B_{\mu \nu} = 0$, whereas $C_{\mu \nu}$ and $D_{\mu \nu}$ are not. Moreover, $A_{\mu \nu}$ and $B_{\mu \nu}$ are respectively transverse and longitudinal to the spatial momentum $k$.

In general, in linear gauges ($F_\mu A^\mu = 0$) it is given by

$$F^\mu F^\nu D^{\text{full}}_{\mu \nu} = -\alpha,$$

where $\alpha$ is the gauge parameter in both cases. The identity is the same at $T = 0$ and $T \neq 0$ [11,13–15].

To demonstrate this, we will use the fact that both $A_{\mu \nu}$ and $B_{\mu \nu}$ are transverse to the four-momentum $K^\mu$, i.e. $K^\mu A_{\mu \nu} = 0$ and $K^\mu B_{\mu \nu} = 0$, whereas $C_{\mu \nu}$ and $D_{\mu \nu}$ are not. Moreover, $A_{\mu \nu}$ and $B_{\mu \nu}$ are respectively transverse and longitudinal to the spatial momentum $k$.

With these tensors the self-energy can be written as

$$\Pi_{\mu \nu} = \Pi_T A_{\mu \nu} + \Pi_L B_{\mu \nu} + \Pi_C C_{\mu \nu} + \Pi_D D_{\mu \nu},$$

where $\Pi_T$ and $\Pi_L$ denote the four-momentum transverse components (transverse and longitudinal to the spatial momentum respectively) and $\Pi_C$ and $\Pi_D$ are the nontransverse components. This decomposition of the self-energy in the above basis of tensors is not only valid for the covariant gauge, but also for the temporal and Coulomb gauges and, in general, for linear gauges that do not break rotational invariance [11,13–15].

From the Ward identity (2) and the decomposition (3) above one can derive a relation between the different transverse and nontransverse components of the self-energy (see, for example, [15]):

$$[\Pi_C(K)]^2 = \left[ K^2 + \Pi_L(K) \right] \Pi_D(K).$$

We note that this result does not imply that the self-energy is transverse. Indeed, it is well known that already at one-loop the self-energy is not transverse [14]. However, remarkably, the hard thermal loop
part of the self-energy is, i.e. $K^\mu \Pi^\mu_{\mu'}^{\text{HTL}} = 0$. This is due to the fact that HTLs satisfy abelian-like Ward identities [5].

The Ward identity (4) will be a starting point in our discussion on the transversality of the divergent parts of the classical self-energy.

### 2.1. Linear divergences

Let us now consider the classical approximation of SU(N) gauge theory, which is obtained by taking the $\hbar \to 0$ limit of the quantum theory. The classical theory is expected to be a good approximation at low energies because the classical and low-energy limit of the Bose–Einstein distribution function $n(\omega_k)$ yield the same result:

$$n(\omega_k) = \frac{1}{e^{\omega_k/T} - 1} \rightarrow n_{\text{cl}}(\omega_k) \equiv \frac{T}{\omega_k \hbar},$$

where $\omega_k = |k|$ is the frequency at wavenumber $k$. As mentioned in the introduction, the classical theory as an effective theory requires the introduction of a cut-off $\Lambda$ which appears in the calculation of diagrams, reflecting directly the divergences of the theory.

The linearly divergent terms in the classical theory correspond to the HTLs in the quantum theory [2]. This is so because the HTLs are proportional to the $\omega_0^2$, where $\omega_0 \sim g T \hbar^{-1/2}$ is the plasmon frequency. Thus, the HTLs behave as $1/\hbar$, which become the linear divergences in the classical theory as we take $\hbar \to 0$.

The fact that HTLs are transverse indicates that the linearly divergent terms should also be so. This can be checked by making use of (4). Consider the case $K^2 = K_\mu K^\mu \neq 0$ and let us start at one-loop. Since $\Pi_\mathcal{C} = 0$ at tree level, it begins at order $O(g^2)$ (one-loop). Now, from (4) we notice that $\Pi_\mathcal{D}$ should start at $O(g^4)$ (two-loops). The two-loop contribution $\Pi_\mathcal{D}^{[2]}$ (the superscript denotes loop order) is superficially log divergent, containing at most a linear subdivergence. Hence by (4) we see that the one-loop contribution $\Pi_\mathcal{C}$ cannot have a linear divergence. Thus, at one-loop both $\Pi_\mathcal{C}$ and $\Pi_\mathcal{D}$ vanish, and therefore,

$$K^\mu \Pi^{[1],\text{ln}}_{\mu
u} = 0,$$

as we expected from our considerations of hard thermal loops above. In fact, this is another way of showing that HTLs are transverse.

### 2.2. Logarithmic divergences

At one-loop there are no logarithmic divergences [2], so we consider the case of two-loop, i.e. $O(g^4)$. We first split the two-loop self-energy component $\Pi_\mathcal{D}^{[2]}$ in a logarithmically divergent part, a part that may contain a linear subdivergence and a finite part:

$$\Pi_\mathcal{D}^{[2]} = \Pi_\mathcal{D}^{[2],\text{log}} + \Pi_\mathcal{D}^{[2],\text{sublin}} + \Pi_\mathcal{D}^{[2],\text{fin}}.$$  \hspace{1cm} (7)

We insert this expression into the Ward identity (4). The terms in $\Pi_\mathcal{C}$ at the right-hand side that match with the $O(g^4)$ at the left-hand side are those corresponding to one-loop, which does not have logarithmic divergences, and therefore

$$\Pi_\mathcal{D}^{[2],\text{log}} = 0.$$  \hspace{1cm} (8)

We saw already that $\Pi_\mathcal{C}$ does not contain a linear divergence, thus also

$$\Pi_\mathcal{D}^{[2],\text{sublin}} = 0.$$  \hspace{1cm} (9)

Next, we consider $\Pi_\mathcal{C}$. Analogously to (7), we split it in terms of the different types of divergences

$$\Pi_\mathcal{C}^{[2]} = \Pi_\mathcal{C}^{[2],\text{log}} + \Pi_\mathcal{C}^{[2],\text{sublin}} + \Pi_\mathcal{C}^{[2],\text{fin}}.$$  \hspace{1cm} (10)

We use the Ward identity (4) at $O(g^8)$, which we may write as

$$(\Pi_\mathcal{C}^{[2],\text{log}} + \Pi_\mathcal{C}^{[2],\text{sublin}} + \Pi_\mathcal{C}^{[2],\text{fin}})^2 + 2 \Pi_\mathcal{C}^{[1],\text{fin}} \Pi_\mathcal{C}^{[3]}$$

$$= K^2 \Pi_\mathcal{D}^{[4]} + \Pi_\mathcal{A}^{[1]} \Pi_\mathcal{D}^{[3]} + \Pi_\mathcal{L}^{[2]} \Pi_\mathcal{D}^{[2]}.$$  \hspace{1cm} (11)

We now focus on the terms that could lead to a logarithmic divergence. We keep from (11) terms proportional to $(\log \Lambda)^2$. This results in

$$(\Pi_\mathcal{C}^{[2],\text{log}})^2 + 2 \Pi_\mathcal{C}^{[1],\text{fin}} \Pi_\mathcal{C}^{[3]}$$

$$= K^2 \Pi_\mathcal{D}^{[4]} + \Pi_\mathcal{A}^{[1]} \Pi_\mathcal{D}^{[3]}.$$  \hspace{1cm} (12)

As a consequence of (8), $\Pi_\mathcal{D}^{[2]}$ does not contain a log-divergence, therefore the last term on the r.h.s. of (11) does not contribute to (12). Let us consider the products of one- and three-loop contributions. Since at one-loop there are no log-divergences, the three-loop diagrams must contain a double log-divergence for these products to contribute. Now, schematically, the expression for a three-loop diagram is

$$\Pi_\mathcal{D}^{[3]}(P) = g^6 T^3 \int \frac{d^4 K}{(2\pi)^4} \frac{d^4 K'}{(2\pi)^4} \frac{d^4 K''}{(2\pi)^4} \times f^{(3)}(K, K', K'', P),$$  \hspace{1cm} (13)
where $K$, $K'$ and $K''$ are the internal momenta, $P$ is the external momentum, $g$ the coupling constant and $T$ the temperature. The result after the integration over any two arbitrary internal momenta can be regarded as either an expression for two disjunct one-loop diagrams or a two-loop diagram, with external lines depending on the other momenta. Consider for example the three-loop diagram in Fig. 1.

In the case that the integration over two arbitrary internal momenta ($K_0$ and $K_{00}$ in Fig. 1) corresponds to a two-loop diagram (as in Fig. 1b), it can at most give a single logarithmic divergence, which we denote as $\log \Lambda$. When it does, the integration over the remaining momentum ($K$ in Fig. 1) cannot give an extra $\log \Lambda$, since the superficial degree of divergence of the total diagram is $d = 2 - \ell = -1$. In the case that the integration over $K'$ and $K''$ does not give a logdivergence, the integration over $K$ could still lead to one $\log \Lambda$. Hence, a three-loop diagram can at most give a single log-divergence. Therefore, the product of one- and three-loop diagrams cannot contribute to (12).

The above argument can be repeated for the four-loop contribution to the self-energy. In this case there are four internal momenta. The result after integration over any three given internal momenta can be regarded as the expression for a disjunct two and one-loop diagram or three disjunct one-loop diagrams. Therefore it can at most give a single logarithmic divergence, and since a four-loop contribution to the self-energy is finite, the remaining integration cannot give an extra log-divergence and as in the case above, $\Pi^{[4]}_C$ cannot contribute to (12). Thus, we find from (12) that

$$\Pi^{[2],\log}_C = 0.$$  

(14)

Note that we cannot say that the two-loop contribution to $\Pi_C$ containing a linear divergence from one-loop subdiagrams equals zero, as we could for $\Pi_D$.

Since both $\Pi^{[2]}_C$ and $\Pi^{[2]}_D$ vanish, then we conclude that the logarithmically divergent part of the two-loop classical self-energy is transverse

$$K^{\mu} \tilde{\Pi}^{[2],\log}_{\mu\nu} = 0.$$  

(15)

Our argument does not hold for three- or higher-loop log-divergences. However, since those diagrams are superficially finite, the divergences can only come through two-loop subdiagrams and in general, we do not expect the whole contribution to the self-energy to be transverse. The important point is that, as we mentioned in the introduction, the essential divergences that appear in the classical theory are linear (at one-loop) and logarithmic (at two-loop) and they are both transverse.

### 3. Conclusions and outlook

Here we showed that the logarithmically divergent parts of the classical finite temperature $SU(N)$ self-energy are transverse. This also holds for the hard thermal loops, which correspond to classical linear divergences. Therefore we see that all divergences appearing in the classical self-energy are transverse. We would like to comment on the importance of this result regarding the construction of a low-energy effective theory beyond the HTL approximation.

Consider for instance the effective action which results from integrating out the hard modes with momenta $P > \mu$, with $\mu$ an intermediate scale such that $\omega_{\pi h} < \mu < T/h$. In a $h$ or high $T$ expansion the effective action for the soft modes would schematically be written as
\[ \Gamma_{\text{eff}} = g^2 T \left( \frac{T}{\hbar} - c_1 \mu \right) \mathcal{T}_{\text{HTL}} \]
\[ + (g^2 T)^2 \log \left( \frac{c_2 T}{\hbar \mu} \right) \mathcal{T}_{\log} \]
\[ + S_{\text{cl}} + O \left( g^2 \hbar \frac{c_2}{\mu} \frac{h \mu}{T} \right), \tag{16} \]

where \( c_1 \) and \( c_2 \) are constants that depend on the regularization scheme. The first term in the expansion, which corresponds to the HTLs, is proportional to \( \hbar^{-1} \), being thus linearly divergent in the classical limit \( \hbar \to 0 \). The second term is proportional to \( \log (T/\hbar) \) and so corresponds to the logarithmic divergences in the classical theory. The third term is the classical action and the other terms in the expansion are unimportant contributions in either a high-\( T \) or classical regime. Thus we see that in a \( \hbar \) or high-temperature expansion the next-to-leading order terms are given by the classical log-divergences.

A consistent scheme to include hard-mode contributions beyond HTLs in the classical theory seems to require the inclusion of terms that diverge in the classical limit. An effective action of the form (16) gives rise to currents in the equations of motion for the classical \( SU(N) \) gauge field

\[ \delta A_\mu S_{\text{cl}} = j^{\mu}_{\text{HTL}} + j^{\mu}_{\log}, \tag{17} \]

where \( j^{\mu}_{\text{HTL}} \sim \delta A_\mu \mathcal{T}_{\text{HTL}} \) and \( j^{\mu}_{\log} \sim \delta A_\mu \mathcal{T}_{\log} \). The current \( j^{\mu}_{\text{HTL}} \) generated by the HTLs is conserved. For consistency, it is necessary that the current \( j^{\mu}_{\log} \) generated by the log-divergences is also conserved. Our result (15) shows that the logarithmic divergent part of the self-energy is transverse, which implies that \( j^{\mu}_{\log} \) is indeed conserved. We stress that this is a special property at finite temperature that should not generally be expected, and in fact, this result encourages the study of the classical logarithmic divergences towards

the construction of a feasible low-energy effective theory beyond hard thermal loops.

References

Saturation of low-energy antiproton annihilation on nuclei

A. Gal a,*, E. Friedman a, C.J. Batty b

a Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel
b Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, UK

Received 13 July 2000; received in revised form 12 August 2000; accepted 30 August 2000

Editor: L. Montanet

Abstract

Recent measurements of very low-energy \( p_L < 100 \text{ MeV}/c \) \( \bar{p} \) annihilation on light nuclei reveal apparent suppression of annihilation upon increasing the atomic charge \( Z \) and mass number \( A \). Using \( \bar{p} \)-nucleus optical potentials \( V_{\text{opt}} \), fitted to \( \bar{p} \)-atom energy-shifts and -widths, we resolve this suppression as due to the strong effective repulsion produced by the very absorptive \( V_{\text{opt}} \). The low-energy \( \bar{p} \)-nucleus wavefunction is kept substantially outside the nuclear surface and the resulting reaction cross section saturates as function of the strength of \( \text{Im} V_{\text{opt}} \). This feature, for \( E > 0 \), parallels the recent prediction, for \( E < 0 \), that the level widths of \( \bar{p} \) atoms saturate and, hence, that \( \bar{p} \) deeply bound atomic states are relatively narrow. Antiproton annihilation cross sections are calculated at \( p_L = 57 \text{ MeV}/c \) across the periodic table, and their dependence on \( Z \) and \( A \) is classified and discussed with respect to the Coulomb focussing effect at very low energies. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 24.10.Ht; 25.43.+t; 25.60.Dz
Keywords: Antiproton annihilation; Low-energies; Optical potentials; Saturation

1. Introduction

Experimental results for antiproton annihilation cross sections at very low energies \( (p_L < 100 \text{ MeV}/c) \), below the \( \bar{p}p \rightarrow \bar{p}n \) charge-exchange threshold, have recently been reported for light nuclei \([1–3]\). At these energies the total \( \bar{p} \) reaction cross section consists only of \( \bar{p} \) annihilation. Bianconi et al. \([3]\) reported the first ever measurements of \( \bar{p} \) annihilation on Ne in the momentum range of 53–63 MeV/c. Comparing with other data, they showed that whereas at relatively higher energies \( (p_L \approx 200–600 \text{ MeV}/c) \) ratios of \( \bar{p} \) annihilation cross sections on different nuclei exhibit the well-known \( A^{2/3} \) strong-absorption dependence, such ratios at very low energies defy any simple, obvious regularity. It has been demonstrated that the ‘expected’ \( ZA^{1/3} \) dependence of these cross sections on the atomic charge \( Z \) and mass number \( A \) is badly violated \([3,4]\). Antiproton annihilation cross sections at very low energies simply do not rise with \( A \) as fast as is anticipated. For example, over the whole mass range studied so far (H, D, \(^4\text{He}, \text{Ne})\), the Ne/H ratio of total annihilation cross sections is at least 6 times smaller than expected.

In the present letter we study low-energy \( \bar{p} \) annihilation on nuclei, using the optical model approach. Optical potentials have been very successful in describing strong-interaction effects in hadronic atoms \([5]\), including \( \bar{p} \) atoms \([6]\). It has been noted, for pions, that the total reaction cross sections at low energies
are directly related to the atomic-state widths, and that once a suitable optical potential is constructed by reasonably fitting it to the atomic level shifts and widths in the negative energy bound-state domain, these total reaction cross sections are reliably calculable [7,8]. The recent publications [1 – 3] of experimental results of total cross sections for $\bar{p}$ annihilation on nuclei at very low energies raise the intriguing possibility of connecting these two energy regimes in a systematic way also for antiprotons. However, most of the data on annihilation cross sections for $\bar{p}$ are for very light nuclei, where the concept of a rather universal optical potential that depends on $A$ and $Z$ only through the nuclear densities is questionable. For this reason we use optical potentials, in the present work, mostly for crossing the $E = 0$ borderline within the same atomic mass ($A$) range, from bound states to scattering. This optical model approach is different from the allegedly model-independent scattering length approximation [9] which we have found to be less useful. We defer a discussion of the latter statement to a separate publication.

Among the targets used in the $\bar{p}$ annihilation measurements [1 – 3], the only ‘real’ nuclei are $^4$He and Ne, in the sense that the conventional optical model is not expected to be applied to the lighter targets of hydrogen or deuterium. In the present work we show that the very low-energy $\bar{p}$ total annihilation cross sections on $^4$He [2] and on Ne [3] are reproduced well by reasonable optical potentials fitted separately to the $\bar{p}$ atomic data at the relevant atomic mass range. These potentials are strongly absorptive, which leads to a remarkable saturation of the total reaction cross section with increasing $A$. Strong absorption has very recently been shown [10,11] to lead also to saturation of the widths of $\bar{p}$ atomic states and to the prediction of relatively narrow deeply bound $\bar{p}$ atomic states. In view of the close analogy between bound-state widths and total reaction cross sections (see Eqs. (1), (2)), it should come as no surprise that the underlying physics is the same. In fact, several $\bar{p}D$ calculations [12 – 14] have found that the $\bar{p}$ absorptivity in deuterium, as suggested by the magnitude of the imaginary part of the $\bar{p}D$ s-wave scattering length, is weaker than in hydrogen due partly to the repulsive effect of absorption. Lastly, we use $\bar{p}$ optical potentials fitted to a comprehensive set of atomic data across the periodic table to predict $\bar{p}$ total annihilation cross sections at very low energies for a wide range of $A$ values, with the exception of very light targets. We show such calculated cross sections at $p_L = 57$ MeV/c and discuss their $A$ and $Z$ dependence, particularly with respect to the $ZA^{1/3}$ dependence arising from Coulomb focussing in the limit of very low energies. We also demonstrate the extent to which this dependence is violated at 57 MeV/c.

2. Optical potentials

In the present work we aim at connecting $\bar{p}$ at energies slightly below threshold, with $\bar{p}$ annihilation on nuclei at very low energies above threshold, using an optical potential $V_{\text{opt}}$. Assuming for simplicity a Schrödinger-type equation, the width $\Gamma$ of an atomic level is given (non-perturbatively) by:

$$\frac{\Gamma}{2} = -\frac{\int \text{Im} \ V_{\text{opt}}(r)|\psi(r)|^2 \, dr}{\int |\psi(r)|^2 \, dr}.$$  

(1)

Here $\psi(r)$ is the $\bar{p}$ full atomic wavefunction. The corresponding expression for the total reaction cross section at positive energies is

$$\sigma_R = -\frac{2}{\hbar v} \int \chi(r)^2 \text{Im} \ V_{\text{opt}}(r) \, dr.$$  

(2)

Here $\chi(r)$ is the $\bar{p}$-nucleus elastic scattering wavefunction and $v$ is the c.m. velocity.

There exist several optical model potentials [5,6] parameterized in terms of nuclear densities which are quite successful in reproducing the strong interaction effects in not too light nuclei, e.g., fitting all data for nuclei heavier than carbon. However, most of the good quality data on $\bar{p}$ annihilation at very low energies are for very light nuclei where a straightforward application of the potential deduced from heavier targets is not expected to yield good fits. We have therefore started the present analysis by studying $\bar{p}$ atoms of $^3$He.

The $\bar{p}$ nucleus optical potential used here is given by the ‘$1p$’ expression [5]

$$2\mu V_{\text{opt}}(r) = -4\pi \left(1 + \frac{\mu}{m} \right) b_0 \rho(r),$$  

(3)

where $m$ is the nucleon mass, $\mu$ is the reduced $\bar{p}$-nucleus mass, $b_0$ is a complex parameter obtained from fits to the data and $\rho(r)$ is the nuclear density.
distribution normalized to $A$. Trying to fit level shift and width data in $^3$-$^4$He [15] for the $2p$ and $3d$ states simultaneously, we always ended up with the calculated width of the $2p$ state in $^4$He being too small, thus contributing an unacceptably large value to the total $\chi^2$ of the fits. Handling each isotope separately did not change this situation and we then decided to fit only the data for the $2p$ states for the two isotopes. This is somewhat unfortunate as one expects to be able to use $l$-independent potentials, as is indeed the case with heavier targets [5,6]. However, since our main concern here was to use atomic potentials at (very low) positive energies where $d$ and higher partial waves contribute very little in light nuclei, we considered this procedure acceptable. Note that we are using the same potential for all the partial waves in the positive energy regime.

Very good fits to the data with reasonable values for the complex parameter $b_0$ could be obtained only when a ‘finite range’ was introduced in the form of a Gaussian folding of a $\vec{p}N$ interaction into the nuclear density distributions. This is in contrast to the case of heavier targets where such a procedure was not necessary. Nevertheless, we consider the resulting potential, with $b_0 = 0.49 + i3.0$ fm and a Gaussian with a range parameter of 1.4 fm folded into the nuclear density distribution (hereafter referred to as potential (a)), as quite reasonable. It fits within the errors the measured strong interaction shifts and widths of the $2p$ state in antiprotonic $^3$He and $^4$He.

3. Results and discussion

Turning to positive energies, we first calculate the total reaction (annihilation) cross section for 57 MeV/c $\vec{p}$ on $^4$He, in order to make it possible to compare results for He and Ne [3]. Using the above potential (a), the calculated cross section is 901 mb, in excellent agreement with the interpolated value of 915 $\pm$ 39 mb [3]. For incident momenta of 47.0 and 70.4 MeV/c we calculate values of 1060 and 771 mb, respectively, where the experimental values [2] are 979 $\pm$ 145 and 827 $\pm$ 38 mb, respectively. Reasonably good agreement is therefore established between experiment and predictions made with a potential derived from fits to $\vec{p}$ atoms of $^3$-$^4$He.

Fig. 1 demonstrates the extreme strong-absorption conditions which are relevant to the $\vec{p}$ nucleus interaction at very low energies (and for $\vec{p}$ atoms). It shows calculated reaction cross sections for $\vec{p}$ at 57 MeV/c on $^4$He and Ne as function of the strength parameter $\text{Im}b_0$ of the optical potential (a).

Fig. 1. Calculated total reaction cross sections for 57 MeV/c $\vec{p}$ on $^4$He and Ne as function of the strength parameter $\text{Im}b_0$ of the optical potential (a).
Fig. 2. Calculated $\bar{p}$ total reaction cross sections (open circles) at 57 MeV/c for potentials (a) and (b), and for potential (b) but without the Coulomb interaction. Also shown are the two data points for $^4\text{He}$ and Ne.

Fig. 2 shows calculated $\bar{p}$ reaction cross sections at 57 MeV/c across the periodic table. The dotted-dashed line is for the above mentioned potential (a) which is expected to be valid only in the immediate vicinity of He. The dashed line (b) is for the first potential from Table 7 of Ref. [5] which fits $\bar{p}$ atom data over the whole periodic table, starting with carbon. This potential is not expected to fit data for very light nuclei, and indeed it does not fit the $^4\text{He}$ annihilation cross section. However, it is noteworthy that for $A > 20$ the two potentials predict almost the same cross sections and certainly display a very smooth and similar dependence on $A$. A smooth $A$ dependence of the calculated cross sections is generally expected once several partial waves contribute, as we elaborate below. Also shown in the figure is the recent experimental result [3] for Ne, with very limited accuracy. Furthermore, for $A > 20$ the points along the solid line are the calculated $\bar{n}$-nucleus total reaction cross sections, obtained from potential (b) by switching off the Coulomb interaction. The solid line is a fit to an $A^{1/3}$ power law which appears to be appropriate to strong absorption of uncharged particles at these very low energies, in contrast to the prediction of approximate $A$ independence made in Ref. [16] for $\bar{n}$ annihilation at ultra low energies where only $s$ waves contribute. We checked that by reducing the absorptivity by 3–4 orders of magnitude, a proportionality of $\sigma_R$ to $A$ as expected for weak absorption, is indeed observed. Comparing the dashed line for negatively charged particles with the solid line for uncharged particles, it is clear that the $\sigma_R$ values obtained by including the Coulomb interaction are considerably enhanced with respect to the ones obtained without it. Attempting to fit the enhancement by a $Z^\alpha$ power law, we find that $\alpha$ varies roughly between 1/3 for Ne to 1/2 for Pb. Attempts to identify more precisely this enhancement across the periodic table have been recently addressed in Ref. [17].

Finally, we discuss the $ZA^{1/3}$ scaling considered in Refs. [3,4] and in references quoted therein. We note that an attractive Coulomb potential causes focussing of partial-wave trajectories onto the nucleus, an effect which at very low energies may be evaluated semiclassically (see Ref. [18] for the applicability of the semiclassical approximation to Coulomb scattering). The maximal orbital angular momentum $l_{\text{max}}$ for which the $\bar{p}$-nucleus distance of closest approach is smaller than the nuclear radius $R$, at sufficiently low energy is given by

$$l_{\text{max}} + 1/2 \approx 2\eta k R,$$

(4)

where $k$ is the c.m. momentum and

$$\eta = 2Z/k_L a_B^{(p)}$$

(5)

is the corresponding Coulomb parameter. Here $k_L = p_L/h$, where $p_L$ is the $\bar{p}$ laboratory momentum, and $a_B^{(p)}$ is the Bohr radius of the $\bar{p}$ atom. Since $2\eta \gg k R$ at very low energies and for high values of $Z$, the value of $l_{\text{max}}$ greatly exceeds the value $k R$ which is appropriate to uncharged particles ($\eta = 0$). For
example, on Pb at $p_L = 57$ MeV/c, $l_{\text{max}}$ is about 6 for $\bar{p}$, but is less than 2 for $\bar{n}$. Therefore, for strong absorption, where all the lower partial waves are totally absorbed, one obtains

$$\sigma_R \approx \frac{\pi}{k^2} \sum_{l=0}^{l_{\text{max}}} (2l + 1) \approx \frac{2\pi}{kR} R^2 \gg \pi R^2,$$

(6)

which we rewrite as

$$\sigma_R \approx (4\pi r_0 / a_B^{(p)})(ZA^{1/3} / k k_L),$$

(7)

with $r_0$ defined by $R = r_0 A^{1/3}$. A similar approach was used by Fälldt and Pilkuhn [19] to calculate Coulomb corrections to $\pi^+$-nucleus total cross sections near the 3,3 resonance. In the present case of very low energies their correction becomes the dominant term, and the resulting reaction cross section on the r.h.s. of Eq. (6) is substantially higher than the black-disk value $\sigma_R = \pi R^2$ which for uncharged projectiles may be perceived as the unitarity limit of $\sigma_R$. Indeed, the calculated saturation plateau values of $\sigma_R$ in Fig. 1 are several times larger than the appropriate black-disk values.

In Fig. 3 we plot, for potential (b), ratios of calculated total reaction cross sections at 57 MeV/c to the scaling parameter $S = Z A^{1/3} / k k_L$. It is clear that $\sigma_R$ does not scale with $S$ across the periodic table. Also plotted is the asymptotic value

$$\sigma_R / S \rightarrow 4\pi r_0 / a_B^{(p)} \approx 0.273,$$

(8)

with $r_0 = 1.25$ fm and $a_B^{(p)} = 57.6$ fm. For a given $\bar{p}$ beam energy this asymptotic value, arising from Coulomb focusing, is approached very slowly upon increasing $Z$.

Before concluding we briefly mention other derivations of the $ZA^{1/3}$ dependence of $\sigma_R$. The proportionality to $Z$ is usually derived from the well-known Coulomb enhancement Gamow factor which at low energies is well approximated by $2\pi \eta$ [17]. However, at these very low energies, the Coulomb wavefunctions are far from being constant over the nuclear volume, so that the applicability of the Gamow factor appears questionable. The origin of a possible proportional relation of $\sigma_R$ to $A^{1/3}$ is even less clear: for $s$-wave dominated $\bar{p}$ annihilation, as considered in Refs. [3, 4], it is due to the imaginary part of the $\bar{p}$ scattering length which is assumed to closely follow the nuclear radius $R$. However, for $\bar{p}$ strongly absorptive potentials this imaginary part has invariably been found to be about 1 fm across the periodic table [20] (see also Ref. [9]), considerably less than $R$. It is inconceivable that a proportionality to $R$ can arise unless several partial waves contribute significantly.

4. Conclusions

In conclusion, we have shown that the recently reported annihilation cross sections for $\bar{p}$ on $^4$He and Ne at 57 MeV/c are reproduced very well by optical potentials which fit $\bar{p}$ atomic data in the respective mass ranges (Fig. 2). For these strongly absorptive potentials, the asymptotic $ZA^{1/3}$ dependence of the total reaction cross section was derived from the Coulomb focusing effect at very low energies, but at 57 MeV/c it was found not to be satisfied across the periodic table except for unrealistically high values of $A$ (Fig. 3). The correct dependence is determined by the saturation property of total reaction cross sections.
at very low energies (Fig. 1). The apparent suppression of $\bar{p}$ total annihilation cross sections measured at $p_L < 100\text{ MeV}/c$ is a direct consequence of the strong absorption, and in particular its saturation property. Predictions have been given for $\bar{p}$ total annihilation cross sections at 57 MeV/c across the periodic table which would complement and enrich the information deduced from the measurements already available for $\bar{p}$ atoms.

A more extended discussion of optical model calculations and their implications for $\bar{p}$ atoms and $\bar{p}$ annihilation on nuclei, including a connection to $\bar{p}p$ annihilation at $p_L < 200\text{ MeV}/c$, is included in a forthcoming publication [21].

Acknowledgements

CJB wishes to thank the Hebrew University for support for a visit during which this work was started.

This research was partially supported by the Israel Science Foundation.

References

Isomer spectroscopy of neutron rich $^{190}$W$^{116}$

Zs. Podolyák$^{a,*}$, P.H. Regan$^a$, M. Pfützner$^b$, J. Gerl$^c$, M. Hellström$^d$, M. Caamaño$^a$, P. Mayet$^c$, Ch. Schlegel$^c$, A. Aprahamian$^a$, J. Benlliure$^e$, A.M. Bruce$^f$, P.A. Butler$^g$, D. Cortina Gil$^c$, D.M. Cullen$^h$, J. Döring$^c$, T. Enqvist$^c$, F. Rejmund$^h$, C. Fox$^g$, J. Garcés Narro$^a$, H. Geissel$^c$, W. Gelletly$^a$, J. Giovannazzo$^i$, M. Göriska$^c$, H. Grawe$^c$, R. Grzywacz$^j$, A. Kleinböhl$^c$, W. Korten$^k$, M. Lewitowicz$^l$, R. Lucas$^k$, H. Mach$^m$, M. Mineva$^d$, C.D. O’Leary$^g$, F. De Oliveira$^l$, C.J. Pearson$^a$, M. Rejmund$^n$, M. Sawicka$^b$, H. Schaffner$^c$, K. Schmidt$^c$, Ch. Theisen$^k$, P.M. Walker$^a$, D.D. Warner$^o$, C. Wheldon$^a$, H.J. Wollersheim$^c$, S.C. Wooding$^c$, F.R. Xu$^a$

$^a$ Department of Physics, University of Surrey, Guildford, GU2 7XH, UK
$^b$ Institute of Experimental Physics, Warsaw University, PL-00661 Warsaw, Poland
$^c$ GSI, Planckstrasse 1, D-64291 Darmstadt, Germany
$^d$ Division of Cosmic and Subatomic Physics, Lund University, Lund, SE-22100, Sweden
$^e$ Departamento de Física de Partículas, University of Santiago de Compostela, Santiago de Compostela, Spain
$^f$ School of Engineering, University of Brighton, Brighton BN2 4GJ, UK
$^g$ Oliver Lodge Laboratory, Department of Physics, University of Liverpool, Liverpool, L69 7ZE, UK
$^h$ IPN, 91406 Orsay Cedex, France
$^i$ CEN Bordeaux-Gradignan/CNRS, F-33175 Gradignan Cedex, France
$^j$ University of Tennessee, Knoxville, TN 37996, USA
$^k$ CEA Saclay, DSM/DAPNIA/SPbN, F-91191 Gif-sur-Yvette Cedex, France
$^l$ GANIL, BP 5027, F-14021 Caen Cedex, France
$^m$ Department of Neutron Research, Uppsala University, S-61182, Nyköping, Sweden
$^n$ CSNSM, 91405 Orsay Cedex, France
$^o$ CLRC Daresbury Laboratory, Warrington, WA4 4AD, UK

Received 19 April 2000; received in revised form 18 July 2000; accepted 11 September 2000
Editor: V. Metag

Abstract

Gamma-rays de-exciting a millisecond isomer in the neutron-rich nucleus $^{190}$W$^{116}$ have been observed following relativistic projectile fragmentation of a 1 GeV per nucleon $^{208}$Pb beam. The isomeric decay populates the ground-state rotational band, with energies that indicate a significant deformation change at $Z = 74$ for the $N = 116$ isotones. The successful application of

* Corresponding author.
E-mail address: z.podolyak@surrey.ac.uk (Zs. Podolyák).

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.
PESC0370-2693(00)01051-0
A significant amount of information on the global behaviour of nuclei can be obtained from the energy spacing of the lowest lying states in even–even systems. For example, it is well established that the excitation energy of the first \( 2^+ \) state can be used to infer the extent of quadrupole deformation [1,2]. Similarly, the ratio of the excitation energies of the first \( 4^+ \) and \( 2^+ \) states can be used in a simple model [3], to distinguish between an axially symmetric deformed rotor (with an energy ratio of 3.33), a spherical, vibrational nucleus (2.0) and a triaxial rotor (\( \sim 2.5 \)). A number of attempts have been made to correlate this macroscopic nuclear behaviour with the valence numbers of protons and neutrons [4–7]. In general, for nuclei, where the spectroscopic information is available, the systematics follow smooth trends, providing sub-shell closures are taken into account [5–7].

The global properties of nuclei are tested by obtaining information on the low lying excitations of nuclei across as wide a range as possible. In practice, the heavy, neutron rich, rare earth nuclei are particularly difficult to access experimentally. These nuclei are inaccessible with stable beam/target fusion–evaporation reactions and are too heavy to be populated in fission. Although deep inelastic reactions have recently been used to access high spins in nuclei with a few neutrons more than the most neutron-rich stable isotopes [8], this technique is limited by a general difficulty in channel selection. Projectile fragmentation at intermediate and relativistic energies has proven to be an efficient and selective method of populating nuclei far from the valley of stability [9,10] and is an ideal tool for the study of high-spin metastable states in heavy neutron-rich nuclei [10].

Discrete \( \gamma \)-ray spectroscopy cannot be carried out at the production target in the case of exotic nuclei, since the intensity of the background is many orders of magnitude higher than that of the radiation emitted by the fragments of interest. However, these fragments can be transported and identified on an ion-by-ion basis. This method allows the study of the excited states only in those nuclei which are produced during the fragmentation in isomeric states, with lifetimes long enough to survive the time of transportation between the production target and the \( \gamma \)-ray detection system. The deformed rare-earth nuclei in the A \( \sim 170–190 \) region are well known for their so-called high-\( K \) isomeric states [11]. These isomers occur due to the maximal angular momentum coupling of a number of single nucleon orbitals with large angular momentum projections, \( \Omega \), on the nuclear symmetry axis. Such high-\( K \) \( (K = \sum_i \Omega_i) \) intrinsic states are often hindered in their decay to the low-\( K \) rotational states since this involves a large alteration in the total angular momentum orientation, which is forbidden via low multipolarity transitions if the \( K \) projection is a good quantum number. This region of \( K \)-isomerism in the rare earth nuclei has long been predicted to extend into the neutron-rich Hf/W/Os region with \( A \sim 180–200 \) [12,13]. However, until now, the spectroscopic information available on such neutron-rich nuclei has been severely limited due to the difficulty in synthesising these systems at the medium to high spins required to populate such states.

This letter reports on our discovery of a high-spin isomeric decay in the neutron-rich nucleus \( ^{190}W_{116} \), populated following a relativistic projectile fragmentation reaction. The isomer is interpreted as decaying into the ground-state band with energy spacings that suggest a significant deformation change. This isotope is four mass units heavier than the most neutron-rich stable tungsten isotope and has eight neutrons more than the heaviest tungsten isotope which can be measured at high spins with fusion–evaporation reactions using stable beam/target combinations [14]. \( ^{190}W \) was the heaviest isotope of this element which had been experimentally synthesised in \((n,n2p)\) and \((p,3p)\) reactions, although no excited states had been observed [15].

In order to investigate isomeric states in neutron-rich nuclei in the Hf/W/Os region, a beryllium target of thickness 1.6 g/cm\(^2\) was bombarded with a 1
GeV/nucleon. \(^{208}\)Pb beam provided by the SIS accelerator in GSI. The typical, on-target beam intensity was \(2 \times 10^8\) lead ions per 12 second beam spill. The nuclei of interest were separated and identified using the FRagement Separator (FRS) \([16]\) operated in standard achromatic mode. An additional degree of selectivity for the ions reaching the final focus of the FRS was achieved by placing a wedge-shaped aluminium degrader in the intermediate focal plane of the separator. Niobium foils of thickness 221 mg/cm\(^2\) were placed after both the target and the degrader in order to maximise the electron stripping. Typically 90\% of the ions of a given isotope were fully stripped.

For the experiment, the mass-to-charge ratio of the ions, \(A/Q\), was determined from their time of flight in the second part of the FRS. The measured change of the magnetic rigidity of ions before and after they passed through this degrader was used to obtain unambiguous charge identification. The energy deposition of the identified fragments was measured as they passed through a gas ionisation chamber. Following this, they were slowed down in a variable thickness aluminium degrader and finally stopped in a 4 mm thick aluminium catcher. The atomic number of the ions was determined using three methods: (i) by measuring how the position of the nuclei in the final focal plane varies with \(A/Q\); (ii) from their energy loss in the ionisation chamber; and (iii) from the emitted X-rays following internal conversion decays of states below isomers. The identification procedure is described in more detail in Ref. \([17]\).

The total transmitted ion rate was kept below 1 kHz in order to minimise dead-time losses. Scintillator detectors were placed both in front of and behind the catcher, allowing the offline suppression of those fragments destroyed in the slowing down process or those which were not stopped in the catcher (totaling \(\approx 20\%\)). The catcher was surrounded by four clover-style germanium detectors. The photopeak \(\gamma\)-ray efficiency of this array was measured to be 8\% at 661 keV. The effective detection efficiency was however reduced in practice due to the ions stopping in the catcher, which gave rise to a prompt burst of low energy X-rays and bremsstrahlung. This had the effect of “blinding” on average 10 of the total 16 detector elements in each event.

At the catcher, the prompt and delayed \(\gamma\)-rays in coincidence with the individually identified fragment were recorded. The time difference was measured between the implantation of the fragment in the stopper (as measured by the time signal from a plastic scintillator placed in front of the stopper) and a subsequently detected \(\gamma\)-ray in the array, over ranges of 8 \(\mu\)s and 80 \(\mu\)s. Since the time of flight through the FRS was approximately 300 ns, this setup allowed the detection of isomeric decays with half-lives in the typical range of 100 ns to several hundred microseconds. Note, however, that shorter lifetimes could also be detected if the decay branch by electron conversion was hindered for specific charge states of the ion \([9]\). This technique is not suitable for investigation of \(\beta\)-decay in this mass region, where half-lives of seconds or more are anticipated.

The results on \(^{190}\)W were obtained from two different magnetic rigidity settings. Approximately 85\% of the data come from a setting optimised to select fully stripped nuclei centred on the maximal transmission of \(^{191}\)W, while the remaining 15\% of the data come from a second setting, centred on the nucleus \(^{184}\)Lu. In this latter case, the hydrogen-like \(^{190}\)W ions (those containing a single atomic electron) were simultaneously transmitted through the FRS. A total of \(4.5 \times 10^4\) \(^{190}\)W ions were collected with an average transmission rate of 0.2 ions/s. The identification plot for the setting optimised to select the fully stripped nuclei around \(^{191}\)W is presented in Fig. 1. The detection of the previously identified \(\gamma\)-rays following the de-excitation of the \(I^+ = 7^-\tau = 20\) ns isomer in \(^{200}\)Pt \([18]\) was used to confirm the calibration of the particle identification. This isomer provides a clear, internal calibration for both \(A/Q\) and \(Z\) values using the time of flight and energy loss in the ionisation chamber, respectively.

The \(\gamma\)-ray spectrum observed in delayed coincidence with \(^{190}\)W ions is shown in Fig. 2. The weak population of this exotic nucleus negates the use of \(\gamma-\gamma\) data to prove the coincidence relationships between these transitions. Within experimental errors, the \(\gamma\)-rays at 207, 357, 485, 591 and 695 keV have the same intensity when corrected for detection efficiency. The systematics of this deformed region strongly suggest that they form a rotational cascade (i.e., they have E2 character), built on the ground state of \(^{190}\)W (see Fig. 3). As Fig. 3 shows, the energies of the assumed rotational cascade in \(^{190}\)W appear to be consistent with a reduction in collectivity compared to the lighter isotopes. The low energy part of the spectrum shown in Fig. 2 shows the tungsten X-rays and a 46 keV \(\gamma\)-
Fig. 1. Identification plot of the fragments by the FRS for (a) hydrogen like ions ($Q = Z - 1$) and (b) fully stripped ions ($Q = Z$) for the setting centred on fully stripped $^{191}$W. Panel (c) shows the gamma-spectrum recorded in coincidence with the events labelled as ‘200Pt’ in (a). All the indicated $\gamma$-ray transitions belong to $^{200}$Pt. The strongest transitions were already known [18]; the others were found in the present experiment and proved to belong to $^{200}$Pt by $\gamma-\gamma$ coincidence measurement. (d) Z spectra obtained from the ionisation chamber. From below: total Z spectrum, events corresponding to ‘200Pt’ in (a), and events corresponding to ‘190W’ in (b). As the latter two spectra show, there is a small contamination from nuclei with $Z - 1$, namely $^{190}$X [10], which can be removed by gating on the ionisation chamber.

The mean lifetime for the isomeric state was determined to be in the range between 135 $\mu$s and 4.5 ms. The time distribution of the $\gamma$-ray transitions identified in $^{190}$W is shown in the inset of Fig. 2. The lifetime is clearly longer than the 80 $\mu$s range of the Time-to-Amplitude Converter (TAC). By taking the ratio of counts in the first and last third of the 80 $\mu$s time window, an estimate of the mean lifetime for the isomer of $\tau = 390^{+35}_{-25}$ ms was obtained. This lifetime was obtained for the gamma-ray transitions with the background subtracted, therefore it is excluded that the long lifetime is related to the decays of long-lived activities accumulated in the catcher. An upper limit of
Fig. 2. Delayed $\gamma$-ray spectrum for $^{190}\text{W}$ with the condition that the $\gamma$-rays were detected between 1 and 70 $\mu$s after the ion implantation. The dispersion on the X-axis is 1 keV per channel. The insets show the relative intensities and the summed time spectrum associated with the labelled $\gamma$-ray transitions. The two lines on the time spectrum correspond to the upper and lower limits for the lifetime of $\tau = 4.5$ ms and $\tau = 135$ $\mu$s, respectively.

Fig. 3. Proposed decay scheme for $^{190}\text{W}$ as obtained from the present work, with the systematics of the yrast band energies of the even–even tungsten isotopes from mass 180 upto 188.

4.5 ms for the mean lifetime can be obtained from the absolute $\gamma$-ray intensities, supposing that all the $^{190}\text{W}$ nuclei are produced in the isomeric state.

Assuming that the observed 207 keV transition represents the yrast $2^+ \rightarrow 0^+$ decay, the Grodzins empirical estimate [1] for the ground-state deformation of $^{190}\text{W}$ yields a value of $\beta_2 = 0.17$. Fig. 4(a) shows the results of deformed Woods–Saxon–Strutinsky potential-energy-surface calculations for $^{190}\text{W}$ (as described in [19]) which predict a ground-state deformation of $\beta_2 = 0.17$ ($\beta_4 = -0.06$ and $\gamma = 0^\circ$), remarkably consistent with that obtained from the Grodzins estimate. The calculations also predict a shallow nature to this minimum with respect to the triaxial de-
Fig. 4. Potential energy surface calculations for (a) the ground-state configuration and (b) the $K^\pi = 10^-$ configuration for $^{190}$W. The energy difference between two successive contour lines is 200 keV. (c) Blocked BCS calculations for $^{190}$W, showing the favoured nature of the $K^\pi = 10^-$ two quasineutron configuration.

gree of freedom, which is discussed below. In order to assign a single particle configuration to the isomeric state, blocked-BCS (Bardeen–Cooper–Schrieffer) calculations as described in reference [13] were performed for $^{190}$W, using the same deformation parameters as obtained from the deformed Woods–Saxon–Strutinsky calculations. The quasiparticle energies and the pairing force strengths were fitted to known states of the neighbouring $^{190,191,192}$Os and $^{191}$Re nuclei (consistent with extrapolation from Ref. [20]). As shown in Fig. 4, there is predicted to be a low-lying two-quasineutron state with a two-quasineutron $K^\pi = 10^-$, $9/2^-$[505] $\otimes$ $11/2^+$[615] Nilsson configuration. Configuration-constrained potential-energy-surface calculations [21] have been performed for this $K^\pi = 10^-$ state. The results, shown in Fig. 4, predict almost identical shapes for the isomeric and the ground state, justifying the usage of these deformation parameters in the blocked-BCS calculation. Therefore, we suggest a $K^\pi = 10^-$, $9/2^-$[505] $\otimes$ $11/2^+$[615] configuration for the observed isomer. An isomer with the same proposed structure has been observed in the $N = 116$ isotope $^{192}$Os$_{116}$ [22].

The heavier $N = 116$ isotones, i.e., $^{192}$Os and $^{194}$Pt, are well known examples of $\gamma$-soft nuclei [23]. Therefore it is expected that the $\gamma$-degree of freedom plays an important role also in $^{190}$W. The deduced ratio of the energies of the $4^+$ and $2^+$ states is close to the asymptotic limit of 2.5 for a $\gamma$-soft nucleus [3,4]. This interpretation is consistent with the potential energy surface calculations shown in Fig. 4, which suggest a significant $\gamma$-softness.

The implied $\gamma$-softness in $^{190}$W is consistent with the lifetime for the decay out of the isomer. The reduced hindrance is defined as $f = (\tau^\gamma / T^W)^{1/(\Delta K - \lambda)}$, where $\tau^\gamma$ is the partial $\gamma$-ray mean-life, $T^W$ is the Weisskopf single-particle estimate and $\lambda$ is the transition multipolarity. For the $K^\pi = 10^-$ isomer in $^{190}$W, assuming an E1, $\Delta K = 10$ decay with mean lifetime limits between 135 $\mu$s and 4.5 ms, results in a reduced hindrance, $f$, of between 6 and 10. This corresponds to a much enhanced E1 decay compared to the values for the sequence of $K^\pi = 8^-$ states in this region [24], which all have $f$-values of greater than 30. The small $f$-value for $^{190}$W indicates a significant reduction in the purity of the $K$ quantum number for either the yrast band and/or the isomeric state, which may be ascribed to the $\gamma$-softness. (Note that an E1-hindrance comparison with the isotone $^{192}$Os is not possible since, in that case, the $K^\pi = 10^-$ isomer lies
lower in energy than the $10^+$ member of the ground-state band.)

While the general feature of $\gamma$-softness is according to expectations in this mass region, a closer examination of the $4^+/2^+$ energy ratios is revealing, as shown in Fig. 5. The new data point for $^{190}$W$_{116}$ as deduced from the current work shows a striking deviation from the pattern for the lighter isotonic chains, which asymptote to the rotational limit value of 3.33 with decreasing proton number. Although the energies of the first two excited states of $^{190}$W can be explained in terms of triaxiality, their ratio compared to the systematics is not fully understood. The clear bifurcation in this plot, arising from the new data point, is reminiscent of the systematics representing the breakdown of the $Z = 64$ shell gap for $N < 78$ and $N > 88$ [4, 6]. Mach [7] has discussed the alteration of effective shell gaps in this neutron rich region and proposed the possibility of a sub-shell gap for proton number 76. More data on the ground-state bands of neutron rich even–even nuclei in this region are clearly required to address this question fully. The $^{190}$W data point suggests this may be fertile ground for future spectroscopic studies.

In conclusion, the yrast sequence of the neutron rich isotope, $^{190}$W, has been deduced following the decay of a proposed $K^T = (10^-)$ isomer, populated in the fragmentation of a relativistic energy $^{208}$Pb beam. The excitation energies of the first two excited states suggest a deformation effect. The present results open up the prospect of exploiting $K$-isomer decays to study the structure of nuclei inaccessible so far.

The excellent work of the technical and accelerator staff at GSI is acknowledged. This work is supported by EPSRC (UK), Polish Committee of Scientific Research under grant KBN 2 P03B 036 15, Department of Energy contract DE-FG02-96ER40983 (USA), and the EU Access to Large Scale Facilities Programme. The array of segmented Clover Ge detectors is jointly funded by CEA (France), EPSRC (UK), GSI (Germany) and NBI (Denmark).

References

Search for violation in $D^0$ and $D^+$ decays

FOCUS Collaboration


a University of California, Davis, CA 95616 USA
b Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, RJ, Brazil
c CINVESTAV, 07000 México City, DF, Mexico
d University of Colorado, Boulder, CO 80309, USA
e Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
f Laboratori Nazionali di Frascati dell’INFN, Frascati, I-00044, Italy
g University of Illinois, Urbana-Champaign, IL 61801, USA
h Indiana University, Bloomington, IN 47405, USA
i Korea University, Seoul, 136-701, South Korea
j INFN and University of Milano, Milano, Italy
k University of North Carolina, Asheville, NC 28804, USA
l Dipartimento di Fisica Nucleare e Teorica and INFN, Pavia, Italy
m University of Puerto Rico, Mayaguez, PR 00681, USA
n University of South Carolina, Columbia, SC 29208, USA
o University of Tennessee, Knoxville, TN 37996, USA
p Vanderbilt University, Nashville, TN 37235, USA
q Vanderbilt University, Nashville, TN 37235, USA
r Vanderbilt University, Nashville, TN 37235, USA
Abstract

A high statistics sample of photoproduced charm particles from the FOCUS (E831) experiment at Fermilab has been used to search for CP violation in the Cabibbo suppressed decay modes $D^+ \rightarrow K^-K^+\pi^+$, $D^0 \rightarrow K^-K^+$ and $D^0 \rightarrow \pi^-\pi^+$. We have measured the following CP asymmetry parameters: $A_{CP}(K^-K^+\pi^+)=+0.006 \pm 0.011 \pm 0.005$, $A_{CP}(K^-K^+)=−0.001 \pm 0.022 \pm 0.015$ and $A_{CP}(\pi^-\pi^+)=+0.048 \pm 0.039 \pm 0.025$ where the first error is statistical and the second error is systematic. These asymmetries are consistent with zero with smaller errors than previous measurements.

1. Introduction

CP violation occurs if the decay rate for a particle differs from the decay rate of its CP-conjugate particle [1]. CP violation, which in the Standard Model (SM) is a consequence of a complex amplitude in the Cabibbo–Kobayashi–Maskawa (CKM) matrix, has been observed to date only in the neutral–kaon system. In charm meson decays (as well as in $K$ and $B$ decays), two classes of CP violation exist: indirect and direct. In neutral–charm–meson decays, indirect CP violation may arise due to $D^0 – S^0$ mixing. In the case of direct violation, CP violating effects occur in a decay process only if the decay amplitude is the sum of two different parts, whose phases are made of a weak (CKM) and a strong contribution due to final state interactions (FSI) [2]. The weak contributions to the phases change sign when going to the CP-conjugate process, while the strong ones do not. In singly Cabibbo-suppressed $D$ decays, penguin terms in the effective Hamiltonian may provide the different phases of the two weak amplitudes.

Compared to the strange and bottom sectors, the SM predictions of CP violation for charm decays are much smaller [2 – 5], making the charm sector a good place to test the SM and to look for evidence of new physics. In the SM, direct CP violating asymmetries in $D$ decays are predicted to be largest in singly Cabibbo-suppressed decays, at most $10^{-3}$, and non-existent in Cabibbo-favored and doubly Cabibbo-suppressed decays [1]. However, a CP asymmetry could occur in the decay modes $D \rightarrow K_S\pi\pi$ due to interference between Cabibbo-favored and doubly Cabibbo-suppressed decays.

Experimentally, one looks at the Cabibbo-suppressed decay modes which have the largest combination of branching fraction and detection efficiency. For
this reason we select the all-charged decay modes $D^+ \to K^- K^+ \pi^+$, $D^0 \to K^- K^+$, and $D^0 \to \pi^- \pi^+$ (throughout this Letter the charge conjugate state is implied, unless otherwise noted).

In $D$ decays the charged $D$ is self-tagging and the neutral $D$ is tagged as either a $D^0$ or a $\bar{D}^0$ by using the sign of the bachelor pion in the $D^{\pm}$ decay.

Before searching for a CP asymmetry we must account for differences, at the production level, between $D$ and $\bar{D}$ in photoproduction (the hadronization process, in the presence of remnant quarks from the nucleon, gives rise to production asymmetries [6]). This is done by normalizing to the Cabibbo-favored modes $D^0 \to K^- \pi^+$ and $D^+ \to K^- \pi^+ \pi^+$, with the additional benefit that most of the corrections due to inefficiencies cancel out, reducing systematic uncertainties. An implicit assumption is that there is no measurable CP violation in the Cabibbo-favored decays.

The CP asymmetry can be written as:

$$A_{CP} = \frac{\eta(D) - \eta(\bar{D})}{\eta(D) + \eta(\bar{D})},$$

where $\eta$ is (considering for example the decay mode $D^0 \to K^- K^+$):

$$\eta(D) = \frac{N(D^0 \to K^- K^+)}{N(D^0 \to K^- \pi^+)} \frac{\epsilon(D^0 \to K^- \pi^+)}{\epsilon(D^0 \to K^- K^+)},$$

where $N(D^0 \to K^- K^+)$ is the number of reconstructed candidate decays and $\epsilon(D^0 \to K^- \pi^+)$ is the efficiency obtained from Monte Carlo simulations.

The CP asymmetry parameter measures the direct CP asymmetry in the case of $D^+$ and the combined direct and indirect CP asymmetries for $D^0$ [7].

The name FOCUS stands for Photoproduction of Charm with an Upgraded Spectrometer with a lexical license. The word “upgrade” refers to the upgrade of the E687 (the predecessor experiment) spectrometer [8].

Charmed particles were produced by the interaction of high energy photons, obtained by means of bremsstrahlung of electron and positron beams (with typically 300 GeV endpoint energy), with a beryllium oxide target. The mean energy of the photon beam was approximately 180 GeV. The data were collected at Fermilab during the 1996–1997 fixed-target run. More than $6.3 \times 10^9$ triggers were collected from which more than 1 million charmed particles have been reconstructed.

The particles from the interaction are detected in a large-aperture magnetic spectrometer with excellent vertex measurement, particle identification and calorimetric capabilities. The vertex detector consists of two systems of silicon microvertex detectors. The upstream system consists of 4 planes interleaved with the experimental target, while the downstream system consists of 12 planes of microstrips arranged in three views. These detectors provide high resolution separation of primary (production) and secondary (decay) vertices with an average proper time resolution of 30 fs for 2-track vertices. The momentum of the charged particles is determined by measuring their deflections in two analysis magnets of opposite polarity with five stations of multiwire proportional chambers. Kaons and pions in the $D$-meson final states are well separated up to 60 GeV/$c$ of momentum using three multicell threshold Čerenkov counters.

2. Selection criteria

The final states are selected using a candidate driven vertex algorithm [8]. A secondary vertex is formed from the reconstructed tracks and the momentum vector of the $D$ candidate is used as a seed to intersect the other tracks in the event to find the primary vertex. Once the production and decay vertices are determined, the distance $\ell$ between them and the relative error $\sigma_\ell$ are computed. Cuts on the $\ell/\sigma_\ell$ ratio are applied to extract the $D$ signals from the prompt background. The topological configuration of the event is evaluated with four tests: the primary and secondary vertex confidence levels (minimum values of 1% were required) and two measures of vertex isolation, a no point-back isolation and a secondary vertex isolation. The no point-back isolation cut requires that the maximum confidence level for a candidate-$D$ daughter track to form a vertex with the tracks from the primary vertex be less than a certain threshold. The secondary vertex isolation cut requires that the maximum confidence level for another track to form a vertex with the $D$ candidate be less than a certain threshold. The analyses differ mainly in the way the particle identification is handled and, less importantly, in the way the vertex cuts are applied. To minimize the systematic error we use identical vertex cuts on the signal and normalizing modes.
Fig. 1. (a) $K^- K^+ \pi^+$ invariant mass distribution, (b) $K^+ K^- \pi^+$ invariant mass distribution, (c) $K^- \pi^+ \pi^+$ invariant mass distribution, (d) $K^+ \pi^- \pi^-$ invariant mass distribution. The fits (solid curves) are described in the text and the numbers quoted are the yields.

In the $D^+ \rightarrow K^- K^+ \pi^+$ analysis, we require $\ell/\sigma_{\ell} > 10$, the no point-back isolation must be less than 20%, and the secondary vertex isolation less than 0.1%. The vertices (primary and secondary) have to lie inside a fiducial volume, the $D$ momentum must be in the range $25 < P < 250$ GeV/c (a very loose cut) and the primary vertex must be formed with at least two reconstructed tracks, in addition to the seed track. The Čerenkov particle identification cuts used in FOCUS are based on likelihood ratios between the various stable particle identification hypotheses. These likelihoods are computed for a given track from the observed firing response (on or off) of all cells within the track’s ($\beta = 1$) Čerenkov cone for each of our three Čerenkov counters. The product of all firing probabilities for all cells within the three Čerenkov cones produces a $\chi^2$-like variable $W_i = -2 \ln(\text{Likelihood})$ where $i$ ranges over the electron, pion, kaon and proton hypotheses (see Ref. [9] for more details). We require $W_K > W_{\min}$ for the tracks reconstructed as a kaon, and a pion consistency cut for the pion tracks. The pion consistency cut requires that no particle hypothesis is favored over the pion hypothesis with a $\Delta W = W_{\pi} - W_{\min}$ exceeding 2. To remove contamination from the $D_s^+ \rightarrow K^- K^+ \pi^+$ sig-

---

17 The reason for this cut lies in the presence of a trigger counter just upstream of the second microstrip device, therefore we defined the fiducial volume as the target region between the first slab of the experimental target and this trigger counter.
nal due to Čerenkov misidentified background from the decay mode $D^+ \to K^-\pi^+\pi^+$, we employ an anti-reflection cut which rejects candidates which, when reconstructed as $K^-\pi^+\pi^+$, are consistent with the $D^+$ hypothesis (we also reject events whose $K^-K^+$ mass exceeds 1.84 GeV/$c^2$ in order to exclude background due to $D^{0}\pi^+ \to (K^-K^+)\pi^+$). This cut has no effect in the vicinity of the $D^+ \to K^-K^+\pi^+$ signal peak. In Fig. 1, the invariant mass plots obtained with this set of cuts for the decay modes $D^+ \to K^-K^+\pi^+$, $D^- \to K^+K^-\pi^-$, and the normalizing decays $D^+ \to K^-\pi^+\pi^+$ and $D^- \to K^+\pi^-\pi^-$ are shown. In the $D^+ \to K^-\pi^+\pi^+$ analysis there is an additional cut on the $D^0$ mass formed by a kaon and a pion to remove the $D^{*+} \to D^0\pi^+ \to (K^-\pi^+)\pi^+$ decay chain. The $KK\pi$ invariant mass distributions are fit with a Gaussian for the $D^+$ signal, a second Gaussian for the $D^+_s$ signal, and a quadratic polynomial for the background. A binned maximum likelihood fit finds $6860^{+110}_{-115}$ $D^0\pi^+\to (K^-K^+)\pi^+$ events. The fit for the normalizing modes (fit with a Gaussian plus a linear polynomial) gives $6860^{+110}_{-115}$ $D^0\pi^+\to (K^-K^+)\pi^+$ events.

In the $D^0 \to K^-K^+$ analysis, the sign of the bachelor pion in the $D^{*\pm}$ decay chain $D^{*+(-)} \to D^0(D^0_{(*)})\pi^{+(-)}$ is used to identify the neutral $D$ as either a $D^0$ or a $D^*$. We require that the mass difference between the $D^0$ and the $D^*$ mass be within 4 MeV/$c^2$ with respect to the nominal mass.
difference [10]. We use \( \ell/\pi > 8 \), while the no pointback isolation and the secondary vertex isolation cuts are unnecessary because the \( D^* \) tag sufficiently reduces the background. All the other cuts, except the anti-reflection cuts, are the same as those used in the \( D^+ \to K^- K^+ \pi^+ \) analysis.

In Fig. 2, we show the invariant mass plots, obtained with this set of cuts, for the decay modes \( D^0 \to K^- K^+ \), \( \bar{D}^0 \to K^+ K^- \), and the normalizing decays \( D^0 \to K^- \pi^+ \) and \( \bar{D}^0 \to K^+ \pi^- \). The peak in the \( K \bar{K} \) invariant mass plots at \( \approx 1.95 \text{ GeV}/c^2 \) is due to the reflection of the \( D^0 \to K^- \pi^+ \) mode when the pion is misidentified as a kaon. A Monte Carlo simulation of this reflection reproduces the shape observed in the data. Consequently, the \( K^- K^+ \) invariant mass distributions are fit with a Gaussian for the \( D^0 \) signal, a function obtained by smoothing the reflection peak (only the shape of this reflection peak is modeled by our Monte Carlo simulation), and a quadratic polynomial for the background. A binned maximum likelihood fit gives 1623 \( \pm 47 \) \( D^0 \to K^- K^+ \) and 1707 \( \pm 53 \) \( \bar{D}^0 \to K^+ K^- \) events. The fit for the normalizing modes (fit with a Gaussian plus a linear polynomial and excluding the low mass region to avoid possible contamination due to other charm hadronic decays involving an additional \( \pi^0 \)) gives 18501 \( \pm 144 \) \( D^0 \to K^- \pi^+ \) and 19633 \( \pm 149 \) \( \bar{D}^0 \to K^+ \pi^- \) events.

The \( D^0 \to \pi^- \pi^+ \) analysis is identical to the \( D^0 \to K^- K^+ \) analysis with the exception of the Čerenkov identification. In order to reduce the large reflection peak from the \( D^0 \to K^- \pi^+ \) mode, a tight Čerenkov identification requirement (\( W_K - W_\pi > 1 \)) for the pion tracks is implemented.

In Fig. 3 the invariant mass plots for the decay modes \( D^0 \to \pi^- \pi^+ \) and \( \bar{D}^0 \to \pi^+ \pi^- \) are shown. As in the \( D^0 \to K^- K^+ \) case, the peak at \( \approx 1.75 \text{ GeV}/c^2 \) is due to the reflection of the \( D^0 \to K^- \pi^+ \) mode. Again our Monte Carlo simulation reproduces the shape observed in the data. The \( \pi^- \pi^+ \) invariant mass distributions are fit with a Gaussian for the \( D^0 \) signal, a function obtained by smoothing the reflection peak, and a quadratic polynomial for the background. A binned maximum likelihood fit gives 606 \( \pm 31 \) \( D^0 \to \pi^- \pi^+ \) and 571 \( \pm 33 \) \( \bar{D}^0 \to \pi^+ \pi^- \) events.

3. \( CP \) asymmetry measurements

Because the efficiency is strongly dependent on the \( D \) momentum, it is necessary to verify that the observed momentum spectrum is reproduced by the Monte Carlo simulation. A mismatch could generate a false asymmetry. Fig. 4 shows the \( D^0 \) momentum for the decay mode \( D^0 \to K^- \pi^+ \) for real data (points with errors) and Monte Carlo data (histogram).
The measured $CP$ asymmetries are reported in Table 1 along with a comparison to previous measurements. The analysis of the decay mode $D^+ \rightarrow K^- K^+ \pi^+$ is complicated by the possibility of intermediate resonant states, such as $K^{*0} K^+$ and $\phi \pi^+$. We think that in this case a Dalitz plot analysis is the appropriate tool to investigate $CP$ asymmetry effects. This topic will be the subject of a future paper.

Our asymmetry measurements have been tested by modifying each of the vertex cuts individually; the results are always consistent.

The systematic errors on these measurements reflect uncertainties in reconstruction efficiency, Čerenkov particle identification, and hadronic absorption of secondaries in the target and spectrometer materials. The estimates were obtained by splitting our data into independent samples depending on $D$ momentum and the different periods in which the data were collected. The main reason for this time dependence is the insertion of the upstream silicon system (in the target region) during the 1997 fixed-target run period. A technique modeled after the $S$-factor method from the Particle Data Group [10] was used to separate true systematic variations from statistical fluctuations. The asymmetry is evaluated for each of the statistically independent subsamples and a scaled variance is calculated; the split sample variance is defined as the difference between the reported statistical variance and the scaled variance if the scaled variance exceeds the statistical variance. For all decays studied, the scaled variance is less than the statistical variance, so it does not contribute to the systematic uncertainty. The evaluation of systematic effects related to different fit procedures is performed on the whole sample. The asymmetry is calculated using various fit conditions (these include the choice of the estimator, the background shape and, in the case of the $D^0$ analysis, the shape of the reflection peak) and the sample variance is used because the fit variants are all a priori likely. To obtain the final systematic error, the variance from the different fitting procedures and a further contribution, due to the uncertainties in the efficiency calculation and finite statistics of the Monte Carlo events, are then added in quadrature. Table 2 shows the contribution of each of these sources to the total systematic uncertainty for the $CP$ asymmetry measurements.

The measured asymmetries are consistent with zero within the errors.
4. Summary and conclusions

We have searched for $CP$ violation in the Cabibbo-suppressed decay modes $D^+ \rightarrow K^- K^+ \pi^+$, $D^0 \rightarrow K^- K^+$ and $D^0 \rightarrow \pi^- \pi^+$ using a high statistics sample of photoproduced charm particles from the FOCUS (E831) experiment at Fermilab. We have measured the following $CP$ asymmetry parameters:

$$A_{CP}(K^- K^+ \pi^+) = +0.006 \pm 0.011 \pm 0.005,$$
$$A_{CP}(K^- K^+) = -0.001 \pm 0.022 \pm 0.015,$$
$$A_{CP}(\pi^- \pi^+) = +0.048 \pm 0.039 \pm 0.025,$$

where the first error is statistical and the second error is systematic.

These asymmetries are consistent with zero and represent a substantial improvement over previous measurements.

Acknowledgement

We wish to acknowledge the assistance of the staffs of Fermi National Accelerator Laboratory, the INFN of Italy, and the physics departments of the collaborating institutions. This research was supported in part by the U.S. National Science Foundation, the U.S. Department of Energy, the Italian Istituto Nazionale di Fisica Nucleare and Ministero dell’Università e della Ricerca Scientifica e Tecnologica, the Brazilian Conselho Nacional de Desenvolvimento Científico e Tecnológico, CONACyT-México, the Korean Ministry of Education, and the Korean Science and Engineering Foundation.

References

Single particle spectrum and binding energy of nuclear matter

M. Baldo\textsuperscript{a,*}, A. Fiasconaro\textsuperscript{b}

\textsuperscript{a} INFN, Sez. Catania, Corso Italia 57, 95129 Catania, Italy
\textsuperscript{b} Dipartimento di Fisica, Università di Catania, Corso Italia 57, 95129 Catania, Italy

Received 14 April 2000; received in revised form 10 July 2000; accepted 17 August 2000

Abstract

In non-relativistic Brueckner calculations of nuclear matter, the self-consistent single particle potential is strongly momentum dependent. To simplify the calculations, a parabolic approximation is often used in the literature. The variation in the binding energy value introduced by the parabolic approximation is quantitatively analyzed in detail. It is found that the approximation can introduce an uncertainty at best of 1–2 MeV already around the saturation density, and therefore it should be avoided in Brueckner calculations.

\textcopyright 2000 Elsevier Science B.V. All rights reserved.

PACS: 21.65.+f; 24.10.Cn; 45.50.Jf; 21.30.-x; 26.60.+c

1. Introduction

It is one of the fundamental issue in nuclear physics to evaluate the nuclear matter binding energy and saturation properties, starting from a realistic nucleon–nucleon (NN) interaction with no free parameter. This old project requires the solution of a complex many-body problem, and has received several contributions and improvements along the years, beginning as far back as the middle of the last century. One of the main approaches to this long standing problem is the so-called hole-expansion or Bethe–Brueckner–Goldstone (BBG) theory [1]. The first real breakthrough in this scheme was the introduction of the self-consistent single particle potential at the two hole-line level of approximation, which is then usually referred as the Brueckner–Hartree–Fock (BHF) approximation [1,2]. The introduction of the self-consistent potential drastically improves the results. In particular, the binding energy and saturation density, which otherwise would turn out unreasonable, move to values which can be considered an acceptable starting approximation. The remaining discrepancy could be summarized in the celebrated “Coester band” [4], along which the saturation points for different NN interactions were clustering and which misses the empirical region (corresponding approximately to a binding energy per nucleon of $-16$ MeV and a nucleon density of $0.17 \text{ fm}^{-3}$). Later, the Liége group stressed [5] the relevance of the choice of the single particle potential. In particular they suggested the use of the “continuous choice”, which indeed appears to move the saturation point towards the empirical one, but still missing it [5,6]. A period of major developments took place in the latest two decades. Starting from the works by Day [3], the hole-line expansion was analysed up to the three hole-line level of approximation. A strong indication of convergence of the expansion was obtained [3]. Furthermore, BHF calculations with the...
continuous choice seem to get a substantially smaller corrections from three-body correlations [7]. The results confirm that the empirical saturation point is still missed, and therefore that three-body forces are needed in the nuclear hamiltonian [3]. In the meanwhile the relativistic Dirac–Brueckner (DB) method was developed [8], which already at the two hole-line level of approximation appears to be able to reproduce the empirical saturation point. The main relativistic correction introduced by the DB method is due to the structure of the Dirac 4-spinors, which in the medium appear “rotated” with respect to the free ones. The non-relativistic three-body forces and this relativistic effect of the DB approach are probably two faces of the same dynamical effect [9]. The many-body theory has reached, therefore, such a precision that it is possible to test the nuclear hamiltonian. Because of that, the time seems to be appropriate to check the reliability of the approximations which are commonly employed in BBG calculations of nuclear matter.

In this letter we consider the BHF in the continuous choice and we analyze quantitatively the uncertainty of the results which comes out by approximating the single particle self-consistent potential with a parabolic form. This approximation allows to calculate the single particle self-consistent potential with a parabolic form. This approximation allows to calculate the potential at each iteration, only for few momenta, thus reducing drastically the computer time. It is an approximation that is systematically used by almost all groups working in the field. We will try to optimize at best the parabolic approximation and estimate the lower limit of the error thereby introduced. Indeed, the single particle potential, as obtained from fully self-consistent BHF calculations, is strongly momentum dependent and not necessarily so simple as a parabola. On the basis of our results, we strongly recommend to avoid the parabolic approximation.

2. Sketch of the formalism

In the BHF approximation, the nuclear matter total energy $E$ is obtained from the Brueckner G-matrix $G(\omega)$ according to the equation

$$
E = \sum_{k_1 < k_F} \frac{\hbar^2 k_1^2}{2m} + \frac{1}{2} \sum_{k_1, k_2 < k_F} \langle k_1 k_2 | G(e_{k_1} + e_{k_2}) | k_3 k_4 \rangle_A
$$

(1)

with $|k_1 k_2\rangle_A = |k_1 k_2\rangle - |k_2 k_1\rangle$. Here $k_F$ is the Fermi momentum, the summation over the momenta $k_i$ include spin and isospin variables. The single particle energies $e_{k_i}$, appearing in the entry energy of the G-matrix, are given by

$$
e(k) = \frac{\hbar^2 k^2}{2m} + U(k),$$

(2)

where the single particle potential $U(k)$ is determined by the self-consistent equation

$$
U(k) = \sum_{k' < k_F} \langle kk' | G(e_{k_1} + e_{k_2}) | kk' \rangle.
$$

(3)

The self-consistency is coupled with the integral equation for the G-matrix

$$
\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | v | k_3 k_4 \rangle + \sum_{k'_3 k'_4} \langle k_1 k_2 | v | k'_3 k'_4 \rangle \frac{(1 - \Theta_F(k'_3))(1 - \Theta_F(k'_4))}{\omega - e_{k'_3} - e_{k'_4}}
$$

$$
\times \langle k'_3 k'_4 | G(\omega) | k_3 k_4 \rangle,
$$

(4)

where $\Theta_F(k)$ is 1 for $k < k_F$ and is zero otherwise. The product $Q(k, k') = (1 - \Theta_F(k))(1 - \Theta_F(k'))$, appearing in the kernel of Eq. (4), enforces the scattered momenta to lie outside the Fermi sphere, and it is commonly referred as the “Pauli operator”. The self-consistent set of equations are usually solved by an iteration procedure. The G-matrix can be expanded in partial waves, according to the classification of two-nucleon channels [1]. To avoid coupling between different two-body channels, the Pauli operator $Q$, as well as the two-body energies $e_{k'_3} + e_{k'_4}$ in the denominator, are averaged over the angle between the relative momentum $q = (k'_3 - k'_4)/2$ and the total momentum $P = k'_3 + k'_4$. Despite this approximation, which has been tested recently in Ref. [10], the numerical solution of the coupled equations (3), (4) is quite time consuming, since the single particle potential $U(k)$ must be calculated in a wide range of momenta with a fine enough grid. If one assumes that the potential $U(k)$, or equivalently the single particle energy $e(k)$, has approximately a quadratic form

$$
e(k) \approx e_0 + \frac{\hbar^2 k^2}{2m*},
$$

(5)

then one can calculate the potential, at each iteration step, in few points only and interpolate the obtained values with a parabola. The approximation of Eq. (5) is
usually called the effective mass approximation, since then the spectrum has the same shape as the free one but with an “effective mass” $m^*$. It is an approximation so often used that it appears worth making a test of its reliability.

To this aim, we have performed a set of BHF calculations fully self-consistently without any assumption about the potential shape, as well as by forcing the potential to a parabolic shape by means of a fitting procedure, and then compared the results. Since the procedure of approximating the single particle potential with a quadratic form is not unique, we tried different possibilities with the purpose of reducing the error to a minimum. We get, therefore, a lower limit of the error. In actual applications of the parabolic approximation, the error is expected to be substantially larger.

### 3. Results and discussion

The performed BHF calculations include all two-body channels up to total angular momentum $J = 11\hbar$. In a set of calculations we adopted the Argonne $v_{18}$ [11] potential as the NN interaction. This potential belong to a new generation of realistic NN potentials, with an improved fit of the scattering data, which give similar results and cluster closely together in the Coester band [12]. The self-consistent single particle potential $U(k)$ was calculated up to the momentum cut-off $k_{\text{max}} = 7.5 \text{ fm}^{-1}$, which turns out to be large enough in the considered density range [13]. The potential, for the Fermi momentum $k_F = 1.4 \text{ fm}^{-1}$, is displayed in Fig. 1 (full circles). It is numerically calculated with a grid step of 0.1 fm$^{-1}$ from the G-matrix, Eq. (3), and inserted as the entry potential at each iteration step, until convergence is reached, i.e., the potential and the binding energy are stable under iteration with good accuracy. Stability within few keV of the binding was systematically reached. The numerical method is described in Ref. [6].

The quadratic approximation, at each step of the iteration procedure, is introduced by fitting the potential up to a certain maximum momentum $k_{\text{FIT}}$. For definiteness, we have considered in detail two choices, namely $k_{\text{FIT}} = 2k_F$ and $k_{\text{FIT}} = k_{\text{max}}$. At each iteration step, the potential $U(k)$ coming directly from the G-matrix calculation is fitted with a parabola, which is then used as the entry potential for the next iteration. Convergence is reached when both potentials remain stable under this procedure. Of course other methods for approximating the actual potential by a quadratic form can be used, but the fitting procedure is obviously expected to "minimize" the deviation and therefore the corresponding error. In this procedure one obtains, therefore, two potentials, one calculated from the G-matrix with the parabolic input, and one from the parabolic fit to this potential. Of course, if the potential coming directly from the G-matrix were indeed parabolic, the two potentials would closely agree. For the choice $k_{\text{FIT}} = 2k_F$, in Fig. 1 the two potentials at convergence are displayed. In principle, one can calculate the nuclear matter binding energy from both potentials, but the result will be in general slightly different. As one can see, the fully self-consistent potential, obtained without any fitting procedure, as specified above, does not coincide with anyone of the two previous potentials, and these differences give a quantitative indication of the uncertainty introduced by the parabolic approximation. The corresponding saturation curves are reported in Fig. 2(a). The parabolic potential produces a saturation curve in fair agreement with the one reported, e.g., in Ref. [10]. Around saturation the parabolic approximation introduces a shift in the binding of 1–2 MeV. The two choices for the potentials, discussed above, give different binding, since $U(k)$ is not really parabolic, and the fitting proce-

![Fig. 1. Single particle potential as function of momentum. The full circles indicate the results of the fully self-consistent calculation, where the potential is taken at each iteration step as calculated from the Brueckner G-matrix. The solid line is the result of the parabolic approximation. The parabolic potential, used as input for the G-matrix, produces the potential indicated by the squares.](image)
dure, despite the minimization of the error, introduces necessarily an approximation. Another uncertainty is coming from the choice of \( k_{\text{FIT}} \), as can be seen in Fig. 2(b), where the results for \( k_{\text{FIT}} = k_{\text{max}} \) are reported. In this case the discrepancy are larger for lower density, since then the potential \( U(k) \) becomes indeed flatter at momenta below \( k_F \). A more complete account of the dependence on the fitting range is reported in Fig. 3, where the binding at \( k_F = 1.4 \text{ fm}^{-1} \) is reported as a function of \( k_{\text{FIT}} \). Since the actual potential is anyhow not parabolic, the discrepancy is present for any value of \( k_{\text{FIT}} \), and cannot be reduced below a certain value.

In all cases the saturation curves appear distorted, and the saturation point shifted. Even if in some cases the saturation point seems to be “improved”, this does not have any physical meaning, since, anyhow, it is mainly a spurious effect, introduced by the parabolic approximation.

Completely similar results are obtained with the “old” potential Argonne \( \nu_{14} \) [14].

In conclusion, we have shown that the parabolic approximation for the single particle potential \( U(k) \) in the self-consistent Brueckner scheme introduces an uncertainty of 1–2 MeV near the saturation density, and therefore it cannot be used in accurate calculations. This warning appears relevant, since it is one of the most used approximation. The full momentum dependence has to be retained, which prevents the use of a constant effective mass approximation. It can be used only for quite rough estimate of the nuclear binding.

References
Pairing-induced localization of the particle continuum in weakly bound nuclei

S.A. Fayans a,1, S.V. Tolokonnikov a,1, D. Zawischa b, *

a Russian Research Centre — Kurchatov Institute, 123182 Moscow, Russian Federation
b Institut für Theoretische Physik, Universität Hannover, D-30060 Hannover, Germany

Received 10 April 2000; received in revised form 14 June 2000; accepted 15 August 2000

Abstract

The Hartree–Fock–Bogolyubov (HFB) problem for the cutoff local energy-density functional is solved numerically by using the Gor’kov formalism with an exact treatment of the particle continuum. The contributions from the resonant and “gas” continuum to the spectral density of the HFB eigenstates as well as the shifting and broadening of the discrete HF hole orbitals are clearly demonstrated with the illustrative example of the drip-line nucleus 70Ca. The structure of the neutron density distribution in the localized ground state is analyzed, and the formation of its extended tail (“halo”) is shown to be a collective pairing effect.

PACS: 21.60.-n; 21.90.+f; 24.10.Cn

Keywords: Local energy-density functional; Pairing correlations; Localized states in the continuum; Drip-line nuclei; Neutron halo

Pairing correlations significantly influence the properties of finite nuclei producing, e.g., the observed odd–even staggering of the binding energies and radii. In the mean-field models, the pairing is introduced either at the BCS or the HFB level [1]. The zone of the active phase space around the Fermi surface, in which the major pairing effects are developed, depends on the effective interaction used. Whenever this zone includes states from the particle continuum, and this is obviously inevitable when approaching the drip line, the HFB approximation breaks down, yielding unphysical nucleonic gas [2]. A correct treatment of the pairing problem with continuum is provided by the HFB theory which, for the chemical potential $\mu < 0$, always gives a localized ground state [3]. In principle, an exact solution of the HFB equations should be found with the physical boundary conditions both for the bound and scattering states (examples of such solutions are given in [4,5]). However, in most applications of the HFB theory to nuclei, with different kinds of the energy-density functionals, the particle continuum is discretized in a box (see, e.g., [6,7] and references therein), and it is still questionable whether or not the particle level density can be well reproduced with such a prescription, particularly near the threshold. Continuum effects have been studied very recently in [8] within the HFB method using a fixed, analytically soluble mean field potential and keeping self-consistency only in the pairing channel; again, the continuum states have been discretized in a spatial box. An attempt to take into account the resonant con-
where \( x \) is the strength parameter \( f \) of states at \( E > |\mu| \approx \frac{S_{2n}}{2} \) (\( S_{2n} \) is the two-neutron separation energy) belong to the continuum, and these solutions contain both local and non-localized HFB resonances. Their positions and widths, as well as the genetic relation to the Hartree–Fock (HF) single-particle spectrum, and also the contribution from the nonresonant continuum are worth to be revealed with numerically “exact” HFB calculations. Of particular interest is the situation on the drip line where the nuclei have only one bound state — the ground state, and the whole HFB spectrum is continuous. Our study is motivated by the great activity in physics of radioactive nuclear beams which have lead already, e.g., to the discovery of halo structure in some loosely bound nuclei.

Our approach is based on the generalized variational principle applied to the cutoff (local) energy-density functional [11 – 13] and on the physically transparent and mathematically elegant Green’s function principle applied to the cutoff (local) energy-density functional [11 – 13] and on the physically transparent and mathematically elegant Green’s function applied to the cutoff (local) energy-density functional [11 – 13].

Spherical systems, after separating the spin-angular variables one gets the equation for the radial component \( \tilde{g}_{jk} \) of the generalized Green’s function \( G \):

\[
\begin{pmatrix}
E - h_{jl} + \mu & -\Delta \\
-\Delta & E + h_{jl} - \mu
\end{pmatrix}
\begin{pmatrix}
g_{jl}^{11} \\
g_{jl}^{21}
\end{pmatrix}
= \begin{pmatrix}
g_{jl}^{12} \\
g_{jl}^{22}
\end{pmatrix}
\times \begin{pmatrix}
\delta(r - r') \\
0
\end{pmatrix},
\]

where

\[
h_{jl} = \frac{\hbar^2}{2m} \left( -\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} \right)
+ U_c(r) + U_{sl}(r) \langle \tilde{\sigma} \rangle_{jl}
\]

is the Hartree–Fock (HF) Hamiltonian in the \( jl \) channel with \( U_c \) and \( U_{sl} \) the central and spin-orbit potentials, respectively,

\[
\langle \tilde{\sigma} \rangle_{jl} = j(j+1) - l(l+1) - \frac{3}{4}, \quad E \geq 0.
\]

The solution of this matrix equation is constructed by using the four linearly independent solutions \( u_i, v_i \), \( i = 1 \ldots 4 \), which satisfy the homogeneous system obtained from (2) by replacing the right hand side by zero [4,13]. Two of them are regular at \( r \to 0 \) and the other two are regular at \( r \to \infty \). The latter are chosen with the asymptotic momenta \( k_\pm = \sqrt{\mu \pm E} \) such that \( \text{Im} k_\pm \geq 0 \) which provides, in particular, the correct asymptotics for the particle scattering states at the \( \text{Im} E = 0 \) axis. Such boundary conditions ensure the spatial localization of the nuclear ground state at \( \mu < 0 \) [3,4].

It is convenient to study the pairing effects in terms of the spectral distributions over the eigenstates of the HFB Hamiltonian, i.e., in terms of the spatial integrals of the imaginary parts of the Gor’kov Green’s function.
Fig. 1. Neutron even-$l$ partial spectral densities in drip-line nucleus $^{70}$Ca calculated by integrating the imaginary parts of the corresponding Green’s functions up to $R = 10$ fm. A smearing parameter $\gamma = 1$ keV is used to visualize the $\delta$-like peaks and very narrow resonances. Upper panel: level densities for the “HF” potential; the $\epsilon < 0$ part for the discrete spectrum with indicated single-particle levels, the $\epsilon > 0$ part for continuum states. Lower panel: the HFB results; all states are in the continuum, the left part ($\epsilon < 0$) is for the localized $v^2$-spectral density, the right ($\epsilon > 0$) for the non-normalizable $u^2$-components (see text).

functions. Here, for the given $jl$ quantum numbers, we shall consider two partial spectral distributions:

$$
\frac{dN^{22}_{jl}}{dE} = -\frac{2j + 1}{\pi} \int_0^R \text{Im} \, g^{22}_{jl}(r; E) \, dr, \tag{3}
$$

and

$$
\frac{dN^{11}_{jl}}{dE} = -\frac{2j + 1}{\pi} \int_0^R \text{Im} \, g^{11}_{jl}(r; E) \, dr. \tag{4}
$$

The first and the second we shall call the $v^2$- and $u^2$-spectral density, respectively. The corresponding analytical expressions can be obtained by using the partial Green’s functions which incorporate the generalized Wronskian of the four linearly independent solutions as given in Refs. [4,13]. We add that the expressions for the canonical-basis spectral distributions and the numerical results with discretized continuum for a few tin isotopes can be found in Ref. [6].

In Eq. (3), the spatial integral converges for $R \to \infty$ since it includes only the localized components $\propto v^2$. At $E > |\mu|$, all HFB eigenstates are in the continuum and, since $\text{Im} \, g^{11}_{jl}$ contains non-localized components $\propto u^2$, the integration in Eq. (4) is meaningful only up to a certain finite range $R$. One may present the two above expressions by one function $dN^{\text{HFB}}_{jl}/dE$ if (3) and (4) are considered as functions of $\epsilon = \mu - E$ and $\epsilon = \mu + E$, respectively, though both are for the same HFB eigenvalue $E$. We show in Figs. 1 and 2 the partial HFB neutron spectral densities for even and odd $l$, respectively, calculated for the drip-line nucleus $^{70}$Ca, as functions of $\epsilon$ with $R = 10$ fm (lower panels) and also present the corresponding “HF” level densities (upper panels). The “HF” refers to the results obtained by solving Eq. (2) but keeping only the diagonal part of $\hat{H}$ which emerges after solving the HFB problem ($h_{jl}$ includes the contribution from pairing correlations). In other words, in these figures we...
show the level densities for the basis which diagonal-
izes the mean field part of the HFB Hamiltonian ver-
sus the spectral densities which arise when the pair-
ing field $\Delta$ is present explicitly in the left matrix in
Eq. (2). We point out that the spherical volume of ra-
dius $R = 10$ fm is chosen here as the physical volume
in which the mean-field and pairing correlations are
mostly active to produce the localized ground state, it
is not a normalization box; within this volume we in-
tegrate the imaginary parts of the Green’s functions
which have the correct asymptotic behavior (the nor-
amal and anomalous densities, when solving the HFB
problem, have been evaluated up to 25 fm, and, again,
the wave functions are not zero at and beyond this ra-
dius, but the contribution from this far region to the
bulk ground state properties is negligible). For the vi-
dual and anomalous densities, when solving the HFB
effect, the term $\Delta(k)\approx 2\pi$.

Let us discuss first the peculiarities in the “HF”
level densities. For a finite-depth potential well, such
as our neutron HF potential, it can be shown that the
partial single particle level density in the continuum
within a spherical volume with radius $R$ larger than
the potential range is given by

$$
\frac{dN_{j\ell}^{HF}}{d\epsilon} = \frac{2j + 1}{\pi} \left\{ \frac{d\delta_{j\ell}/d\epsilon}{d\epsilon} + \frac{k^3 R^3}{2\epsilon} \left[ (Y_{j\ell}(kR))^2 - Y_{j\ell-1}(kR)Y_{j\ell+1}(kR) \right] \right\},
$$

(5)

where $Y_{j\ell}(kR) = (\cos \delta_{j\ell})_{ij}(kR) - (\sin \delta_{j\ell})n_i(kR)$,
$i = l, l \pm 1; k = \sqrt{2m\epsilon}/\hbar$. This formula is valid for
$R$ big enough so that the radial wave function can be
expressed through spherical Bessel and Neumann func-
tions ($j_i$ and $n_i$, respectively) and the phase shift
$\delta_{jj}(\epsilon)$ can be calculated accurately. The resonance-
like bumps seen in the upper panels in Figs. 1 and
2 at $\epsilon > 0$ are associated with the term $d\delta_{jj}/d\epsilon$ in
(5). These bumps are imposed on a rather flat non-
resonant continuum which weakly oscillates with an
energy-dependent wavelength defined by $\Delta(kR) \approx
2\pi$. Without potential, when $\delta_{jj}(\epsilon) = 0$, Eq. (5) gives
the free-gas partial level density. Subtracting the latter
from Eq. (5), at $kR \gg l$ one gets

$$
\frac{dN_{j\ell}^{HF}}{d\epsilon} - \frac{dN_{j\ell}^{gas}}{d\epsilon} = \frac{2j + 1}{\pi} \left\{ \frac{d\delta_{j\ell}}{d\epsilon} - \cos(2kR + \delta_{j\ell})\sin \delta_{j\ell} \right\}/2\epsilon.
$$

(6)

Averaging this expression over small energy range
$\Delta\epsilon > \pi \sqrt{2h^2\epsilon/mR^2}$ eliminates the oscillating co-
sine term, and one is left with the usual level density
$\propto d\delta_{jj}/d\epsilon$ associated with the potential well it-
self (see also Ref. [16]). However, this procedure can
not be applied at low energies, i.e., in the region near
the continuum threshold which, in drip-line nuclei,
is very close to the Fermi surface. This zone of the
phase space is of importance since the major pairing
effects are developed around the Fermi surface. At
any finite $R$, when $\epsilon \rightarrow 0$, the leading term $\propto k^{2l-1}$
of the expansion of $d\delta_{jj}/d\epsilon$ in powers of $k$ can-
cels in Eq. (5) due to the contribution coming from the
term $\propto \sin(2\delta_{jj})_{ij} - (kR)n_{i+1}(kR)$. The resulting
level density near the continuum threshold turns out
to be proportional to $C_{ij}k^{2l-1}$, i.e., it has the free-gas
behavior, the coefficients $C_{ij}$ depend, however, on the
potential well. For the $s$-wave level density, using the
effective range approximation, we get from (5) and (6)
an analytical expression:

$$
\frac{dN_{s0}^{HF}}{d\epsilon} = \frac{2}{3\pi} \frac{k^3}{\epsilon} \left\{ 1 - \frac{3a}{R} \left[ 1 - \frac{a}{R} \left( 1 - \frac{r_{eff}}{2} \right) \right] \right\},
$$

(7)

where $a$ is the scattering length and $r_{eff}$ is the effective
range. In the curly brackets here is a factor which
could give an enhancement of the HF level density as
compared to the free-gas one. For our “HF” potential
we found $a = -14.8$ fm and $r_{eff} = 14.4$ fm, which
gives, with $R = 10$ fm, an enhancement factor of
about 7. These parameters correspond to the virtual
$s$-state at $\epsilon_0 = -51$ keV which shows up as a sharp
peak at very low positive $\epsilon$ in the upper panel in
Fig. 1. The width of this very asymmetric peak at
half maximum is about 23 keV. At larger energies,
one can see in this panel two partners of the $d$-wave
resonance which is split due to the spin-orbit potential,
and also the $g_{7/2}$ resonance whose $g_{9/2}$ partner is
in the discrete spectrum at $\epsilon = -276$ keV. Besides,
a negative-parity $h_{11/2}$ resonance can be seen in the
Table 1

Characteristics of some “HF” states and localized \(v^2\)-components of the HFB resonances in \(^{70}\text{Ca}\)

<table>
<thead>
<tr>
<th>State</th>
<th>(\lambda)</th>
<th>(\epsilon_{\lambda}^{0})</th>
<th>(f_{\lambda}^{0})</th>
<th>(\epsilon_{\lambda})</th>
<th>(f_{\lambda})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3\text{s}_{1/2})</td>
<td>(-0.051^b)</td>
<td>(\approx 0.023^b)</td>
<td>(-3.467)</td>
<td>(4.574)</td>
<td></td>
</tr>
<tr>
<td>(2\text{d}_{3/2})</td>
<td>(+0.659)</td>
<td>(0.192)</td>
<td>(-4.369)</td>
<td>(2.150)</td>
<td></td>
</tr>
<tr>
<td>(2\text{d}_{1/2})</td>
<td>(+2.097)</td>
<td>(2.038)</td>
<td>(-4.859)</td>
<td>(4.483)</td>
<td></td>
</tr>
<tr>
<td>(1\text{g}_{9/2})</td>
<td>(-0.276)</td>
<td>(-5.132)</td>
<td>(0.144)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1\text{f}_{5/2})</td>
<td>(-2.666)</td>
<td>(-5.559)</td>
<td>(0.115)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2\text{p}_{1/2})</td>
<td>(-4.261)</td>
<td>(-6.285)</td>
<td>(0.122)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2\text{p}_{3/2})</td>
<td>(-6.193)</td>
<td>(-7.621)</td>
<td>(5 \times 10^{-5})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1\text{g}_{7/2})</td>
<td>(+0.064)</td>
<td>(1.419)</td>
<td>(-8.236)</td>
<td>(2.115)</td>
<td></td>
</tr>
<tr>
<td>(1\text{h}_{11/2})</td>
<td>(+8.106)</td>
<td>(0.915)</td>
<td>(-9.896)</td>
<td>(1.330)</td>
<td></td>
</tr>
<tr>
<td>(1\text{f}_{7/2})</td>
<td>(-8.980)</td>
<td>(-10.071)</td>
<td>(0.145)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1\text{d}_{3/2})</td>
<td>(-12.814)</td>
<td>(-13.663)</td>
<td>(0.277)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2\text{s}_{1/2})</td>
<td>(-14.514)</td>
<td>(-15.265)</td>
<td>(0.306)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1\text{d}_{5/2})</td>
<td>(-17.467)</td>
<td>(-18.103)</td>
<td>(0.086)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1\text{p}_{1/2})</td>
<td>(-22.962)</td>
<td>(-23.328)</td>
<td>(1 \times 10^{-4})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1\text{p}_{3/2})</td>
<td>(-25.564)</td>
<td>(-25.894)</td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Energies and widths are in MeV.
\(^b\) Virtual state (see text).

The resonance characteristics and the energies \(\epsilon_{\lambda}^{0}\) of some bound “HF” states are listed in Table 1 (the first three columns).

The graphical presentation of Figs. 1 and 2 has the advantage that the HFB spectral densities from the lower panels would be smoothly transformed to the corresponding upper “HF” ones if the \(N_{jl}\)’s in the HFB Hamiltonian of Eq. (2) could be smoothly switched off keeping the “HF” part \(h_{jl}\) unchanged and defining \(\mu\) by the particle number condition. In this way, one can easily trace the genetic origin of the irregularities seen in the HFB spectral densities. In particular, the resonance contributions from the particle continuum to the localized \(v^2\)-spectral density can be recognized. The characteristics of the picked-out localized \(v^2\)-parts of the HFB resonances are listed in Table 1 attributing them the genetic “HF” \(jl\) quantum numbers (the fourth and fifth columns). The widths of the “HF” resonance states and of the localized \(v^2\)-components of the HFB resonances have been estimated from a 3-point no-background Breit–Wigner fit of the corresponding maxima. One can see that the discrete “HF” states get widths, i.e., they become the HFB resonances whose \(v^2\)-components are shifted, as expected, to lower \(\epsilon\). The HFB nonresonant background can be also seen in the lower panels of Figs. 1 and 2. The non-localized \(u^2\)- and localized \(v^2\)-components of the HFB resonances are located symmetrically with respect to \(\epsilon = \mu\) (i.e., \(E = 0\)). We remark that all these belong to the same HFB eigenstate, however we do not discuss here the properties of the \(u^2\)-components due to lack of space.

As seen in Fig. 1, the \(1\text{g}_{9/2}\) “HF” state at \(\epsilon_{\lambda}^{0} = -276\) keV becomes, with pairing, a HFB resonance whose localized \(v^2\)-component lies well below the continuum threshold, at \(\epsilon \approx -\Delta\) where \(\Delta\) is the diagonal matrix element of the pairing field over the “HF” states near the Fermi surface. In our case, the energy gap \(2\Delta\) between the two components of this resonance is rather big, \(\approx 10\) MeV. This could lead to the noticeable reduction of the tail of the neutron density distribution (the so-called pairing anti-halo effect [7,8]). However, in the drip-line nuclei, when \(|\mu| \ll \Delta\), near the threshold there appear strong localized \(v^2\)-components from the resonant and non-resonant parts of the particle continuum with different \(jl\). This is clearly seen in the lower panels of Figs. 1 and 2.

The “HF” and HFB partial neutron occupation numbers \(N_{jl}\) in \(^{70}\text{Ca}\) are listed in Table 2. Also given in this table are the numbers of neutrons \(N_{jl}(r > R_1)\) and \(N_{jl}(r > R_2)\) in the tail of \(\rho_n\) beyond the nuclear surface; \(R_1\) and \(R_2\) are defined by \(\rho_n(R_1) = 0.01\) fm\(^{-3}\) and \(\rho_n(R_2) = 0.001\) fm\(^{-3}\), respectively. The choice of \(R_1\) roughly corresponds to the radius \(R_F + t/2\) where \(t = 4(\ln 3)d\) is the width of the diffuse zone approximated in the usual way by the Fermi function \(\propto \{1 + \exp[(r - R_F)/d]\}^{-1}\). The calculated densities for Ca isotopes with \(A = 38\) to 70 (a few of them are shown in Fig. 3) can be well described in the surface region by such a Fermi distribution. For \(^{70}\text{Ca}\), we found \(R_F = 4.60\) fm and \(d = 0.73\) fm which gives \(R_1 = 6.2\) fm (for comparison: in \(^{48}\text{Ca}\), \(R_F = 3.85\) fm, \(d = 0.52\) fm and \(R_1 = 5.0\) fm; thus, approaching the drip line, the size of the diffuse surface zone is significantly increasing). Using a larger radius \(R_2\), one can characterize the \(jl\) composition of the outmost tail of \(\rho_n\) in the “halo” region. Table 2 shows that...
Table 2

Partial neutron occupation numbers in $^{70}$Ca

<table>
<thead>
<tr>
<th>$l_j$</th>
<th>$N_{jl}$</th>
<th>$N_{jl}(r &gt; R_1)^a$</th>
<th>$N_{jl}(r &gt; R_2)^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“HF”</td>
<td>HFB</td>
<td>“HF”</td>
</tr>
<tr>
<td>$s_{1/2}$</td>
<td>4</td>
<td>4.740</td>
<td>0.079</td>
</tr>
<tr>
<td>$p_{3/2}$</td>
<td>8</td>
<td>7.861</td>
<td>0.612</td>
</tr>
<tr>
<td>$p_{1/2}$</td>
<td>4</td>
<td>3.774</td>
<td>0.408</td>
</tr>
<tr>
<td>$d_{5/2}$</td>
<td>6</td>
<td>8.207</td>
<td>0.094</td>
</tr>
<tr>
<td>$d_{3/2}$</td>
<td>4</td>
<td>4.725</td>
<td>0.088</td>
</tr>
<tr>
<td>$f_{7/2}$</td>
<td>8</td>
<td>8.012</td>
<td>0.362</td>
</tr>
<tr>
<td>$f_{5/2}$</td>
<td>6</td>
<td>4.599</td>
<td>0.589</td>
</tr>
<tr>
<td>$g_{9/2}$</td>
<td>10</td>
<td>5.274</td>
<td>1.267</td>
</tr>
<tr>
<td>$g_{7/2}$</td>
<td>0.877</td>
<td>0.211</td>
<td>0.035</td>
</tr>
<tr>
<td>$h_{11/2}$</td>
<td>0.925</td>
<td>0.191</td>
<td>0.026</td>
</tr>
<tr>
<td>$b_{9/2}$</td>
<td>0.292</td>
<td>0.097</td>
<td>0.018</td>
</tr>
<tr>
<td>$i_{13/2}$</td>
<td>0.321</td>
<td>0.087</td>
<td>0.014</td>
</tr>
<tr>
<td>$i_{11/2}$</td>
<td>0.136</td>
<td>0.059</td>
<td>0.011</td>
</tr>
<tr>
<td>$j_{15/2}$</td>
<td>0.144</td>
<td>0.052</td>
<td>0.009</td>
</tr>
<tr>
<td>$j_{13/2}$</td>
<td>0.054</td>
<td>0.035</td>
<td>0.008</td>
</tr>
<tr>
<td>$k_{17/2}$</td>
<td>0.040</td>
<td>0.026</td>
<td>0.006</td>
</tr>
<tr>
<td>$k_{15/2}$</td>
<td>0.019</td>
<td>0.016</td>
<td>0.005</td>
</tr>
<tr>
<td>$\Sigma_{lj}$</td>
<td>50</td>
<td>50.00</td>
<td>3.500</td>
</tr>
</tbody>
</table>

$a$ $R_1$ is defined by the condition $\rho_m(R_1) = 0.01$ fm$^{-3}$.

$b$ $R_2$ is defined by the condition $\rho_m(R_2) = 0.001$ fm$^{-3}$.

The pairing makes the $N_{jl}$ distribution more “uniform” than in the “HF” case, the major effect is due to the depletion of the discrete “HF” orbitals $g_{9/2}$, $f_{7/2}$ and $f_{5/2}$ and filling the resonance structures from the “HF” continuum ($3s_{1/2}$, $2d_{5/2}$, etc., see Table 1). The partial neutron numbers in the “halo” region are also distributed more uniformly, the contributions from the states with higher angular momenta, up to $l = 8$, are not negligible. Thus, in our calculations, the composition of the extended tail of the neutron density distribution in $^{70}$Ca is determined by a collective pairing effect.

The numbers of neutrons beyond the diffusive surface, $N_{\text{halo}}$, calculated both for even and odd Ca isotopes with $A = 38$ to 70 are shown in the right insert in Fig. 3. Depending on the choice of the upper limit for the tail density, 0.01, 0.003 or 0.001 fm$^{-3}$, $N_{\text{halo}}$ varies from 1.41 to 5.30, from 0.47 to 2.35 and from 0.19 to 1.04, respectively; the relative increase of $N_{\text{halo}}$ for the latter choice reaches a factor of 5.5. Irregularities seen in the behavior of $N_{\text{halo}}$ are due to the odd-even effect. Odd isotopes are calculated with blocking [13], the heaviest one is predicted to be $^{55}$Ca. The neutron density in this drip-line nucleus is also plotted in Fig. 3, the one-neutron “halo” tail of $\rho_n$ in this case originates from the $2p_{1/2}$ blocked level with $\varepsilon_n = -820$ keV.

The rms matter radii $r_m$ are shown in the left insert in Fig. 3. The staggering seen in the lighter Ca isotopes is generated by the gradient pairing force (1) whose parameters were chosen to reproduce both the neutron separation energies $S_n$ and anomalous behavior of charge radii [13]. One may suspect that this force yields too strong pairing near the drip line where the surface becomes more diffuse and the repulsive gradient term in (1) decreases; as the result we get $\Delta \approx 5$ MeV for $^{70}$Ca and $\Delta \approx 1.8$ MeV around $^{44}$Ca, the latter being in agreement with experiment. Unfortunately, the density dependence of the effective pair-

![Figure 3](image-url)
Fig. 4. Binding energies \( E_b \) (lower curves, left scale) and two-neutron separation energies \( S_{2n} \) (upper curves, right scale) in even Ca isotopes. Solid (dashed) lines connect the points resulting from the spherical HFB calculations with functional DF3 and gradient (“constant”) pairing force (see text). Experimental data and those derived from systematics are from [17].

The pairing force is not well known, and the extrapolation to the drip line with parametrization (1) may be questionable. Quite different results could be obtained with another kind of the pairing force. One can use, for example, instead of (1), a simple \( \delta \)-interaction without \( \rho \)-dependence, with \( \mathcal{F}_\delta = -0.66C_0 \), which also reproduces well the experimental \( S_0 \) values, and calculate then the whole Ca chain. The matter radii obtained with such a “constant” force are shown by the dotted line in the left insert in Fig. 3, and the binding energies together with two-neutron separation energies are presented in Fig. 4 in comparison with the results for the force (1). The behavior of \( r_m \) versus \( A \) for “constant” \( \mathcal{F}_\delta \) is relatively smooth, without noticeable staggering; the odd- and even-A drip-line nuclei occur to be \( ^{57}\text{Ca} \) and \( ^{70}\text{Ca} \), respectively. Around these nuclei, weak irregularities can be seen in the behavior of \( r_m \). \( ^{70}\text{Ca} \) comes out with \( \Delta = 0 \), i.e., pairing correlations do not appear for “constant” \( \mathcal{F}_\delta \) in this nucleus; besides, the calculated \( S_{2n} \) value is negative, \(-236\) keV (see Fig. 4), so \( ^{70}\text{Ca} \) could be a two-neutron drip-line double-magic emitter. As seen in Fig. 4, the binding energies \( E_b \) and \( S_{2n} \) values are reproduced fairly well with both kinds of the pairing force, though a better description is obtained with the gradient force (1). Systematic deviations start at the mass numbers right beyond the measured region. Clearly, new experimental data and more extensive theoretical studies are needed to find still better parametrizations of the effective pairing force and improve the predictive power of self-consistent mean-field models.

In conclusion: we have presented a numerically exact solution of the HFB problem for the cutoff local energy-density functional. The Green’s function formalism with physical boundary conditions has been used. The localization of the particle continuum in the drip line nucleus \( ^{70}\text{Ca} \) has been analyzed in terms of the partial spectral densities for the HFB eigenstates whose spectrum is fully continuous. The genetic relation to the “HF” (mean-field) partial level densities has been discussed, and the contributions from the resonant and nonresonant particle continuum to the localized ground state, as well as the shifting and broadening of the discrete hole orbitals have been clearly revealed. The composition of the extended neutron distribution in the “halo” region beyond the diffuse surface has been analyzed in terms of the partial occupation numbers, and its formation has been shown to be a collective pairing effect.

Acknowledgements

Partial support of this work by the Deutsche Forschungsgemeinschaft and by the Russian Foundation for Basic Research (project 98-02-16979) is gratefully acknowledged.

References

Particle densities in heavy ion collisions at high energy
and the dual string model

J. Dias de Deus, R. Ugoccioni *

CENTRA and Departamento de Física (I.S.T.), Av. Rovisco Pais, 1049-001 Lisboa, Portugal

Received 12 August 2000; received in revised form 30 August 2000; accepted 5 September 2000

Editor: R. Gatto

Abstract

We analyse recent results on charged particle pseudo-rapidity densities from RHIC in the framework of the Dual String Model, in particular when including string fusion. The model, in a simple way, agrees with all the existing data and is consistent with the presence of the percolation transition to the Quark–Gluon Plasma already at the CERN–SPS.

Recent results on charged particle pseudo-rapidity densities in central Au + Au collisions, at √s = 56 and √s = 130 A GeV, presented by the PHOBOS Collaboration, at RHIC, [1], give very interesting information that may help to clarify the way the expected Quark–Gluon Plasma (QGP) is approached as the energy increases. Those data also allow to select among different models of particle production. As in this experiment the charged particle densities and the average number of participating nucleons are simultaneously measured, that provides additional strong constraints to models.

As nuclei are made up of nucleons, it is natural to start by building nucleus–nucleus collisions as resulting from superposition of nucleon–nucleon collisions, in the way it is done in the Glauber model approach and generalisations of it. In one (low energy) limit the nucleons are seen as structureless and emit particles only in their first collision: this is the wounded nucleon model [2]. The prediction for particle density, when N_A nucleons from each one of the nuclei in a AA collision participate, is

\[ \frac{dN}{dy}_{N_A N_A} \cong \frac{dN}{dy}_{pp} N_A, \tag{1} \]

where \( \frac{dN}{dy} \) is the particle rapidity (or pseudo-rapidity) density (for \( N_A N_A \) and nucleon–nucleon collisions). If the nucleon is seen as made up of quarks and gluons, with a growing number of participating sea quarks and gluons as the energy increases, one anticipates dominance of multi-collision processes [3] and the relation

\[ \frac{dN}{dy}_{N_A N_A} \cong \frac{dN}{dy}_{pp} v_N, \tag{2} \]

to hold, where \( v_N \) is the number of nucleon–nucleon collisions when \( N_A \) nucleons participate. Elementary multi-scattering arguments [4] give

\[ v_N = N_A^{4/3}, \tag{3} \]

In Fig. 1, together with the PHOBOS data, we have presented the quantity \( \frac{1}{N_A} \frac{dN}{dy}_{N_A N_A} \) as function...
Fig. 1. Pseudo-rapidity density normalised per participant pair as a function of c.m. energy. The lines give predictions for the wounded nucleon model Eq. (1) (solid line), the pure multicollision approach Eq. (2) (dotted line), and the Dual String Model, without fusion Eq. (7) (dash-dotted line) and with fusion Eq. (14) (dashed line). AA points are taken from [1, 19, 20], pp and pN from [5 – 8].

of the c.m. energy $\sqrt{s}$ for the bounds (1) — solid line, and (2) with (3) — dotted line. We used for $dN/dy_{pp}$ the parametrisation $0.957 + 0.0458 \ln(\sqrt{s}) + 0.0494 \ln^2(\sqrt{s})$, with $\sqrt{s}$ in GeV, which fits data from pp and $p\bar{p}$ non-single-diffractive collisions for c.m. energies $\sqrt{s} \geq 22$ GeV. The parametrisation used in [1, 8] could not be used here because it does not fit NA22 data.

In the Dual String Model (DSM), i.e., the Dual Parton Model [9] with the inclusion of strings [10], the limits referred to above appear in a natural way. The valence quarks of the nucleon produce particles, via strings, only once — this is the wounded nucleon model case — and production is proportional to the number $N_A$ of participant nucleons (Fig. 2(a)). As the energy and $N_A$ increase the role of sea quarks and gluons increases, they interact and produce, again via strings, particles, and the number of collisions $\nu$ becomes the relevant parameter (Fig. 2(b)).

One should notice that the diagram of Fig. 2(b) may be interpreted as multiple inelastic scattering, either internally within a given nucleon–nucleon collision or externally involving interactions with different nucleons. On the other hand, this diagram may appear repeated several times.

Following [4], and taking into account the basic diagrams of Figs. 2(a) and 2(b), we now write an expression for the particle pseudo-rapidity density,

$$
\frac{dN}{dy}_{|N_A \nu h} = N_A [2 + (2k - 1)\alpha] h + (\nu_{N_A} - N_A) 2k \alpha h,
$$

(4)

where $h$ is the height of the valence–valence rapidity plateau, $\alpha$ is the relative weight of the sea–sea (including gluons) plateau and $k$ is the average number of string pairs per collision. The diagrams of Figs. 2(a) and 2(b) correspond to $k = 1$. However, as we mentioned above, the diagram of Fig. 2(b) can be iterated with $k \geq 1$ being, in general, a function of energy. The number of nucleon–nucleon collisions is, of course,

$$
N_A + (\nu_{N_A} - N_A) = \nu_{N_A},
$$

(5)
and the number $N_s$ of strings is
\[ N_s = N_A[2 + 2(k - 1)] + (v_{N_A} - N_A)2k \]
\[ = 2k\nu_{N_A}. \] (6)

The first term on the right-hand side of Eq. (4) is just a sum over nucleon–nucleon scattering contributions (including internal parton multiple scattering) and we can thus write
\[ \left. \frac{dN}{dy} \right|_{N_A N_A} = \left. \frac{dN}{dy} \right|_{pp} N_A + (\nu_{N_A} - N_A)2k\alpha h, \] (7)
with
\[ \left. \frac{dN}{dy} \right|_{pp} = [2 + 2(k - 1)\alpha']h. \] (8)

If external multiple scattering is absent, by putting $\nu_{N_A} = N_A$, one obtains the wounded nucleon model limit, Eq. (1). If multiple scattering dominates, $k \gg 1$, we obtain the limit of Eq. (2).

In order to make more transparent the comparison with data, we shall rewrite Eq. (7), by using Eq. (8) and Eq. (3), in the form
\[ \frac{1}{N_A} \left. \frac{dN}{dy} \right|_{N_A N_A} = \left. \frac{dN}{dy} \right|_{pp} N_A^{1/3} - (N_A^{1/3} - 1)(1 - \alpha)h. \] (9)

We show the result of this model in Fig. 1 (dash-dotted line). From comparison of Eq. (8) with $pp$ data at low energy, $k \approx 1$, one obtains $h \approx 0.75$. The parameter $\alpha$ in Eq. (9) was put equal to 0.05.

In the Dual String Model the strings interact, the simplest interaction being fusion due to overlap in the transverse plane [10]. This is the mechanism that leads to percolation and to the Quark–Gluon Plasma formation [11–13]. When strings fuse, the strength of the colour field is reduced in comparison with the colour field generated by the same number of independent strings. This is essentially due to the random sum of colour vectors [14]: $Q_\eta^2 = \sum_{i=1}^n Q_i^2$ and $Q_\eta = \sqrt{n} \bar{Q}$ if all the $n$ strings are of the same type.

Introducing the dimensionless transverse density percolation parameter $\eta$,
\[ \eta = \frac{\eta^2 N_s}{R_{NA}^2}, \] (10)
where $r_s$ is the string transverse radius (we shall take $r_s = 0.2$ fm, see [11,15]), $R_{NA}$ the radius of the interaction area ($R_{NA} \approx 1.14 N_A^{1/3}$) and $N_s$ the number of strings, the effective reduction factor in particle production is [16],
\[ F(\eta) = \sqrt{\frac{1 - e^{-\eta}}{\eta}}. \] (11)

As $\eta \to 0$, $F(\eta) \to 1$ (no fusion) and as $\eta \to \infty$, $F(\eta) \to 1/\sqrt{\eta} \approx 1/\sqrt{N_s}$ (all the strings fuse).

We can consider the parameter $\eta$ in two situations. In nucleon–nucleon internal interactions, and then
\[ \eta_{pp} = \frac{r_s^2}{R_{pp}^2}[2 + (2k - 1)] = \frac{r_s^2}{R_{pp}^2}2k. \] (12)

At present energies $\eta_{pp}$ is negligible, $\eta_{pp} \approx 10^{-2} \div 10^{-1}$. But we can also consider $\eta$ in external interactions, with
\[ \eta_{N_A N_A} = \frac{r_s^2}{R_{NA}^2}2k(\nu_{N_A} - N_A) \]
\[ \approx \left( \frac{r_s}{1.14} \right)^2 2k(N_A^{1/3} - 1)N_A^{1/3}. \] (13)

For $N_A \approx 10^2$, as in [1], $\eta_{N_A N_A} > 10\eta_{pp}$ and we shall then only consider $\eta_{N_A N_A}$.

Eq. (4) with string fusion becomes
\[ \frac{1}{N_A} \left. \frac{dN}{dy} \right|_{N_A N_A} = \left. \frac{dN}{dy} \right|_{pp} \left[ 1 - F(\eta_{N_A N_A}) \right] \]
\[ + F(\eta_{N_A N_A}) \left. \frac{dN}{dy} \right|_{pp} N_A^{1/3} \]
\[ - (N_A^{1/3} - 1)(1 - \alpha)h. \] (14)

In Fig. 1 we have also shown the prediction of the DSM with string fusion (dashed line) again with $h = 0.75$ and $\alpha = 0.05$. The deviation from the wounded nucleon model limit becomes weaker and the agreement with PHOBOS data is quite satisfactory.

We would like now to make a few comments:

1. The predictions for particle densities in central Pb + Pb collisions of the DSM without fusion and of the DSM with fusion are very different at $\sqrt{s} = 200$ A GeV (RHIC) and at $\sqrt{s} = 5.5$ A TeV (LHC) as can be seen in the following table, showing the average pseudo-rapidity density in the interval $[-1, 1]$:
2. The models considered here are essentially soft models. The parameters of the elementary collision densities, \( h \) and \( \alpha \), were assumed constant, all the energy dependence being attributed to the parameter \( k \), the average number of string pairs per elementary collision. If \( h \) and \( \alpha \) are allowed to grow with energy, as a result, for instance, of semi-hard effects, the parameter \( k \) may then have a slower increase than the one obtained here.

3. The value found for \( \alpha \), \( \alpha \approx 0.05 \), means that the height of the sea–sea plateau is much smaller than the height of the valence–valence plateau. By noticing that for valence–valence collisions the two strings stretch all over forward/backward rapidity without much overlap, while for sea–sea collisions the two strings do overlap, the value found for \( \alpha \) means

\[
\frac{dN}{dy}_{sea-sea} \approx 0.1 \frac{dN}{dy}_{val-val}
\]

(15)

4. In our Dual String Model with fusion, the parameter \( \eta N_A N_A \) at the CERN-SPS has the value \( \eta N_A N_A \approx 1.8 \), larger than the critical density (\( \eta_c \approx 1.12 \pm 1.17 \)) which means that `percolation transition is already taking place at \( \sqrt{s} = 20 \) GeV, even allowing for non-uniform matter distribution in the nucleus (\( \eta_c \approx 1.5 \)) [17]; this result is valid even with \( k = 1 \). The observed anomalous \( J/\psi \) suppression [18] may then be a signature of the percolation transition to the Quark–Gluon Plasma [13].

After the submission of this paper, we became aware of two papers on the same subject [19,20].

1. From the paper of Wang and Gyulassy [19] we realised that the HIJING point at 200 A GeV in Ref. [1] was 20% too high. This point was corrected in our figure.

2. In the WA98 Collaboration paper on scaling of particle and transverse energy production [20], results on \( dN/dy \) were presented in \( Pb + Pb \) collisions at 158 A GeV. This point was included in Fig. 1 but not taken into account in the calculations. It somewhat disagrees with the NA49 point presented in [1].

Acknowledgements

We thank X.-N. Wang and U. Heinz for comments on recent work. R.U. gratefully acknowledges the financial support of the Fundação Ciência e Tecnologia via the “Sub-Programa Ciência e Tecnologia do 2º Quadro Comunitário de Apoio”.

References

The gluon spin in the chiral bag model

Hee-Jung Lee a,*, Dong-Pil Min a, Byung-Yoon Park b, Mannque Rho c, d, Vicente Vento e

a Department of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-742, South Korea
b Department of Physics, Chungnam National University, Daejon 305-764, South Korea
c Service de Physique Théorique, CE Saclay 91191 Gif-sur-Yvette, France
d School of Physics, Korea Institute for Advanced Study, Seoul 130-012, South Korea
e Departamento de Física Teorica and Institut de Física Corpuscular, Universitat de València and Consejo Superior de Investigaciones Científicas, E-46100 Burjassot (València), Spain

Received 5 June 2000; accepted 22 August 2000
Editor: J.-P. Blaizot

Abstract

We study the gluon polarization contribution at the quark model renormalization scale to the proton spin, \( \Gamma \), in the chiral bag model. It is evaluated by taking the expectation value of the forward matrix element of a local gluon operator in the axial gauge \( A^0 \). It is shown that the confining boundary condition for the color electric field plays an important role. When a solution satisfying the boundary condition for the color electric field, which is not the conventionally used but which we favor, is used, the \( \Gamma \) has a positive value for all bag radii and its magnitude is comparable to the quark spin polarization. This results in a significant reduction in the relative fraction of the proton spin carried by the quark spin, which is consistent with the small flavor singlet axial current measured in the EMC experiments. © 2000 Published by Elsevier Science B.V.

Keywords: Quark; Gluon; Polarization; Chirality; Bag model

The EMC experiment [1] revealed the surprising fact that less than 30% of the proton spin may be carried by the quark spin. This is at variance with what one expects from non-relativistic or relativistic constituent quark models. This discrepancy — so-called “the proton spin crisis” — can be understood as an effect associated with the axial anomaly [2]. For example it has been argued [3] that the experimentally measured quantity is not merely the quark spin polarization \( \Delta \Sigma \) but rather the flavor singlet axial charge, to which the gluons contribute through the axial anomaly. Another interpretation [4] is that the anomalous gluons can induce a sea-quark polarization through the axial anomaly, which cancels the spin from the valence quarks, if the gluon spin component is positive. These explanations, and possibly others, could be reconciled if one would establish that they are gauge dependent statements, while the measured quantity is gauge-invariant [5].

Although not directly observable, an equally interesting quantity related to the proton spin is the fraction
of spin in the proton that is carried by the gluons. In Ref. [6], the gluon spin \( \Gamma \) is introduced as
\[
\frac{1}{2} = \frac{1}{2} \Sigma + L_Q + \Gamma + L_G, \tag{1}
\]
where \( L_{Q,G} \) is the orbital angular momentum of the corresponding constituent and \( \Gamma \) is defined as the integral of the polarized gluon distribution in analogy to \( \Sigma \). The spin of course is gauge-invariant but the individual components in (1) may not be. \( \Gamma \) can be expressed as a matrix element of products of the gluon vector potentials and field strengths in the nucleon rest frame and in the \( \Lambda^+ = 0 \) gauge. When evaluated with the gluon fields responsible for the \( N - \Delta \) mass splitting, \( \Gamma \) turns out to be negative, \( \Gamma \approx -0.1 \text{mag} \), in the MIT bag model and even more so in the non-relativistic quark model.

By contrast, there are several other calculations that give results with opposite sign. For example, the QCD sum rule calculation [7] yields a positive value \( 2\Gamma \approx 2.1 \pm 1.0 \) at 1 GeV\(^2\). In Ref. [8], it is suggested that the negative \( \Gamma \) of Ref. [6] could be due to neglecting “self-angular momentum”. The authors of [8] show that when self-interaction contributions are included, one obtains a positive value \( \Gamma \approx 0.12 \) in the Isgur–Karl quark model at the scale \( \mu_0^2 \approx 0.25 \text{ GeV}^2 \). Once the gluons contribute a significant fraction to the proton spin, due to the normalization Eq. (1), the relative fraction of the proton spin lodged in the quark spin changes. Thus, the positive gluon spin seems to be consistent with the EMC experiment.

In this letter, we address this issue in the chiral bag model and pay special attention to the confining boundary condition for the gluon fields.

Let us start by briefly reviewing how the gluon spin operator is derived in Refs. [6,11]. From the Lagrangian
\[
\mathcal{L} = -\frac{1}{2} \text{Tr} (F^{\mu\nu} F_{\mu\nu}), \tag{2}
\]
with \( F^{\mu\nu} = \frac{2}{i} F_{\mu\nu} \), one gets the gluon angular momentum tensor
\[
M^{\mu\nu;k} = 2 \text{Tr} \left( x^\nu F^{\mu\alpha} F_a^\lambda - x^\lambda F_{\mu\alpha} F_a^\nu \right) - (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}) L. \tag{3}
\]
Integrating by parts, we have
\[
M^{\mu\nu;k} = 2 \text{Tr} \left( -F^{\mu\alpha} (x^\nu \delta^\lambda - x^\lambda \delta^\nu) A_a + F^{\mu\lambda} A^\nu \right.
+ F^{\nu\lambda} A^\mu + \partial_a (x^\nu F_{\mu\alpha} A^\lambda - x^\lambda F_{\mu\alpha} A^\nu)
\left. + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}) \right). \tag{4}
\]
It seems reasonable to interpret the first term as the gluon orbital angular momentum contribution and the second as that of the gluon spin, while recalling that this is a gauge dependent statement. We will not consider the fourth term hereafter, since it contributes only to boosts. In Refs. [6,11], the third term is also dropped as is done in the open space field theory. When finite space is involved, as in the bag model, dropping this term requires that the gluon fields satisfy boundary conditions on the surface of the region, as we next show. Let us express the gluon angular momentum operator in terms of the Poynting vector, i.e.,
\[
\mathbf{J}_G = 2 \text{Tr} \int_V d^3 r \left[ \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) \right]. \tag{5}
\]
Now doing the partial integration for \( \mathbf{B} = \nabla \times \mathbf{A} \), we have
\[
J_G^k = 2 \text{Tr} \left\{ \int_V d^3 r \left( E^k (\mathbf{r} \times \nabla)^k A_l + (\mathbf{E} \times \mathbf{A})^k \right)
- \int_{\partial V} d^2 r \left( \mathbf{r} \cdot \mathbf{E} \right) (\mathbf{r} \times \mathbf{A})^k \right\}. \tag{6}
\]
The surface term is essential to make the whole angular momentum operator gauge-invariant, but the surface term only vanishes, if the electric field satisfies the boundary condition on the surface,
\[
\mathbf{r} \cdot \mathbf{E} = 0. \tag{7}
\]
This is just the MIT boundary condition for gluon confinement. However, the static electric field traditionally used [9] does not satisfy this condition. Instead color singlet nature of the hadron states is imposed to assure confinement globally.

We next show that the negative \( \Gamma \) of Ref. [6] results if this procedure to confine color is imposed. To proceed, we choose the \( \Lambda^+ = 0 \) gauge and write the gluon spin in a local form as
\[
\Gamma = \langle p, \uparrow | 2 \text{Tr} \int_V d^3 x \left( (\mathbf{E} \times \mathbf{A})^3 + \mathbf{B}_\perp \cdot \mathbf{A}_\perp \right) | p, \uparrow \rangle, \tag{8}
\]
where \( \perp \) denotes the direction perpendicular to the proton spin polarization and the superscript + indicates the light cone coordinates defined as \( x^\pm = x^0 \pm x^3 \). We shall evaluate this expression by incorporating the exchange of the static gluon fields between \( i \) th and \( j \) th quarks (\( i \neq j \)) which are responsible for the \( N - \Delta \) mass splitting in the bag model.

In the chiral bag model, the static gluon fields are generated by the color charge and current distributions of the \( i \) th valence quark given by [10]

\[
J^\alpha_i(r) = \frac{gs}{4\pi} \rho(r) \frac{\lambda^\alpha_i}{2},
\]

where \( \rho(r) \) and \( \mu_i(r) \) are, respectively, the quark number and current densities determined by the valence quark wave functions. (See Ref. [10] for their explicit formulas.) They are very similar in form to those of the MIT bag model. There is, however, an essential difference, namely, that the spin in the chiral bag model is given by the collective rotation of the whole system while in the MIT bag it is given by an individual contribution of each constituent, i.e., there is no index \( i \) in the spin operator in Eq. (10).

The charge and current densities yield the color electric and magnetic fields as

\[
E^\alpha_i = \frac{gs}{4\pi} \frac{Q(r)}{r^2} \frac{\lambda^\alpha_i}{2} \hat{r},
\]

\[
B^\alpha_i = \frac{gs}{4\pi} \left( 2M(r) + \frac{\mu_i(r)}{R^3} \right) \frac{\lambda^\alpha_i}{2} + 3\hat{r} \cdot \frac{\mu_i(r)}{r^3} \frac{\lambda^\alpha_i}{2},
\]

where

\[
\mu_i(r) = \int_0^r \! dr' \mu_i(r'),
\]

\[
M(r) = \int_r^\infty \! dr' \frac{\mu_i(r')}{r'^3}.
\]

The quantity \( Q(r) \) can be determined from Maxwell’s equations. The most general solution can be written as

\[
Q_i(r) = 4\pi \int_0^r \! dr' r'^2 \rho(r'),
\]

with an arbitrary \( \lambda \). The standard procedure is to choose \( \lambda = 0 \) so that the electric field is regular at the origin. This has been the adopted convention in the early days [9]. We will refer to this solution as \( Q_0(r) \). However, \( Q_0(r) \) does not satisfy the local boundary condition (7), since it is normalized as \( Q_0(R) = 1 \). In Ref. [9], the fact that hadrons are color singlet states, had to be imposed in order to justify the use of this solution.

Another solution is obtained by setting \( \lambda = R [10] \) and we will look for its consequences here. This choice satisfies the local boundary condition but requires the relaxation of the continuity of the electric fields inside the bag. It has been shown in [10], and will be shown here again, that these two solutions, \( Q_0(r) \) and \( Q_R(r) \), lead to dramatic differences for certain observables.

By using the static Green functions and the Coulomb gauge condition, one can obtain time-independent scalar and vector potentials from the charge and current densities, (9) and (10),

\[
\phi^\alpha_i(r) = \frac{gs}{4\pi} \frac{Q_i(r)}{r} \frac{\lambda^\alpha_i}{2},
\]

\[
\psi^\alpha_i(r) = \frac{gs}{4\pi} \frac{h_i(r)(S \times r)}{r^2} \frac{\lambda^\alpha_i}{2},
\]

where

\[
h_i(r) = \left( \frac{\mu_i(r)}{2R^3} + \frac{\mu_i(r)}{r^3} + M(r) \right).
\]

From these, the appropriate scalar and vector potentials satisfying the \( A^+ = 0 \) gauge condition can be constructed:

\[
A_i^{\alpha 0}(r) = \phi^\alpha_i(r),
\]

\[
A_i^\alpha(r) = \psi^\alpha_i(r) - \nabla \int_0^\infty \! d\xi \, \phi^\alpha_i(x, y, \xi),
\]
where the direction of the proton polarization is taken as that of the $z$-axis. Finally, we obtain

$$\Gamma_\lambda = \sum_{i \neq j} \sum_{a=1}^{8} \langle p, \uparrow \mid \left( 2 \int_{V} d^3 x \left( E_i^a \times U_j^a \right)^3 \right) + \int_{V} d^2 x \, \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} \left( U_i^{a1}(x) \int_{0}^{\zeta} d\xi \, E_i^{a2}(x, y, \zeta) \right) - U_i^{a2}(x) \int_{0}^{\zeta} d\xi \, E_i^{a1}(x, y, \zeta) \right)\rangle |p, \uparrow \rangle$$

$$= \frac{4}{3} \alpha \int_{0}^{R} r \, dr \, Q_\lambda(r) (h(R) - 2h(r)),$$

(20)

where $\alpha = g_s^2 / 4\pi$. The numerical factor in front of the final formula comes from the fact that $\sum_{a} \lambda_i^a \lambda_j^a \lambda_j^a \lambda_i^a$ for baryon $= -8/3$ for $i \neq j$ so that

$$\sum_{i \neq j} \sum_{a=1}^{8} \langle p, \uparrow \mid |p, \uparrow \rangle = -2,$$

(21)

and the integration over the angle yields $1/3$. It is different from $8/9$ of the MIT bag model [6], which comes from the expectation value

$$\sum_{i \neq j} \sum_{a=1}^{8} \langle p, \uparrow \mid |p, \uparrow \rangle = -4/3.$$

(22)

It is interesting to note that, if we naively substitute the static gluon fields $\Phi_i^a$ and $U_i^a$ of Eqs. (16) and (17) satisfying the Coulomb gauge condition into the second term of Eq. (6), we get

$$\Gamma' = -\frac{4}{3} \alpha \int_{0}^{R} r \, dr \, Q_\lambda(r) h(r),$$

(23)

which is the same expression that was used in Refs. [10,12] to evaluate the anomalous gluon contribution to the flavor singlet axial current $a_0$ with the extra factor $(-N_f/2\pi)$, i.e., $a_0 = \Sigma - (N_f/2\pi) \Gamma'_\lambda$. On the other hand, in Ref. [8], the gluon spin $\Gamma'$ instead of $\Gamma$ is used for the anomaly correction term because the calculation is performed in the $A^+ = 0$ gauge.

If the gluons can contribute to the proton spin, then the collective coordinate quantization scheme of the

\begin{align*}
(a) & \\
(b) &
\end{align*}

Fig. 1. The moment of inertia associated with the collective rotation as a function of the bag radius and the proton spin fraction carried by each constituents. In the calculation, we have used (a) the “confined” color electric field with $Q_R(r)$ and (b) the conventional one with $Q_0(r)$. 
chiral bag model has to be modified to incorporate their contribution. That is because there is a natural sum rule namely that the total proton spin must come out to be $\frac{1}{2}$, whatever the various contributions are. In the chiral bag model, where the mesonic degrees of freedom also play an important role, the proton spin is described by the following contributions

$$\frac{1}{2} = \frac{1}{2} \sum + L_Q + \Gamma + L_G + L_M,$$  

where $L_M$, the orbital angular momentum of the mesons, has to be added to Eq. (1). The proton spin is generated by quantizing the collective rotation associated with the zero modes of the classical soliton solution of the model Lagrangian. To the collective rotation, each constituent responds with the corresponding moment of inertia. The moments of inertia of the quarks and mesons, $I_Q$ and $I_M$, have been extensively studied in the literature [14]. Substitution of the color electric and magnetic fields, given by Eqs. (11) and (12) respectively, into Eq. (5) defines a new moment of inertia of the static gluon fields with respect to the collective rotation as

$$\langle J_G \rangle = -I_G \omega,$$  

where the expectation value is taken keeping only the exchange terms, and $\omega$ is the classical angular velocity of the collective rotation.

We show in Figs. 1(a) and (b) the gluon moment of inertia evaluated by using the color electric fields with $Q_R(r)$ and $Q_0(r)$. In the case of $Q_R(r)$, $I_G$ is positive for all bag radii and comparable in size to $I_Q$, the quark moment of inertia. On the other hand, $Q_0(r)$ results in a negative $I_G$. This “negative” moment of inertia may appear to be bizarre but it may not be a problem from the conceptual point of view. The $I_G$ defined by Eq. (25) can be interpreted as the one-gluon exchange correction to the corresponding quantity of the quark phase, which is still positive anyway. The point is that the spin fractionizes in the same way as the moment of inertia does. This means that we have

$$L_Q + \frac{1}{2} \sum = \frac{I_Q}{2(I_Q + I_G + I_M)},$$

$$L_G + \Gamma = \frac{I_G}{2(I_Q + I_G + I_M)},$$

$$L_M = \frac{I_M}{2(I_Q + I_G + I_M)}.$$  

Each fraction as a function of the bag radius is presented in the small boxes inside each figure. Note in the case of adopting $Q_R(r)$ that at the large bag limit the proton spin is equally carried by quarks and gluons somewhat like the momentum of the proton. The negative $I_G$ obtained with $Q_0(r)$, thus, yields a scenario where the gluons are anti-aligned with the proton spin.

Fig. 2. The gluon spin $\Gamma$ as a function of the bag radius. (a) and (b) are obtained with the color electric fields explained in Fig. 1.
The dashed and dash-dotted curves in Figs. 2(a) and (b) show the values for $\Gamma_\lambda$ and $\Gamma_\lambda'$. For comparison, we draw $\frac{1}{2} \Sigma$ by a solid curve. Note that, because of the difference in $I_G$, even $\frac{1}{2} \Sigma$ is different according to which $Q_3$ is used. Again, both $I_0$ and $I_0'$ are anti-aligned with the proton spin. Note of course that the negative $I_0'$ is apparently at variance with the general belief that the anomaly is to cure the proton spin problem.

To conclude, we show in Figs. 3(a) and (b) the flavor-singlet axial current including the $U_A(1)$ anomaly given by

$$a_0 = \Sigma - \langle N_f \alpha / 2\pi \rangle \Gamma_\lambda' . \tag{27}$$

For simplicity, we neglect other contributions to $a_0$ studied in [13]. They show that the positive $\Gamma$ is consistent with the small $a_0$ measured in the EMC experiments. The radius dependence of each component may be viewed as gauge dependence both in color gauge symmetry and in the “Cheshire Cat” gauge symmetry discussed by Damgaard, Nielsen and Sollacher [15].

Acknowledgements

The work of HJL, DPM and BYP is supported in part by the KOSEF grant 1999-2-111-005-5. The work of VV is partially supported by DGICYT grant PB97-1127. We would like to thank KIAS where this work was initiated for the generous support.

References

Abelian dominance in low-energy gluodynamics due to dynamical mass generation

Kei-Ichi Kondo \textsuperscript{a,b,*}, Toru Shinohara \textsuperscript{b}

\textsuperscript{a} Department of Physics, Faculty of Science, Chiba University, Chiba 263-8522, Japan
\textsuperscript{b} Graduate School of Science and Technology, Chiba University, Chiba 263-8522, Japan

Received 25 April 2000; received in revised form 8 August 2000; accepted 6 September 2000
Editor: T. Yanagida

Abstract

We show that off-diagonal gluons and off-diagonal ghosts acquire their masses dynamically in QCD if the maximal Abelian gauge is adopted. This result strongly supports the Abelian dominance in low-energy region of QCD. The mass generation is shown to occur due to ghost–anti-ghost condensation caused by attractive quartic ghost interactions within the Abelian projected effective gauge theory (derived by one of the authors). In fact, the quartic ghost interaction is indispensable for the renormalizability due to nonlinearity of the maximal Abelian gauge. The ghost–anti-ghost condensation is associated with the spontaneous breaking of global $SL(2, \mathbb{R})$ symmetry recently found by Schaden at least for $SU(2)$ case. Moreover we write down a new extended BRS algebra in the maximal Abelian gauge which should be compared with that of Nakanishi–Ojima for the Lorentz gauge. Finally, we argue that the mass generation may be related to the spontaneous breaking of a supersymmetry $OSp(4|2)$ hidden in the maximal Abelian gauge. © 2000 Published by Elsevier Science B.V.

PACS: 12.38.Aw; 12.38.Lg
Keywords: Quark confinement; Topological field theory; Spontaneous symmetry breaking; Magnetic monopole

1. Introduction

The purpose of this Letter is to justify the Abelian dominance in the low-energy region of QCD defined in the maximal Abelian (MA) gauge. This story begins with the idea of ‘t Hooft [1] called the Abelian projection. Immediately after this proposal, a hypothesis of Abelian dominance in low-energy physics of QCD was claimed by Ezawa and Iwazaki [2]. By adopting the MA gauge invented by Kronfeld et al. [3], actually, Abelian dominance was discovered by Suzuki and Yotsuyanagi [4] a decade ago based on numerical simulation on a lattice and has been confirmed by the subsequent simulations, see [5] for reviews. However there is no analytical derivation or proof of Abelian dominance so far. How can one justify or prove the Abelian dominance in low-energy physics in QCD?

In a previous paper [6], we tried to give an answer by constructing an effective Abelian gauge theory which is considered to be valid in the low-energy region of QCD. We called it the Abelian-projected effective gauge theory (APEGT), although this name is somewhat misleading as will be explained below. Before this work, a number of low-energy effective gauge theories were already proposed based on the idea of Abelian-projection. However, we should keep...
in mind that these models were constructed by ignoring all the off-diagonal gluon fields from the beginning under the assumption of the Abelian dominance and/or the Abelian electro-magnetic duality, even if they can well describe some features of confinement physics in QCD. In fact, they could not be derived by starting with the QCD Lagrangian. Therefore, one can neither answer how the off-diagonal gluon fields influence the low-energy physics, nor how the Abelian electro-magnetic duality could appear from the non-Abelian gauge theory.

In contrast to these models, the APEGT is a first-principle derivation of effective theory from QCD. It was shown [6] that the off-diagonal gluons do affect the low-energy physics in the sense that off-diagonal gluons renormalize the resulting effective Abelian gauge theory. Moreover, the coupling constant of the effective Abelian gauge theory has the renormalization-scale dependence governed by the renormalization group $\beta$-function which is exactly the same as the original QCD, thereby, exhibiting the asymptotic freedom. In this sense, the APEGT reproduces a characteristic feature of the original QCD, asymptotic freedom, even if it is an Abelian gauge theory. In addition, it was demonstrated how the dual Abelian gauge theory (magnetic theory) can in principle be obtained in the low-energy region of QCD. Actually, it is possible to show [6] that monopole condensation leads to a dual Ginzburg–Landau theory supporting the dual superconductor picture [7] of QCD vacuum. A version of the non-Abelian Stokes theorem indicates that the Wilson loop operator can be expressed in terms of diagonal gluon fields, see, e.g., [8]. Combining these results, we are able to explain the Abelian dominance in quark confinement, see [8].

In the derivation of APEGT, however, we have treated the off-diagonal gluons as if they are massive in the MA gauge. This assumption was necessary to justify the procedure of integrating out the off-diagonal gluon fields based on the functional integral, since this integration was interpreted as a step of the Wilsonian renormalization group (RG) of integrating out the massive (high-energy) modes. In view of this, the resultant APEGT is regarded as the low-energy effective theory which is meaningful at least in the length scale $R > M_A^{-1}$ with $M_A$ being the mass of off-diagonal gluons. In the derivation, moreover, we have integrated out the off-diagonal ghosts and anti-ghosts. This step was also necessary to reproduce the correct coefficient of the $\beta$-function.

To really justify the Abelian dominance, therefore, we need to show that the off-diagonal gluons and ghosts become massive in the MA gauge within the same framework as the APEGT. The main purpose of this Letter is to demonstrate that this is indeed the case. Actually, the derivation of off-diagonal gluon mass and ghost mass can be performed within the setting up of the previous paper [6]. In the previous work, we have ignored the ghost self-interactions in the derivation of the APEGT, simply because they were not necessary to obtain the asymptotic freedom. In this Letter we properly take the ghost self-interaction into account. We show that the quartic ghost self-interaction among off-diagonal ghosts leads to two kinds of ghost condensation. As a result, the off-diagonal gluons and off-diagonal ghosts (anti-ghosts) acquire non-zero masses.

It should be remarked that the quartic ghost interaction term is generated by integrating out the off-diagonal gluon fields, even if such an interaction term is absent in the original Lagrangian of QCD. This is due to the nonlinearity of the MA gauge. In general, quartic ghost self-interaction terms are generated in the nonlinear gauge due to radiative corrections. For the theory to be renormalizable, therefore, we need to incorporate quartic ghost self-interaction in the bare Lagrangian via the gauge-fixing and FP ghost term, as pointed out already in Appendix B of [6]. If so, how one can specify the quartic ghost interaction? As a possibility, we introduce it so as to keep the supersymmetry [9] which is quite different from that of supersymmetric theory in theoretical particle physics. It is hidden in the gauge fixing and ghost part of the MA gauge, while there is no supersymmetry in the Yang–Mills Lagrangian, since we are dealing with the usual QCD without supersymmetry. This requirement determines almost uniquely the ghost self-interaction. A special case was already examined in the previous paper [8 – 13] in a slightly different context. In this point, this Letter supplements the previous paper [6] by taking into account the ghost self-interactions properly.
2. QCD in the modified MA gauge

For the gauge group \( G = SU(N) \), we consider the Cartan decomposition of the gauge potential into the diagonal and off-diagonal components,

\[
A_\mu(x) = A_\mu^D(x) + A_\mu^O(x),
\]

where \( A = 1, \ldots, N^2 - 1 \). Then the maximal Abelian (MA) gauge [3] is defined as follows. We define the functional of off-diagonal gluon fields, \( R[A] := \int d^4x \frac{1}{2} A_\mu^O(x) A^{\mu O}(x) \). The MA gauge is obtained by minimizing the functional \( R[A] \) with respect to the local gauge transformation \( U(x) \) of the functional of off-diagonal gluon fields. The MA gauge is a partial gauge fixing which fixes the covariant action of QCD in the MA gauge is given by

\[
S_{\text{MA}} = \frac{1}{4} \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu}(x) F_{\rho \sigma}(x),
\]

where \( \epsilon^{\mu \nu \rho \sigma} \) is the completely antisymmetric tensor.

The modified MA gauge is different from the naive MA gauge by the ghost self-interaction, since

\[
S_{\text{GF+FP}} = - \int d^4x i \delta_B \left[ \bar{C} \left( \partial_\mu A^\mu + \frac{\alpha}{2} B \right) \right] - i \frac{\eta}{4} \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu}(x) F_{\rho \sigma}(x).
\]

This is nothing but the background-field gauge with the background field \( a_{\mu}^D \). After the MA gauge is adopted, the original gauge group \( G = SU(N) \) is broken to the maximal torus group \( H = U(1)^{N-1} \). The MA gauge is a partial gauge fixing which fixes the gauge degrees of freedom for the coset space \( G/H \).

Following the well-known procedure, the manifest covariant action of QCD in the MA gauge is given by

\[
S_{\text{QCD}} = S_{\text{YM}} + S_{\text{GF+FP}} + S_F,
\]

\[
S_{\text{YM}} = \int d^4x \frac{1}{4} F_{\mu \nu}(x) F^{\mu \nu}(x),
\]

\[
S_F = \int d^4x \bar{\psi} i\gamma^\mu \left( \partial_\mu - i g A_\mu^D \right) \psi,
\]

\[
S_{\text{GF+FP}} = - \int d^4x i \delta_B \left[ \bar{C} \left( \partial_\mu A^\mu + \frac{\alpha}{2} B \right) \right].
\]

Here \( S_{\text{GF+FP}} \) is the gauge fixing (GF) and Faddeev–Popov (FP) ghost term where \( \delta_B \) is the Becchi–Rouet–Stora–Tyutin (BRST) transformation, \( \alpha \) is the gauge fixing parameter and \( B \) is the Nakanishi–Lautrup (NL) Lagrange multiplier field. Of course, we can add the gauge fixing term for the residual symmetry \( H \) which we don’t discuss in this Letter.

In this paper, we adopt the modified MA gauge proposed in [9],

\[
S_{\text{GF+FP}} = \int d^4x i \delta_B \left[ \frac{1}{2} A_\mu^O(x) A^{\mu O}(x) - \frac{\alpha}{2} \bar{C} \left( \partial_\mu A^\mu + \frac{\alpha}{2} B \right) \right].
\]

where \( \delta_B \) is the BRST (anti-BRST) transformation. The \( \alpha = -2 \) case has been already investigated in [9,11,13]. The modified MA gauge is different from the naive MA gauge by the ghost self-interaction, since

\[
C^A \rightarrow \pm \bar{C}^A, \quad \bar{C}^A \rightarrow \mp C^A,
\]

\[
B^A \rightarrow -B^A, \quad \bar{B}^A \rightarrow -B^A, \quad A_\mu^O \rightarrow A_\mu^O.
\]

Moreover, our choice (4) leads to a renormalizable theory and preserves the hidden supersymmetry as discussed in the final part of this Letter. The Lagrangian (5) should be compared with the Lagrangian in the Lorentz gauge which is invariant under the FP conjugation only in the Landau gauge,

\[
\mathcal{L}_{\text{GF+FP}} = -i \delta B \left[ \bar{C} \left( \partial_\mu A^\mu + \frac{\alpha}{2} B \right) \right] = +i \delta_B \left[ \frac{1}{2} A_\mu^O \right] + \frac{\alpha}{2} \delta B \left( B^A C^A \right),
\]

although it is also invariant under the BRST and anti-BRST transformations.

By performing the BRST transformation explicitly, we obtain

\[
S_{\text{GF+FP}} = \int d^4x \left[ B^a D_\mu [a]^{ab} A_\mu^b + \frac{\alpha}{2} B^a B^a + i \bar{C} A_\mu^a [a]^{abc} D^\mu [b]^{cd} C^b + g f^{abc} f^{dfe} \bar{C} A_\mu^a [a]^{bcd} A_\mu^e + i \bar{C} g f^{abc} (D^\mu [a]^{bcd} C^b) + \frac{\xi}{2} \bar{g} f^{abc} f^{dfe} \bar{C} C^b C^c C^d \right].
\]
where the choice (4) specifies the strength of the quartic ghost interactions,
\[ \zeta = \alpha. \]  
(9)

An implication of this choice will be discussed later. The gauge fixing term (8) is the most general type we consider in the following.

In particular, the \( G = SU(2) \) case is greatly simplified as
\[
S'_{\text{GF+FP}} = \int d^4x \left\{ B^a D_{\mu} \bar{a}^{ab} A^{\mu b} + \alpha \frac{1}{2} B^a B^a \\
+ i \bar{c}^a D_{\mu} [a] \bar{c}^b D^{a\mu} [b] B^b \\
- \frac{1}{8} \bar{g}^2 \epsilon^{abcd} \bar{c}^a B^b C_\mu C^\mu \\
+ \bar{c}^a g \epsilon^{ab} \left( D_{\mu} [a] \bar{c}^b A^{\mu \mu} \right) C^3 \\
- \frac{1}{2} \bar{c} c B^a \bar{c}^b C^3 \\
+ \frac{\zeta}{4} \bar{c}^a \epsilon^{abcd} \bar{c}^b B^a C^c C^d \right\}. 
\]
(10)

Integrating out the NL field \( B^a \) leads to
\[
S'_{\text{GF+FP}} = \int d^4x \left\{ -\frac{1}{2\alpha} \left( D_{\mu} [a] \bar{a}^{ab} A^{\mu b} \right)^2 \\
+ \left( 1 - \frac{\zeta}{\alpha} \right) i \bar{c}^a g \epsilon^{ab} \left( D_{\mu} [a] \bar{c}^b A^{\mu \mu} \right) C^3 \\
+ i \bar{c}^a D_{\mu} [a] \bar{c}^b D^{a\mu} [b] B^b \\
- \frac{1}{8} \bar{g}^2 \epsilon^{abcd} \bar{c}^a B^b C_\mu C^\mu \\
+ \frac{\zeta}{4} \bar{c}^a \epsilon^{abcd} \bar{c}^b B^a C^c C^d \right\}. 
\]
(11)

3. Ghost condensation and mass generation due to quartic ghost interaction

The \( \zeta = 0 \) case was considered in the previous paper [6], leaving \( \alpha \) arbitrary. Even in this case, the quartic ghost self-interaction is generated after integrating out the off-diagonal gluons as mentioned above (see Eq. (2.52) and Appendix B of [6]),
\[ z_4 \bar{c} c \epsilon^{abcd} \bar{c}^a B^b C^c C^d, \]
\[ z_4 = \frac{4N}{(4\pi^2)^2} \ln \frac{\mu}{\mu_0} \quad (N = 2), \]
(12)
since the interaction term \(-g^2 \epsilon^{abcd} \bar{c}^a B^b C_\mu C^\mu \) does not vanish even for \( \zeta = 0 \) (or \( \alpha = 0 \)).

If we consider the non-zero \( \zeta \) case, \( \alpha \) leads to the renormalization of \( \zeta \) (together with \( g \)). This is expected from the beginning, since the quartic ghost interaction is a renormalizable interaction. In fact, it has been proven that QCD in the MA gauge is renormalizable by including the quartic ghost interaction [18].

For simplicity, we first discuss the \( G = SU(2) \) case. To incorporate the effect of ghost interaction, we introduce the auxiliary scalar field \( \varphi \) as
\[ \frac{\zeta}{4} \bar{c}^a \epsilon^{abcd} \bar{c}^b B^a C^c C^d \]
\[ \rightarrow - \frac{1}{2\sqrt{2}} \bar{c} c \epsilon^{abcd} \bar{c}^a B^b C^c C^d, \]
(13)
where we have used the identity,
\[ \epsilon^{abcd} \bar{c}^a B^b C^c C^d = 2(\bar{c}^a B^b C^c C^d)^2 \]
\[ = 2(\bar{c}^a C^a)^2 = -4\bar{c} c C^2 C^2. \]
Then the GF+FP term is cast into the form,
\[
S'_{\text{GF+FP}} = \int d^4x \left\{ i \bar{c}^a \partial_{\mu} \hat{\varphi}^{ab} C^a \\
- \frac{1}{2\sqrt{2}} \bar{c} c \epsilon^{abcd} \bar{c}^b B^a C^c C^d \right\} + \ldots. 
\]
(14)

In order to see whether the QCD vacuum chooses a nontrivial \( \varphi \) or not, we consider the effective potential for the \( x \)-independent \( \varphi \) neglecting the kinetic terms. The Coleman–Weinberg type argument (summing up all one-loop ghost diagrams with arbitrary number of external \( \varphi \) fields) or integration over off-diagonal ghosts and anti-ghosts leads to the effective potential \( V(\varphi) \) for \( \varphi \),
\[ V(\varphi) \int d^4x = \int d^4x \left\{ \frac{1}{2\sqrt{2}} \bar{c} c \epsilon^{abcd} \bar{c}^b B^a C^c C^d \right\} + \ldots. 
\]
(15)

1. In the \( SU(2) \) case, the ghost condensation was seriously discussed by Schaden [20] from a different viewpoint from ours.

2. Note that the mathematical identity holds
\[ -u \sum_{n=1}^{\infty} \frac{1}{n^2 \left( \frac{i\varphi}{\varphi^2} \right)^2} = \ln[1 + \varphi^2 (\varphi^2)^2] \]
\[ = \ln(\det(\partial_{\mu} \partial^{\mu} - \varphi \epsilon^{ab})) \\
- \ln(\det(\partial_{\mu} \partial^{\mu} - \varphi \epsilon^{ab})). \]
Hence we obtain
\[ V(\varphi) = \frac{1}{2\xi} \varphi^2 - \int \frac{d^4k}{i(2\pi)^4} \ln(-k^2 + \varphi^2). \]  
(16)
The stationary point is given by the zero of the gap equation,
\[ V'(\varphi) \equiv \varphi \left[ \frac{1}{\xi} \varphi^2 - 2 \int \frac{d^4k}{i(2\pi)^4} \frac{1}{(-k^2 + \varphi^2)} \right] = 0. \]  
(17)
Within the minimal subtraction (MS) scheme of the dimensional regularization, the effective potential is obtained as
\[ V(\varphi) = \frac{1}{2\xi} \varphi^2 + \frac{1}{32\pi^2} \varphi^2 \left[ 2 \ln \left( \frac{4\pi\mu^2}{|\varphi|} \right) + C \right]. \]  
(18)
with \( C := 2\gamma - 3 \) and Euler constant \( \gamma = 0.5772\ldots \).
As far as \( \xi \neq 0 \), the gap equation (17) has nontrivial solutions given by \( \varphi = \pm \varphi_0 \) (besides a trivial one \( \varphi = 0 \)) where
\[ v := \varphi_0 = 4\pi\mu^2 e^{-1-\gamma} \exp \left[ -\frac{8\pi^2}{\xi g^2(\mu)} \right] > 0. \]  
(19)
These solutions correspond to global minima of the effective potential. At the global minimum \( \varphi = \varphi_0 \),
\[ V(\varphi_0) = -\frac{1}{32\pi^2} \varphi_0^2 < 0. \]  
This shows that QCD vacuum prefers a ghost condensate for any value of \( \xi g^2(\mu) \neq 0 \) such that \( \varphi_0 \sim g^2(\mu) e^{4\pi\mu^2} \neq 0 \).

Now we consider the theory around the nontrivial vacuum \( \varphi = \varphi_0 \). The off-diagonal ghost propagator is modified in the ghost-condensed vacuum into
\[ \langle \bar{C}^a(x)C^b(y) \rangle = \frac{i}{16\pi^2} \frac{-k^2 \delta^{ab} - i\epsilon^{ab}}{(-k^2)^2 + v^2} e^{ik(x-y)}. \]  
(20)
When \( \langle \bar{C}^a C^b \rangle = 0 \), i.e., \( v = 0 \), another ghost condensation \( \langle \bar{C}^a C^a \rangle \) is also zero in the dimensional regularization. However, nonzero condensation \( \langle \bar{C}^a C^b \rangle \neq 0 \) leads to another condensation \( \langle \bar{C}^a C^a \rangle \neq 0 \). In the condensed vacuum, the ghost–gluon 4-body interaction,
\[ -ig^2 e^{ai} e^{bj} \bar{C}^a C^b A^{\mu c} A^{\mu d}, \]
leads to a mass term of the off-diagonal gluons,
\[ -ig^2 e^{ai} e^{bj} \bar{C}^a C^b A^{\mu c} A^{\mu d} = \frac{1}{2} g^2 \langle [C^a C^b] A^{\mu a} A^{\mu b} \rangle. \]  
(21)
where we have used
\[ \langle [C^a C^b] \rangle = \frac{1}{2} \langle \epsilon^{ab} (\bar{C}^c C^c) + \epsilon^{ab} (\epsilon^{cd} C^c C^d) \rangle. \]
The condensation is given by
\[ \langle [C^a C^a] \rangle = \int \frac{d^4k}{i(2\pi)^4} \frac{-2k^2}{(-k^2)^2 + v^2} = \frac{v}{16\pi} > 0. \]  
(22)
where the signature, i.e., positivity of \( v \) is determined by analytic continuation to Euclidean region. Thus the off-diagonal gluon acquires the mass given by
\[ M_A^2 = g^2 \langle [C^a C^a] \rangle = \frac{g^2 v}{16\pi} > 0. \]  
(23)
The dynamically generated mass \( m_A \) is finite excluding the mass counter term. Note that the introduction of the explicit mass term \( \frac{1}{2} m^2 A_{\mu} A^{\mu} \) spoils the renormalizability of the theory. Our derivation of off-diagonal gluon mass preserves the renormalizability, see [16] for more details.

Now we proceed to estimate the order of the off-diagonal mass. We impose the renormalization condition at the renormalization point \( M \), i.e., we define the renormalized coupling \( \xi g^2 \) by
\[ V'(\varphi) \bigg|_{\varphi=M} = \left( \xi g^2 \right)_{(M)}^{-1}. \]  
(24)
where \( M \) is nonzero but arbitrary. The renormalizability implies that arbitrary choice of \( M \) should not change the physics. Hence we have the \( \mu \)-independence of \( V(\varphi) \) which means that \( \xi g^2(\mu) - (4\pi)^2 \) \( \ln M \) is a \( M \)-independent constant. Then we have
\[ \left( \xi g^2 \right)_{(M)} = \left( \xi g^2 \right)_{(M_0)} \left[ 1 + \left( \xi g^2 \right)_{(M_0)} (4\pi)^2 \right]^{-1} \ln \frac{M}{M_0}. \]
(25)
Hence \( \xi g^2 \) satisfies the RG equation,
\[ \frac{d}{dM} \left( \xi g^2 \right) = -\frac{1}{4\pi^2} \left( \xi g^2 \right)^2. \]
(26)

---

\[ ^3 \text{Even if one introduces a bare mass term of the form,} \]
\[ m^2(\frac{1}{2} A_{\mu} A^{\mu} + \omega C^{\mu} C^{\mu}), \]
the modified BRST and anti-BRST transformations can be constructed as \( \delta_B B = m^2 C, \) \( \delta_B C = m^2 C - g(C \times B), \) under which the modified Lagrangian is invariant. However, the nilpotency of both transformations is violated as \( \delta_B^2 C = \omega m^2 C, \) \( \delta_B C = -m^2 C, \) leading to the breakdown of physical S-matrix unitarity, see [21].
From the asymptotic freedom,
\[ M \frac{dg^2}{dM} = - \frac{b_0}{8\pi^2} g^4, \]
\[ \zeta \] obeys the RG equation,
\[ M \frac{d\zeta}{dM} = \frac{g^2}{4\pi^2} \zeta (\zeta - b_0/2). \]
Then we find that \( \zeta = 0 \) and \( \zeta = b_0/2 \) are the fixed points. The \( M \) dependence of \( V \) means that \( V \) satisfies the differential equation,
\[ \left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} \right] + \gamma_\zeta (g) \frac{\partial}{\partial \zeta} - \gamma_\varphi (g) \varphi \frac{\partial}{\partial \varphi} \right] V(\varphi) = 0, \quad (25) \]
where \( \gamma_\varphi (g) \) is the anomalous dimension of \( \varphi \) defined by \( \gamma_\varphi (g) := \frac{1}{M} \frac{\partial \ln Z}{\partial g} \) and \( \varphi_R := Z^{-1/2} \varphi \) and \( \gamma_\zeta (g) := M \frac{\partial \zeta}{\partial M} \).
Substituting (18) into (25), we obtain
\[ V_D(\varphi) = V(\varphi) + \left( - \frac{b_0}{16\pi^2} \right) \left( \beta(g) = \frac{g \gamma_\varphi (g)}{16\pi^2}, \quad \gamma_\varphi (g) := \frac{-2g^2}{4\pi^2} \zeta (\zeta - b_0/2). \]
The non-trivial fixed point \( \zeta = b_0/2 \) yields
\[ v = 4\pi e^{1-\gamma} \mu^2 \exp \left( - \frac{16\pi^2}{b_0 \mu^2(\mu)} \right) = 4\pi e^{1-\gamma} \Lambda_{\text{QCD}}^2. \]
Therefore, the condensation \( v \) is a renormalization-group invariant and the order is given by the QCD scale, \( \Lambda_{\text{QCD}} \). Hence, the off-diagonal gluon mass is given by \( M_A = \frac{4\pi^3}{4\pi^2} (1-\gamma)/2 \Lambda_{\text{QCD}} = (\pi a_s)^{1/2} e^{(1-\gamma)/2} \Lambda_{\text{QCD}} \).
This is comparable with the lattice simulation result, \( M_A \approx 1.2 \text{ GeV} \), see [19].
Moreover, the quartic ghost interaction can interact to give a mass for the ghost, since the treatment à la Hartree–Fock approximation leads to
\[ 4 \zeta g^2 e^{a \bar{c} \bar{a} C^a C^b C^c C^d} \]
\[ = \frac{\zeta}{2} g^2 \left( e^{a \bar{c} \bar{a} C^a C^b} \right)^2 = \frac{\zeta}{2} g^2 \left( i \bar{c} \bar{a} C^a C^b \right)^2 \]
\[ \rightarrow \xi g^2 \left[ (i \bar{c} \bar{a} C^a) i \bar{c} \bar{b} C^b \right]. \]
This implies the off-diagonal ghost mass,
\[ M_\xi \equiv 4 \zeta g^2 \left[ (i \bar{c} \bar{a} C^a) \right] = 4 \zeta g^2 \frac{v}{16\pi} = \xi M_A. \quad (26) \]
Thus off-diagonal gluons and ghosts can become massive due to ghost self-interactions. Note that \( \bar{c} \bar{a} C^a \) and \( e^{a \bar{c} \bar{a} C^a C^b} \) are invariant under the residual \( U(1) \).
Even in the presence of the condensation, the residual \( U(1) \) invariance is not broken spontaneously and the diagonal gluon remains massless [16]. These results strongly support the Abelian dominance.
It is possible to extend the above analysis to the \( SU(3) \) case [16]. The potential \( V(\varphi^3, \varphi^8) \) is written in terms of two diagonal combinations,
\[ \varphi^i \sim \zeta g^2 \sqrt{-1} f^{iab} \bar{c} \bar{a} C^b \quad (i = 3, 8). \]
In fact, the effective potential for \( SU(3) \) is given by
\[ V(\varphi) = \frac{1}{2} \frac{\varphi \cdot \bar{\varphi}}{\zeta g^2} \varphi \cdot \bar{\varphi} \]
\[ - \sum_{a=1}^3 \int \frac{d^4k}{(2\pi)^4} \ln \left[ \left( -k^2 \right)^2 + (\epsilon_{\alpha \bar{\alpha}} \cdot \bar{\varphi})^2 \right]. \quad (27) \]
where \( \varphi := (\varphi^3, \varphi^8) \) and \( \epsilon_{\alpha \bar{\alpha}} \) is the root vectors
\[ \epsilon_1 = (1, 0), \quad \epsilon_2 = \left( -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right), \]
\[ \epsilon_3 = \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right). \]
The schematic plot of the potential is given in Fig. 1. It turns out that the potential has the global minima at six points on the three straight lines along the root vectors, i.e., (I) \( \varphi^3 = 0, \varphi^8 \neq 0 \), (II) \( \varphi^3 = \sqrt{3} \varphi^8 \), (III) \( \varphi^3 = -\sqrt{3} \varphi^8 \). We find that the off-diagonal gluons in the \( SU(3) \) case have two different masses as follows.4
\[ I: \quad \frac{1}{\sqrt{2}} m_A \rightarrow \frac{1}{\sqrt{2}} m_A = m_A = m_A^5 = m_A^7, \]
\[ II: \ m_A \rightarrow \frac{1}{\sqrt{2}} m_A = m_A = \frac{1}{\sqrt{2}} m_A^5 = m_A^7, \]
\[ III: \ m_A \rightarrow \frac{1}{\sqrt{2}} m_A = m_A = m_A^5 = \frac{1}{\sqrt{2}} m_A^7. \quad (28) \]
The value of larger mass is given by \( m_A^2 = g^2 V_0 / (16\pi) \), where
\[ V_0 = 4^{1/6} (4\pi \mu^2)^{1-\gamma} \exp \left( - \frac{16\pi^2}{3\zeta g^2} \right). \]
The \( \mu \)-independence of the potential holds when \( \zeta = b_0/2 \) for \( SU(3) \). Hence, we obtain \( V_0 = 4^{1/6} (4\pi \mu^2)^{1-\gamma} \times \]

---

4 This result is obtained up to Weyl symmetry.
269

Fig. 1. The three-dimensional plot of the effective potential $V$ and its contour plot.

Another way to estimate the order of the off-diagonal mass is based on the identity of the trace anomaly,

$$\langle 0| T^{\mu}_{\mu} |0 \rangle = \frac{\beta(\alpha_s)}{4\alpha_s} \langle 0 | \mathcal{F}^A_{\mu \nu} \mathcal{F}^{A \mu \nu} |0 \rangle$$

$$= - \frac{11N_c - 2N_f}{24} \left\{ \frac{\alpha_s}{\pi} \langle 0 | \mathcal{F}^A_{\mu \nu} \mathcal{F}^{A \mu \nu} |0 \rangle \right\}.$$  \hspace{1cm} (29)

Note that the values of gluon condensate obtained on a lattice are as follows.\footnote{The authors would like to thank E.-M. Ilgenfritz for providing this information. In the presence of light quarks, the charmonium sum rules [22] gives $\langle 0 | \mathcal{F}^A_{\mu \nu} \mathcal{F}^{A \mu \nu} |0 \rangle \approx 1.3 \sim 1.9 \times 10^{-2}$ (GeV)$^4$.}

The values of gluon condensate obtained on a lattice are as follows.\footnote{The authors would like to thank E.-M. Ilgenfritz for providing this information. In the presence of light quarks, the charmonium sum rules [22] gives $\langle 0 | \mathcal{F}^A_{\mu \nu} \mathcal{F}^{A \mu \nu} |0 \rangle \approx 1.3 \sim 1.9 \times 10^{-2}$ (GeV)$^4$.} $\langle 0 | T^{\mu}_{\mu} |0 \rangle \approx 4V(\phi_0) = \frac{3V_0^2}{16\pi^2}$ for $SU(3)$,

$$\left( \frac{v^2}{8\pi^2} \right) \text{ for } SU(2).$$ \hspace{1cm} (30)

Equating (29) and (30), we obtain

$$V_0 = \left\{ \langle 0 | T^{\mu}_{\mu} |0 \rangle |16\pi^2/3 \right\}^{1/2} \approx 3.2 \text{ (GeV)}^2,$$

for $N_c = 3$ and $N_f = 0$. Finally, we have $m_A = (\alpha_s V_0/4)^{1/2}$, $(\alpha_s V_0/8)^{1/2} \approx 0.4 \sim 0.5$ GeV. These are our predictions. The full details of $SU(3)$ case will be given in [16].

4. APEGT of QCD in the modified MA gauge

In order to obtain the “effective” theory which is written in terms of the diagonal fields $a_1^\alpha, B^\alpha, C^\alpha, \tilde{C}^\alpha$ alone, we intend to integrate out all the off-diagonal fields $A_1^\alpha, B^\alpha, C^\alpha, \tilde{C}^\alpha$. We call the resultant effective field theory the Abelian-projected effective gauge theory (APEGT). That is to say, the APEGT is defined as

$$\exp(iS_{APEGT}) = \int [dA_1^\alpha][dC^\alpha][d\tilde{C}^\alpha][dB^\alpha] \exp(iS_{QCD}).$$ \hspace{1cm} (31)

Hence the vacuum-to-vacuum amplitude (or the partition function) of QCD reads

$$Z_{QCD} := \int [dA_1^\alpha][dC^\alpha][d\tilde{C}^\alpha][dB^\alpha] \exp(iS_{QCD}) = \int [dA_1^\alpha][dC^\alpha][d\tilde{C}^\alpha][dB^\alpha] \exp(iS_{APEGT}).$$ \hspace{1cm} (32)

In the naive MA gauge, such an attempt was first performed by Quandt and Reinhardt [14] for $\alpha = 0$ and subsequently by one of the authors [6] for $\alpha \neq 0$, in particular, $\alpha = 1$ [6] at least for $G = SU(2)$. (We have found that the $\alpha = 0$ case is very special from the viewpoint of renormalizability.) The generalization to $SU(N)$ is straightforward [15,16].

In the naive MA gauge [6], the off-diagonal gluons were expected to become massive, while the diagonal gluons were believed to behave in rather complicated way. Recently, the massiveness of off-diagonal
gluons has been shown by Monte Carlo simulations on a lattice [19]. An analytical explanation was given at least in the topological sector based on the dimensional reduction of the topological sector to the two-dimensional coset $G/H$ nonlinear sigma (NLS) model, see Section IV.C of [9]. In this paper we have given another evidence of mass generation of off-diagonal gluons and ghosts. In view of these facts, the integration of massive off-diagonal gluon fields can be interpreted as a step of integration of massive modes in the sense of the Wilsonian renormalization group. In this sense, the APEGT obtained in this way is regarded as the low-energy effective theory describing the physics in the length scale $R > m_A^{-1}$ or in the low-energy region $p \ll m_A$.

In order to obtain the explicit form of the APEGT in the modified MA gauge, we repeat the steps performed in [6] to obtain the APEGT. The GF + FP term in the condensed vacuum reads

$$S_{GF+FP} = \int d^4x \left[ -\frac{1}{2g^2} D_\mu [a]^{ab} D^\mu [a]^{ab} - \frac{1}{2} M_A^2 A^{ab}_\mu A_{\mu}^{ab} + i \bar{C} D_\mu [a]^{ac} D^\mu [a]^{cb} - g_i \bar{C} e^{ab} \bar{C} a^{ab} C^{bc} - V(\varphi_0 + \bar{\varphi}) \right],$$

where we have put $\varphi = \varphi_0 + \bar{\varphi}$. Note that $V(\varphi_0 + \bar{\varphi}) = V(\varphi_0) + \frac{1}{2} \bar{\varphi}^2 V''(\varphi_0) + O(\bar{\varphi}^3)$ with $V''(\varphi_0) = 0$ and $V''(\varphi_0) = 1/(8\pi^2)$. We perform the integration over (high-energy) massive modes, i.e., off-diagonal gluons $A^{ab}_\mu$ and off-diagonal ghosts $C^a$ and anti-ghosts $\bar{C}^a$ for the total action $S_{YM} + S_{GF+FP}$. In the process of deriving the APEGT, we have introduced the anti-symmetric auxiliary (Abelian) tensor field $B_{\mu}^{ij}$ to avoid the quartic self-interactions among the off-diagonal gluons appearing in $S_{YM}$ where $B_{\mu}^{ij}$ is invariant under the residual gauge transformation $H = U(1)^{N-1}$. The way of introducing $B_{\mu}^{ij}$ is not unique, see [6] and [16] for more details. In the following we discuss one of the original versions [6]. By repeating the procedures in [16], we can show that the resultant APEGT is written as (up to higher-derivative terms)\(^6\)

$$\mathcal{L}_{ab}[a, B] = -\frac{1}{4g^2(\mu)} f_{\mu\nu}^{ij} f_{\nu\sigma}^{ij} \frac{1}{4} g_0^2 B_{\mu\nu}^{ij} B_{\nu\sigma}^{ij} + \frac{\kappa}{2} B_{\mu
u}^{ij} B_{\mu
u}^{ij},$$

where we have defined $g(\mu) := Z_a^{1/2} g$ with

$$Z_a := 1 - z_a + z_d = 1 + \frac{22}{3} N \frac{g^2}{(4\pi)^2} \ln \frac{\mu}{\mu_0}.$$

Here $f_{\mu\nu}^{ij}$ is the Abelian field strength $f_{\mu\nu}^{ij} := \partial_\mu a_i - \partial_\nu a_i^{\mu} + \ast f_{\mu\nu}^{ij}$, i.e., $f_{\mu\nu}^{ij} := \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} f_{\rho\sigma}^{ij}$. This result shows that the off-diagonal gluons can not be ignored and that they influence the APEGT in the form of renormalization of the Abelian sector. In fact, the renormalization factors $z_a, z_b, z_c, z_d$ are given by

$$z_a = -\frac{20}{3} N \frac{g^2}{(4\pi)^2} \ln \frac{\mu}{\mu_0},$$

$$z_b = +2 N \frac{g^2}{(4\pi)^2} \ln \frac{\mu}{\mu_0},$$

$$z_c = +4 N \frac{g^2}{(4\pi)^2} \ln \frac{\mu}{\mu_0},$$

$$z_d = -\frac{2}{3} N \frac{g^2}{(4\pi)^2} \ln \frac{\mu}{\mu_0},$$

where $\mu$ is a renormalization scale.

A remarkable fact is that the running of the gauge coupling constant $g(\mu)$ is governed by the $\beta$-function,

$$\beta(g) := \mu \frac{dg(\mu)}{d\mu} = -b_0 g^3(\mu),$$

$$b_0 = \frac{11}{3} N > 0,$$

which is the same as the original Yang–Mills theory. So the APEGT is an effective Abelian gauge theory exhibiting the asymptotic freedom. The coupling between $B_{\mu}^{ij}$ and $\ast f_{\mu\nu}^{ij}$ is important to derive the dual Abelian gauge theory which leads to the dual superconductivity. This term is generated through the integration (or radiative corrections) and is absent in the original Lagrangian. In this sense, the APEGT just obtained is non-renormalizable. Nevertheless, the APEGT can be made renormalizable, see [16] for more details. The effect of dynamical quarks can be included into this scheme by integrating out the quark fields. It results in further renormalization leading to the $\beta$-function with a different coefficient, $b_0 = \frac{11}{3} N - \frac{2}{4} f r_f$, where $f$ is the number of quark flavors and $r_f$ is the dimension of fermion representation.

---

\(^6\) The higher-derivative terms are suppressed in the low-energy region, since they are of the order $O(p^2/M_A^2)$. 
5. New extended BRS algebra

It is easy to show that the QCD Lagrangian (3) in the modified MA gauge (10) or (11) has a new global symmetry if it is restricted to \( C^3 = 0 \) subspace or to the parameter \( \zeta = \alpha \), that is to say, the Lagrangian is invariant under the two transformations,

\[
\delta_+ \, \tilde{C}^a(x) = C^a(x), \quad \delta_+ (\text{other fields}) = 0, \quad \delta_+ (\text{other fields}) = 0.
\]

(36) (37)

The existence of this symmetry in the Lagrangian in the maximal Abelian gauge was recently noticed by Schaden [20]. After eliminating \( B^\mu \) (and putting \( \zeta = \alpha \)), (11) agrees with the Lagrangian examined by Schaden [20] from a quite different viewpoint, the equivariant cohomology [23]. These transformations \( \delta_\pm \) for the field \( \Phi \) are defined by the generators \( Q_\pm \) as

\[
\delta_\pm \Phi = [i Q_\pm, \Phi], \quad Q_\pm := \int d^3 x \; J_\pm^0,
\]

where the generators are constructed through the Noether currents,

\[
J^a_+ = -iC^a(D_\mu[a]C)^a + \bar{\delta}B(C^a A^\mu_a),
J^a_- = iC^a(D_\mu[a] \bar{C})^a - \bar{\delta}B(\bar{C}^a A^\mu_a).
\] (38)

They should be compared with the ghost number,

\[
\delta_+ C^A(x) := [i Q_+, C^A(x)] = C^A(x),
\delta_+ \tilde{C}^A(x) := [i Q_+, \tilde{C}^A(x)] = -\tilde{C}^A(x),
\]

(39)

where \( Q_+ \) is the ghost charge defined by

\[
Q_+ = \int d^3 x \; J_+^0,
\]

\[
J_\pm^0 = i\{ (D^\mu[a]\bar{C})^A + \bar{C}^A (D^\mu[a]C)^A \}.
\]

Shaden found that there is a \( SL(2, \mathbb{R}) \) symmetry among \( Q_+ \), \( Q_- \) and \( Q_c \), i.e.,

\[
[i Q_+, Q_+] = 2 Q_+, \quad [i Q_-, Q_-] = -2 Q_-,
\]

\[
[i Q_+, Q_-] = Q_c,
\]

where the diagonal generator is the ghost number \( Q_c \).

It is well known that the BRST transformation, anti-BRST transformation and the ghost number generator form the double BRS algebra among three generators, \( Q_B, Q_B, Q_B \) and \( Q_c \),

\[
[Q_+, Q_+] = 0, \quad [Q_+, Q_] = 0, \quad [Q_+, Q_c] = 0, \quad [Q_B, Q_B] = 0, \quad [Q_B, Q_B] = 0, \quad [Q_B, Q_c] = 0.
\]

(40)

By enlarging the double BRS algebra, we find a new extended double BRS algebra [24] among five generators, \( Q_B, Q_B, Q_B, Q_+ \), and \( Q_c \), supplemented by

\[
[i Q_+, Q_+] = 0, \quad [i Q_+, Q_+] = -Q_B, \quad [i Q_+, Q_+] = Q_P, \quad [i Q_+, Q_+] = -Q_-
\]

(41) (42)

Note that the new extended BRS algebra closes only on the space of functionals which are invariant under the residual \( U(1) \) gauge transformation.

This should be compared with the extended BRS algebra (BRSTNO algebra) found by Nakanishi and Ojima [25] in the manifest covariant gauge of the Lorentz type where the additional symmetry is given by

\[
\delta_{c B} B^A = -ig(C \times C)^A, \quad \delta_{c C} C^A = -2C^A, \quad \delta_{c c} (\text{other fields}) = 0,
\delta_{c C} B^A = +ig(\bar{C} \times \bar{C})^A, \quad \delta_{c c} C^A = +2\bar{C}^A, \quad \delta_{c c} (\text{other fields}) = 0.
\] (43)

Although the BRSTNO algebra holds for arbitrary gauge, their generators are conserved only in the Landau gauge \( \alpha = 0 \). In the new extended algebra given above, the generators are conserved for an arbitrary gauge parameter \( \alpha \), but only on the space which is invariant under the residual gauge group.

6. Spontaneous breaking of a global symmetry and hidden supersymmetry in MA gauge

The non-zero expectation value \( (c^{ab} C^a \bar{C}^b) \) is regarded as the spontaneous breaking of the \( SL(2, \mathbb{R}) \) symmetry as pointed out by Schaden [20], since

\[
(0) [i Q_+, c^{ab} C^a \bar{C}^b] \rangle |0 \rangle
\]

7 We can consider other types of ghost condensations with the non-zero ghost number, i.e., \( (0) [i Q_+, c^{ab} C^a \bar{C}^b] |0 \rangle = (0)c^{ab} C^a \bar{C}^b \rangle |0 \rangle \) and \( (0) [i Q_+, c^{ab} C^a \bar{C}^b] |0 \rangle = (0)e^{ab} C^a \bar{C}^b \rangle |0 \rangle \). Three composite operators \( e^{ab} C^a \bar{C}^b, e^{ab} C^a \bar{C}^b, e^{ab} C^a \bar{C}^b \) are mu-
The non-compact \( SL(2, \mathbb{R}) \) symmetry is spontaneously broken into the non-compact Abelian subgroup, since the ghost charge \( Q_c \) is not broken. The massless Nambu–Goldstone (NG) particles associated with this spontaneous breaking can be confined by the quartet mechanism [26], i.e., decouple from physical observables, since the current \( J_{\mu}^\alpha (x) \) is BRST (anti-BRST) exact. Therefore, we need not to worry about the emergence of massless particles.

In the previous paper we have argued that the non-zero mass for the off-diagonal gluons can be understood from the massive spectrum of the coset NLS model implies the quartet mechanism [26], i.e., decouple from physical observables, since the current \( J_{\mu}^\alpha (x) \) is BRST (anti-BRST) exact. Therefore, we need not to worry about the emergence of massless particles.

It is shown that the action (4) for gauge fixing and FP ghost in the modified MA gauge has the orthosymplectic symmetry \( OSp(4|2) \) among \( A_{\mu}^a, C^a, \bar{C}^a \) when it is written in the superspace \( X^M := (\mu, \theta, \bar{\theta}) \) following the superspace formulation by Bonora and Tonin [28]. This superspace formulation can give a geometric meaning of BRST \( \delta_B \) and anti-BRST \( \delta_{\tilde{B}} \) transformations as translations in the Grassmann variables \( \theta \) and \( \bar{\theta} \) respectively, \( \delta_B \leftrightarrow \frac{1}{2} \int d\theta \frac{d}{d\theta}, \delta_{\tilde{B}} \leftrightarrow \frac{1}{2} \int d\bar{\theta} \frac{d}{d\bar{\theta}} \), where we have employed the equivalence between the differentiation and integration with respect to the Grassmann variable. Then the GF and FP part in the modified MA gauge is rewritten into the manifest \( OSp(4|2) \) invariant form,

\[
S'_{GF+FP} = i \int d^4x \int d\theta \int d\bar{\theta} \left[ \frac{1}{2} \eta_{NM} A_{\mu}^N (x, \theta, \bar{\theta}) A_{\nu}^M (x, \theta, \bar{\theta}) \right],
\]

using the Lie-algebra valued superfield (one-form),

\[
A_M (x) dX^M = A_{\mu} (x, \theta, \bar{\theta}) dA^\mu + C(x, \theta, \bar{\theta}) d\theta + \bar{C} (x, \theta, \bar{\theta}) d\bar{\theta},
\]

and a supermetric \( \eta_{NM} = \delta_{\mu\nu} \) for \( (M, N) = (\mu, \nu) \) and \( -i\alpha/2 \) for \( (M, N) = (\theta, \bar{\theta}) \). Thanks to the \( OSp(4|2) \) invariance of the integrand, it is shown [9] that (45) is reduced to

\[
S_{GF+FP} = \pi \alpha \int d^2z \left[ \frac{1}{2} A_{\mu}^a (z) A_{\nu}^a (z) - \frac{\alpha}{2} i C^a (z) \bar{C}^a (z) \right].
\]

If we restrict the gauge potential to its topological nontrivial piece in the coset \( G/H \),

\[
A_{\mu}^a \rightarrow tr \left[ T^{a} \frac{1}{8} U(\xi) \partial_{\mu} U(\xi)^\dagger \right] := \frac{\alpha}{8} \Omega_{\mu}^a,
\]

the action (47) is nothing but the coset NLS model [8, 9].

\[
S_{GF+FP} = \pi \alpha \int d^2z \Omega_{\mu}^a (z) \Omega_{\nu}^a (z).
\]

Thus, as far as \( \alpha \neq 0 \), the dimensional reduction to the two-dimensional coset NLS model occurs and the massive spectrum in the coset NLS model implies the massive off-diagonal gluon, see Section IV.C of [9]. It is also suggestive for the correspondence between two pictures that the symplectic group \( Sp(2) \) for the Grassmann variables is isomorphic to the \( SL(2, \mathbb{R}) \) mentioned above. The action of the NLS model may have a wrong sign depending on the signature of the parameter \( \alpha \). This might be related to the fact that the ghost condensate does not vanish even in the \( \alpha^2 \) case, the dimensional reduction to the two-dimensional coset NLS model occurs and the massive spectrum in the coset NLS model implies the massive off-diagonal gluon, see Section IV.C of [9].

In the case of \( \alpha = 0 \), the \( OSp(4|2) \) invariance is lost and hence the above mechanism of dimensional reduction does not work. On the other hand, the quartic ghost interaction disappears in this case and the ghost condensation generating the off-diagonal gluon mass does not occur and there is no spontaneous breaking of \( SL(2, \mathbb{R}) \) symmetry. In view of these, the case \( \alpha = 0 \) is rather special and should be discussed separately.
7. Conclusion and discussion

We have shown that the masses of off-diagonal gluons and off-diagonal ghosts are dynamically generated in QCD by adopting the MA gauge. This provides an evidence of the Abelian dominance which is expected to hold in low-energy region of QCD. The MA gauge is a nonlinear gauge and hence the quartic ghost interaction term is inevitably generated by radiative corrections [6]. From the viewpoint of renormalizability of the theory, therefore, we need to add the bare quartic ghost interaction to the original Lagrangian. We have explicitly shown that the quartic ghost interaction leads to ghost–anti-ghost condensations which give the masses of the off-diagonal gluons and ghosts in QCD, although QCD doesn’t have any elementary scalar field.

In this Letter we determined the form of the ghost interaction from the requirement of preserving the hidden supersymmetry (the resulting gauge is called the modified MA gauge). Surprisingly, the resulting Lagrangian in the modified MA gauge exactly coincides with that recently proposed by Schaden [20] (at least for SU(2)) from quite a different point of view. Therefore, the ghost and anti-ghost condensation can be understood as a spontaneous breaking of the global SL(2, R) symmetry recently claimed by Schaden for the SU(2) case. We have proposed an extended BRS algebra which includes the SL(2, R) algebra. However, it is not clear at present whether the SL(2, R) symmetry can be applied to the gauge group SU(N) for N > 2. Finally we argued that the mass generation is also related to the spontaneous breaking of a supersymmetry hidden in the modified MA gauge for arbitrary N.

In this Letter, although we have pointed out the importance of the quartic interaction term from renormalizability point of view, we have not indicated that the APEGT obtained in our scenario is really renormalizable. The totally renormalizable APEGT can be obtained improving the previous work [6], see [16].

Finally, it will be interesting to see how the dynamical mass generation just obtained affects the dual (magnetic) theory. This issue will be discussed from APEGT in a forthcoming paper [16].

Acknowledgements

After submitting this paper for publication, the authors were informed by Martin Schaden that he discussed the ghost condensation and its relation to the trace anomaly and obtained the similar results (in the SU(2) case) to ours presented in this paper. This work is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture (10640249).

References

A. DiGiacomo, hep-th/9603029;
M.I. Polikarpov, hep-lat/9609020;
M.N. Chernodub, M.I. Polikarpov, hep-th/9710205;
G. ’t Hooft, in: A. Zichichi (Ed.), High Energy Physics, Editorice Compositori, Bologna, 1975;
K.-I. Kondo, Y. Taira, hep-th/9911242.
Electroweak Sudakov at two loop level

M. Hori $^1$, H. Kawamura $^2$, J. Kodaira $^*$

Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan

Received 1 August 2000; accepted 6 September 2000

Editor: T. Yanagida

Abstract

We investigate the Sudakov double logarithmic corrections to the form factor of fermion in the $SU(2) \otimes U(1)$ electroweak theory. We adopt the familiar Feynman gauge and present explicit calculations at the two loop level. We show that the leading logarithmic corrections coming from the infrared singularities are consistent with the “postulated” exponentiated electroweak Sudakov form factor. The similarities and differences in the “soft” physics between the electroweak theory and the unbroken non-abelian gauge theory (QCD) will be clarified.

© 2000 Published by Elsevier Science B.V.

1. Introduction

Recently the high energy behavior of the Standard Model (SM) electroweak theory receives much attention from both theoretical and phenomenological viewpoints. At future high energy colliders, the total energy is much bigger than the masses of the SM gauge bosons and large double logarithmic (DL) corrections originating from the infrared behavior of the gauge theory [1] cannot be neglected for the exclusive [2] and also for the inclusive [3] processes.

The infrared behavior of the gauge theory has been one of the main subjects of the particle physics and many investigations have been made mainly for the QED and the unbroken non-abelian gauge theory (QCD). For the form factor of fermion in QED and QCD, it is known that the leading singularities can be exponentiated resulting in the Sudakov form factor [4]. In the case of the SM electroweak theory, the situation is much more complicated than QCD in two aspects. The first is that the symmetry is spontaneously broken. Secondly, the pattern of the symmetry breaking is not diagonal, namely a particular combination of the direct product of the gauge group $SU(2) \otimes U(1)$ is survived as an unbroken gauge group $U(1)_{em}$. These mean that we must carefully examine the non-abelian structure and the mixing effect which leads to “mass gap” between the gauge bosons in the electroweak theory. Therefore it is never trivial that the infrared singularities which appear in the form factor can be exponentiated as in QED and QCD. Several authors have addressed this problem [5–10]. The authors in Ref. [5] used the formalism which has been developed in QCD [11]. The infrared evolution equation [12] has been adopted in Refs. [8,9]. The explicit two loop calculations have been done by the authors in Refs. [6, 10] in the Coulomb or axial gauge. Unfortunately the situation is still somehow ambiguous concerning the possibility of the exponentiation of the infrared singularity in the electroweak theory.
Recently, a part of the mixing effect (mass gap effect) mentioned above is investigated by Melles [13] in the Feynman gauge. In this Letter, we extend the analysis by Melles to the general case which includes also the information on the non-abelian structure of the electroweak theory. Since there are many investigations in the Feynman gauge for QCD, we believe that our explicit two loop calculation of fermion’s form factor in the Feynman gauge is useful and instructive. Furthermore we will be able to clarify the similarities and differences in the “soft” physics between the electroweak theory and QCD. The process we consider is the fermion pair production from the $SU(2) \otimes U(1)$ singlet source. To the accuracy of the DL corrections, the chirality of the fermion is assumed to be conserved and we can discuss the left and right handed fermion separately. Therefore we consider, in this paper, the left handed fermion and the right handed antifermion production. The masses of the $W$ and $Z$ bosons will be approximated to be equal $M_W \approx M_Z \equiv M$. We give a fictitious small mass $\lambda$ to the photon to regularize the “real” infrared divergence and the fermion is assumed to be massless. We assume the situation, $s \gg M^2 \gg \lambda^2$ with $s$ the total energy of the produced fermions.

2. One loop DL contribution

To fix our convention and the calculational framework, we present the Feynman rule (in the Feynman gauge) and the one loop calculation. The Feynman rules for the gauge boson propagators and the fermion gauge boson couplings read,

\[ \gamma : \frac{-ig_{\mu\nu}}{q^2 - \lambda^2} , \quad W^\pm, Z : \frac{-ig_{\mu\nu}}{q^2 - M^2} , \]

and

\[ \gamma ff: \frac{i e Q(y\mu_+ + y\mu_-)}{\sqrt{2}(T^1 \pm iT^2)}y\mu omega_-, \]

\[ W^\pm ff: \frac{ig}{\sqrt{2}}(T^3 - \sin^2 \theta_W Q)y\mu_omega_-. \]

The case of the right handed fermion and the left handed antifermion production is discussed by Melles [13].

Since we assume fermions to be massless, the ghost and Higgs particles do not contribute.

\[ Zff: \frac{ig}{\cos \theta_W} \left[ (T^3 - \sin^2 \theta_W Q)y\mu_omega_- - \sin^2 \theta_W Qy\mu_omega_+ \right] , \]

where $T^a (a = 1, 2, 3)$ is the $SU(2)$ generator, $Q$ is the charge of fermion given by $Q = T^3 + Y$ with $Y$ the hypercharge and $\theta_W$ is the Weinberg angle. $\omega_\pm \equiv \frac{1 \pm \gamma_5}{2}$ is the projection operator. The coupling constants of the $SU(2)$ and $U(1)$ gauge groups are $g$ and $g' = g \tan \theta_W$, respectively, and the electric charge $e (> 0)$ is related to $g$ as $e = g \sin \theta_W$. As explained in the introduction, we concentrate on only the left chiral part in Eq. (1) in what follows.

Let us present the group factor of $SU(2) \otimes U(1)$ and the “kinematical” factor from loop integration separately. The group factors become:

\[ \gamma \text{ exchange: } e^2 Q^2 , \]

\[ W \text{ exchange: } g^2 \sum_{a=1,2} T^a T^a , \]

\[ Z \text{ exchange: } -\frac{g^2}{\cos^2 \theta_W} \left( (T^3 - \sin^2 \theta_W Q) \times (T^3 - \sin^2 \theta_W Q) \right) = g^2 T^3 T^3 + g'^2 Y^2 - e^2 Q^2 . \]

The loop integrations in which the weak bosons and photon are exchanged produce the double logarithmic corrections,

\[ -\frac{1}{16\pi^2} \ln^2 \frac{s}{M^2} , \quad -\frac{1}{16\pi^2} \ln^2 \frac{s}{\lambda^2} , \]

respectively. Therefore, the final result up to the one loop level for the fermion pair production reads,

\[ \Gamma^{(1)} = 1 - \frac{1}{16\pi^2} \left( g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2 \right) \times \ln^2 \frac{s}{M^2} - \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{s}{\lambda^2} , \]

where $C_2(R)$ is the $SU(2)$ Casimir operator for the fundamental representation.

3. Two loop DL contribution

We classify the two loop diagrams into three groups. The first group is composed from the ladder and crossed ladder diagrams shown in Fig. 1. The second includes the diagrams which contain the triple gauge
Fig. 1. The ladder and crossed ladder diagrams. The dashed (wavy) line represents the photon ($W$ and/or $Z$) with the mass ($M$).

Fig. 2. The diagrams which have the triple gauge boson couplings. The meaning of lines is the same as in Fig. 1.

The situation simple [15]. Omitting the coupling constant, the triple gauge boson vertex will be decomposed into two parts.

$$\Gamma_{a\mu\nu}(k, q) = (k - q)_\nu g_{a\mu} + (2q + k)_a g_{\mu\nu} - (q + 2k)_\mu g_{\nu a} = \Gamma_{a\mu\nu}^{P} + \Gamma_{a\mu\nu}^{F},$$

with

$$\Gamma_{a\mu\nu}^{P} = -(k + q)_\nu g_{a\mu} - q_\mu g_{\nu a}, \quad \Gamma_{a\mu\nu}^{F} = (2q + k)_a g_{\mu\nu} + 2k_v g_{a\mu} - 2k_\mu g_{\nu a}. \quad \text{(2)}$$

Eq. (2) gives rise to pinch parts when contracted with $\gamma$ matrices and it is easily seen that the contributions from this term are reduced to the effective diagrams shown in Figs. 3b and 3c. The contribution from Eq. (3) is depicted in Fig. 3a with the blob vertex. By combining the contributions from Fig. 3a (Fig. 3b) with those from the diagrams with the vertex correction (self energy insertion), one will get the “gauge invariant” vertex (propagator). The important fact for our purpose is that these contributions are less singular at least by one power of $\ln s$ compared to diagrams Fig. 1 and Fig. 3c [15]. Therefore, at the DL level, it is sufficient to calculate diagrams Fig. 1 and Fig. 3c configurations of Fig. 2.

We present again the contributions from the group factor of $SU(2) \otimes U(1)$ and the “kinematical” factor from loop integration separately because such presentation is very useful to clarify the similarities and differences between the electroweak theory and QCD.

The group factors for the ladder and crossed ladder diagrams in Fig. 1 become,

$$(1a.) = (1b.) = (e^2 Q^2)^2, \quad \text{(4)}$$

$$(1c.) = e^2 Q^2 (g^2 C_2(R) + g^* Y^2 - e^2 Q^2), \quad \text{(5)}$$

$$(1d.) = e^2 Q^2 (g^2 C_2(R) + g^* Y^2 - e^2 Q^2)$$
\[ -2g^2 e^2 Y T^3, \]

(1e.) \[ = 2 \times \left[ e^2 Q^2 \left( g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2 \right) - g^2 e^2 QT^3 \right], \]

(1f.) \[ = \left( g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2 \right)^2 \]

(1g.) \[ = \left( g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2 \right)^2 \]

Those for the diagrams having the triple gauge boson couplings (Fig. 2) read,

(2a.) \[ = 2 \times \left[ g^2 e^2 Q T^3 \right], \]

(2c.) \[ = 2 \times \left[ -g^2 e^2 Y T^3 + g^2 e^2 T^3 T^3 \right], \]

(2d.) \[ = 2 \times \left[ g^2 e^2 Y T^3 - g^2 e^2 T^3 T^3 \right]. \]

The factor 2 in Eqs. (7), (10)–(12) comes from the symmetric diagram. One can see that the structure of the group factors is slightly more complicated than that of QCD. In QCD, only the Casimir operator \( C_2(R) \) appears in the ladder diagram. The crossed ladder diagram is proportional to \( C_2(R)^2 - \frac{1}{2}C_2(R)C_2(G) \) [\( C_2(G) \) is the Casimir for the adjoint representation] and the second term is a non-exponentiating term. However it is known that this term is canceled by the contribution from Fig. 2. On the other hand, the situation is different in the electroweak theory. The non-abelian nature of SU(2) and the mixing effect between SU(2) and U(1) lead to new contributions compared to QCD. Since the propagation of the electroweak bosons is not proportional to the Casimir operator, there are non-exponentiating terms even in the ladder diagrams. See Eqs. (6), (8) (Figs. 1d, 1f). The crossed ladder diagrams and Fig. 2 receive the contributions which originate from both the mixing effect and the non-abelian nature of SU(2). The latter is the same as in QCD.

To evaluate the loop integrals, we follow the method explained in Ref. [13] for the ladder and crossed ladder diagrams. We apply the standard method of Feynman parametrization in evaluating Fig. 2. The result for each diagram turns out to be,

(1a.) \[ = \frac{1}{8\pi^2} \frac{1}{24} \ln^4 \frac{s}{\lambda^2}, \]

(1b.) \[ = \frac{1}{8\pi^2} \frac{1}{12} \ln^4 \frac{s}{\lambda^2}, \]

\( \frac{1}{8\pi^2} \left[ \frac{1}{8} \ln^4 \frac{s}{M^2} - \frac{1}{3} \ln^3 \frac{s}{M^2} \ln \frac{s}{\lambda^2} + \frac{1}{4} \ln^2 \frac{s}{M^2} \ln^2 \frac{s}{\lambda^2} \right]. \)

(1c.) \[ = \frac{1}{8\pi^2} \frac{1}{12} \ln^4 \frac{s}{M^2}. \]

(1d.) \[ = \frac{1}{8\pi^2} \frac{1}{24} \ln^4 \frac{s}{M^2}. \]

(1e.) \[ = \frac{1}{8\pi^2} \frac{1}{12} \ln^4 \frac{s}{M^2}. \]

A comment is in order concerning the above results. The diagrams Figs. 1d and 1f (Figs. 2c and 2d) lead to the same result Eq. (16) (Eq. (18)). As already pointed out by Melles [13], the physical reason is that the virtuality of the momentum circulating the loop is determined by the singularity of the most external propagator in the diagram in order to produce the DL leading contribution. Therefore the singularity from the photon which propagates inside the W and/or Z loop in Figs. 1d and 2c is already regulated by the W and/or Z mass.

By combining the group factors and the loop integrals, we find that all non-exponentiating terms are canceled out completely and obtain the two loop result which is the second term of the expansion of the exponentiated Sudakov form factor:

\[ \Gamma^{(2)} = 1 - \frac{1}{16\pi^2} \left( g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2 \right) \ln^2 \frac{s}{M^2} - \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{s}{\lambda^2} \]

\[ + \frac{1}{2!} \left( \frac{1}{16\pi^2} \left( g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2 \right) \right) \]

\[ \times \ln^2 \frac{s}{M^2} + \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{s}{\lambda^2}. \]

The cancellation of the non-exponentiating terms from each diagram occurs as follows. The terms coming from the SU(2) \( \otimes U(1) \) mixing effect are canceled by the fact that the accompanying integrals turn out to be the same. The mechanism of cancellation of other terms are the same as in QCD.
4. Summary

We have considered the electroweak form factor at two loop level in the DL approximation. We have used the standard Feynman gauge. Our results have shown the exponentiation of the electroweak Sudakov form factor at two loop level. The cancellation of the non-exponentiating contributions is never trivial. The typical aspects of electroweak theory, the non-abelian nature of $SU(2)$ and the mixing effect between $SU(2)$ and $U(1)$, produce a new situation compared to QCD. We have shown that there appear non-exponentiating terms not only in the crossed ladder diagrams and diagrams with triple gauge boson coupling but also in the ladder diagrams. The reason is that the propagation of the electroweak bosons has the contribution which is not proportional to the Casimir operator. However these new non-exponentiating terms are canceled out each other by the dynamical reason that the virtuality of the momentum circulating the loop is determined by the singularity of the most external propagator to the DL accuracy. The cancellation of the non-exponentiating terms coming from the non-abelian nature of $SU(2)$ occurs in the same way as in QCD.

Acknowledgements

The work of M.H. was supported in part by the Monbusho Grant-in-Aid for Scientific Research No. 11005244. The work of H.K. was supported in part by the Monbusho Grant-in-Aid for Scientific Research No. 10000504.

References

Non-perturbative fermion propagator for the massless quenched QED3

A. Bashir

Instituto de Física y Matemáticas Universidad Michoacana de San Nicolás de Hidalgo, Apdo. Postal 2-82, Morelia, Michoacán, Mexico

Received 9 June 2000; received in revised form 1 August 2000; accepted 11 August 2000
Editor: H. Georgi

Abstract

For massless quenched QED in three dimensions, we evaluate a non-perturbative expression for the fermion propagator which agrees with its two loop perturbative expansion in the weak coupling regime. This calculation is carried out by making use of the Landau–Khalatnikov–Fradkin transformations. Any improved construction of the fermion–boson vertex must make sure that the solution of the Schwinger–Dyson equation for the fermion propagator reproduces this result. For two different gauges, we plot the fermion propagator against momentum. We then make a comparison with a similar plot, using the earlier expression for the fermion propagator, which takes into account only the one loop result.

© 2000 Published by Elsevier Science B.V.

PACS: 11.15.Tk; 12.20.-m
Keywords: Landau–Khalatnikov–Fradkin transformations; Gauge covariance

1. Introduction

A natural starting point for the non-perturbative study of gauge theories is the corresponding set of Schwinger–Dyson equations (SDEs). QED in 3-dimensions (QED3) has been a popular choice for such a study due to its relative simplicity and its confining behaviour in the quenched approximation. It requires knowledge of the non-perturbative form of the fundamental fermion–boson interaction. In the quenched approximation, one can then calculate the fermion propagator. Both the propagator and the vertex must obey essential gauge dependence in accordance with Landau–Khalatnikov–Fradkin (LKF) transformations [1,2]. These transformations are written in the coordinate space representation and they allow us to evaluate a non-perturbative expression for a Greens function in an arbitrary covariant gauge if we know its value in any particular gauge. It seems an insurmountable task to know a Greens function in any gauge. However, progress can be made in perturbation theory by calculating it to a certain order.

Due to the complicated nature of the LKF transformations, it has been difficult to derive analytical conclusions for the vertex. However, the fermion propagator is relatively easier to analyze and related analytical results exist for QED in 3 and 4 dimensions, based upon the knowledge of the fermion propagator at the one loop order. These results are generally assumed to be true to all orders [3,4] (referred to as the transversality condition in [5]), and constraints are derived on the non-perturbative form of the fermion–boson vertex. Realizing that this condition would not hold to all orders, Bashir et al. have derived constraints...
on the vertex in QED4 by demanding general constraints from the multiplicative renormalizability of the fermion propagator [6]. Recently, a two loop calculation has been done for the fermion propagator in the massless quenched QED3 [7,8], showing explicitly that the transversality condition is violated. Exploiting this calculation, we go beyond one loop and present the evaluation of the non-perturbative fermion propagator through the use of LKF transformations. In comparison with the corresponding expression in [3], our result has an added piece which makes sure that in the weak coupling regime, correct two loop behaviour of the fermion propagator is achieved. Our calculation calls for the need to construct an improved vertex which would reproduce our result when used in the corresponding SDE. We also plot these two expressions as a function of fermion momentum and compare the results for two different gauges.

2. Fermion–boson vertex and gauge invariance

The study of the fermion propagator in quenched QED requires making an ansatz for the vertex. An acceptable ansatz must ensure the inclusion of the key features required of it. We shall only focus on the features relevant to the discussion in this paper:

- The vertex \( \Gamma^\mu(k, p) \) must satisfy the Ward–Green–Takahashi identity (WGTI) which relates it to the fermion propagator \( S_F(p^2) \):
  \[
  q_\mu \Gamma^\mu(k, p) = S^{-1}_F(k) - S^{-1}_F(k),
  \]
  where \( q = k - p \).

- It must reduce to the Feynman expansion of the perturbative vertex in the weak coupling regime.

- It must ensure local gauge covariance of the propagators and vertices.

Although the WGTI is a consequence of gauge invariance, it only fixes the longitudinal part \( \Gamma_L(k, p) \) of the complete vertex \( \Gamma^\mu(k, p) = \Gamma_L^\mu(k, p) + \Gamma_T^\mu(k, p) \), whereas the transverse part \( \Gamma_T(k, p) \), defined by the equation \( q_\mu \Gamma_T^\mu(k, p) = 0 \), remains undetermined. Without a proper choice of this part, one cannot ensure the local gauge covariance of the propagators and the vertex as demand the LKF transformations. Unfortunately, the LKF transformation law for the vertex is too complicated to be made use of. However, the corresponding rule for the fermion propagator is relatively simple. A proper choice of the transverse vertex in an arbitrary gauge is essential to satisfy it, as it is related to the fermion propagator through the following SDE in the Euclidean space, Fig. 1:

\[
S^{-1}_F(p) = S^{-1}_F(0) + e^2 \int \frac{d^3k}{(2\pi)^3} \Gamma^\mu(k, p) S_F(k) \gamma^\nu \Delta^{0\nu}_\mu(q),
\]

where \( S^0_F(p) = 1/ip \) and we express \( S_F(p) = F(p^2)/ip \). The photon propagator can be split into the transverse and the longitudinal parts as:

\[
\Delta^{0\nu}_\mu(q) = \Delta^{0\nu}_\mu^T(q) + \xi \frac{q_\mu q_\nu}{q^4},
\]

where \( \Delta^{0\nu}_\mu^T(q) = [\delta_{\mu\nu} - q_\mu q_\nu/q^2]/q^2 \). Burden and Roberts [3] pointed out that the condition

\[
\int \frac{d^3k}{(2\pi)^3} \Gamma^\mu(k, p) S_F(k) \gamma^\nu \Delta^{0\nu}_\mu^T(q) = 0,
\]

the so called transversality condition [5], leads to the correct LKF behaviour of the fermion propagator. This condition ensures that \( F(p^2) = 1 \) in the Landau gauge. The LKF transformations then yield

\[
F(p^2) = 1 - \frac{\alpha \xi}{2p} \tan^{-1} \left[ \frac{2p}{\alpha \xi} \right].
\]

However, it has been shown in [7,8] that although this condition is satisfied at one loop order, it gets violated at the two loop order, leading to the following expression for the fermion propagator:

\[
F(p^2) = 1 - \frac{\pi \alpha \xi}{4p} + \frac{\alpha^2 \xi^2}{4p^2} - \frac{3\alpha^3}{4p^2} \left( \frac{7}{3} - \frac{\pi^2}{4} \right) + \mathcal{O}(\alpha^4).
\]

\[\text{Fig. 1. Schwinger–Dyson equation for fermion propagator in quenched QED.}\]
This expression for the fermion propagator of course does not satisfy the LKF transformations in a non-perturbative fashion. However, making use of the said transformations, we can calculate an expression for the fermion propagator which does transform non-perturbatively as required the LKF transformations, which is an important requirement of gauge covariance. We carry out this exercise in the next section.

3. Propagator and the LKF transformation

Let us use the following notation and definition of the massless fermion propagator in the momentum and coordinate spaces, respectively, in an arbitrary covariant gauge $\xi$:

\[ S_F(p; \xi) = \frac{F(p; \xi)}{ip} , \]
\[ S_F(x; \xi) = \frac{1}{i} \delta x(x; \xi). \] (7)

These expressions are related by the following Fourier transforms:

\[ S_F(p; \xi) = \int d^3x \, e^{ip \cdot x} S_F(x; \xi) , \]
\[ S_F(x; \xi) = \int \frac{d^3p}{(2\pi)^3} \, e^{-ip \cdot x} S_F(p; \xi). \] (8)

The LKF transformation relating the coordinate space fermion propagator in the Landau gauge to the coordinate space fermion propagator in an arbitrary covariant gauge reads:

\[ S_F(x; \xi) = S_F(x; 0) e^{-(\alpha \xi/\alpha)x}. \] (9)

The following are the steps to find the non-perturbative expression for the fermion propagator in momentum space in an arbitrary covariant gauge: (i) Input the perturbative expression for the fermion propagator in the Landau gauge, i.e., $F(p; 0)$. (ii) Evaluate $X(x; 0)$ by taking the Fourier transform. (iii) Calculate $X(x; \xi)$ by using the LKF transformation law. (iv) Fourier transform back the result to $F(p; \xi)$. Eq. (6) implies that

\[ F(p; 0) = a_0 + \frac{\alpha}{p} + \frac{\alpha^2}{p^2} + O(\alpha^3) . \] (10)

where $a_0 = 1$, $a_1 = 0$ and $a_2 = -7/4 + 3\pi^2/16$. Although $a_1 = 0$, we shall keep this term in order to prove a point later. Eq. (8) permits us to carry out the Fourier transform of Eq. (10). On doing that and carrying out the angular integration, we get

\[ X(x; 0) = -\sum_{n=0}^{n=2} \frac{a_n \alpha^n}{2\pi^2 x^3} \]
\[ \times \int_0^\infty \frac{dp}{p^{n+1}} (\sin px - px \cos px). \]

The radial integration then yields:

\[ X(x; 0) = \frac{a_0}{4\pi} \frac{1}{x^3} - \frac{a_1}{8\pi} \frac{\alpha}{x^3} - \frac{a_2 \alpha^2}{8\pi} \] (11)

This result in the Landau gauge is related to that in arbitrary covariant gauge through the LKF transformation, Eq. (9). In order to Fourier transform the result back to the momentum space, we use Eq. (8). Substitute Eq. (9) in it, multiply the equation by $\phi$ and then take the trace:

\[ F(p; \xi) = \int d^3x \, p \cdot x e^{ip \cdot x} X(x; 0) e^{-(\alpha \xi/2)x}. \] (12)

Substituting Eq. (11) in it and carrying out the integrations, we obtain:

\[ F(p; \xi) = a_0 - \frac{\alpha(\pi \xi - 4a_1)}{2\pi p} \tan^{-1} \frac{2p}{\alpha \xi} \]
\[ - \frac{4a_2}{\alpha^2 \xi^2 + 4p^2} \left[ \frac{a_1 \xi}{\pi} - \frac{4a_2 p^2}{\alpha^2 \xi^2 + 4p^2} \right]. \] (13)

Expanding this expression around small values of $\alpha$ we get

\[ F(p, \xi) = a_0 + \frac{\alpha}{p} - \frac{\pi \xi \alpha}{4p} - \frac{a_1 \alpha^2 \xi}{p^2} \]
\[ + \frac{a_2^2 \alpha^2 \xi^2}{4p^2} + O(\alpha^4). \] (14)

For $\xi = 0$, we recuperate Eq. (10). There are some interesting points to note:

- The constant $a_1$ appears as a coefficient of $O(\alpha)$ term as well as $O(\alpha^2 \xi)$ term in Eq. (14). The fact that $a_1 = 0$ thus automatically rules out the presence of $O(\alpha^2 \xi)$ term.
- Apart from the $a_1$ terms, $\xi$ always appears in conjunction with $\alpha$, i.e., in the form $\alpha \xi$.
- In the perturbative expression, Eq. (14), there exists no $\alpha^3$ term, and the same is true for higher odd powers of $\alpha$. This does not of course rule out the
possibility of encountering these terms on perturbative evaluation of $F(p, \xi)$ at the three-loop level, and so on.

Substituting the values of $a_0$, $a_1$, and $a_2$, we arrive at the following final result:

$$F(p; \xi) = 1 - \frac{\alpha \xi}{2p} \tan^{-1} \frac{2p}{\alpha \xi} - \frac{(28 - 3\pi^2)p^2\alpha^2}{(\alpha^2\xi^2 + 4p^2)^2}.$$  \hspace{1cm} (15)

This expression contains an expected additional term in comparison with Eq. (5). This term ensures that the perturbative expansion of Eq. (15) matches correctly on to the two-loop calculation of $F(p; \xi)$. It also has the correct gauge dependence non-perturbatively as demanded by LKF transformations, in contrast with Eq. (6). Moreover, being non-perturbative in nature, it contains exact information of terms of orders higher than $\alpha^2$. For example, the perturbative expansion of Eq. (15) to $O(\alpha^4)$ reads:

$$F(p^2) = 1 - \frac{\pi \alpha \xi}{4p} + \frac{\alpha^2 \xi^2}{4p^2} - \frac{3\alpha^2}{4p^2} \left( \frac{7}{3} - \frac{\pi^2}{4} \right) - \frac{\alpha^4 \xi^4}{48p^4} - \frac{3\alpha^4 \xi^2}{8p^4} \left( \frac{7}{3} - \frac{\pi^2}{4} \right) + O(\alpha^6).$$  \hspace{1cm} (16)

Note that there are no $\alpha^3$ terms in this expression. Therefore, in conjunction with the structure of LKF transformation, we conclude that in the actual perturbative calculation of $F(p^2)$ to $O(\alpha^3)$, there will be no terms of the type $\alpha^3 \xi^3$, $\alpha^3 \xi^2$, or $\alpha^3 \xi$. This information is not contained in Eq. (6), which of course does not know anything about orders higher than $\alpha^2$. Similarly for $O(\alpha^4)$, Eqs. (5) and (15) exactly gives the coefficients of $\alpha^4 \xi^4$, $\alpha^4 \xi^3$ and $\alpha^4 \xi^2$ terms in perturbation theory, and so on for higher order terms.

In perturbation theory, every higher order term is expected to be much smaller than the term in the previous order in a systematic way. Naturally, one wonders what is the relative contribution of the additional piece in Eq. (15) in the non-perturbative regime. In Figs. 2, 3, we have drawn $F(p^2)$ as obtained from Eqs. (5) and (15) for two different values of the gauge parameter. For larger values of the gauge parameter, the two results start merging into each other. However, for low values of the gauge parameter, a bump arises in the $F(p^2)$ given by Eq. (15) at low values of $p$ because of the maximum in the additional piece at $p = \alpha \xi / 2$. Therefore, one concludes that for higher values of the gauge parameters, the additional piece modifies Eq. (5) insignificantly. However, for decreasing values of the gauge parameter, the difference starts increasing for low momenta. In essence, Figs. 2, 3 display the values of the gauge parameter for which the more complete Eq. (15) will deviate significantly from Eq. (5) in the non-perturbative regime.

4. Conclusions

In this paper, we present the calculation of the non-perturbative fermion propagator using the knowledge of its two-loop expansion and the LKF transformations. It is natural to assume that physically meaning-
ful solutions of the Schwinger–Dyson equations must agree with perturbative results in the weak coupling regime. This realization has been used in QED4 [6, 10, 11], and more recently in QED3 [7, 8], to use perturbation theory as a guide towards the non-perturbation truncation of Schwinger–Dyson equations. So far, progress has been made in this context by attempting to make the correspondence of non-perturbative propagators and vertex to their one-loop expansion. In this paper, we have gone beyond the one-loop order and constructed a fermion propagator which agrees with perturbation theory at least up to two-loops, and also has the correct gauge dependence as demanded by its LKF transformations. On the numerical side, it is important to know the contribution of the new piece in Eq. (15) as compared to the rest of the equation. It turns out that for higher values of the gauge parameter we do not need to worry about it. However in the neighbourhood of the Landau gauge, it causes significant deviation of the total result in comparison with the one in its absence for low values of momentum.

As the fermion propagator is related to the 3-point vertex through its SDE, our results put constraints on the possible forms for the unknown transverse part of the vertex. This part is in principle determined by understanding how the essential gauge dependence of the vertex demanded by its LKF transformation is satisfied non-perturbatively. In practice it is not an easy condition to implement. However, a simpler constraint is that any non-perturbative construction of the transverse vertex must ensure that we recuperate Eq. (15) when used in the SDE for the fermion propagator, leading to a more reliable non-perturbative truncation of SDEs.

Acknowledgements

I would like to acknowledge the CIC and CONACYT grants under the project 32395-E.

References

    L.D. Landau, I.M. Khalatnikov, Sov. Phys. JETP 2 (1956) 69;
    DTP-99/76, hep-ph/9907418;
    A. Bashir, A. Kizilersü, M.R. Pennington, ADP-00-29/T412,
    DTP-00/30.
Implication of improved upper bounds on $|\Delta L| = 2$ processes

Laurence S. Littenberg *, Robert Shrock

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Received 30 May 2000; received in revised form 3 August 2000; accepted 28 August 2000

Editor: H. Georgi

Abstract

We discuss implications of improved upper bounds on the $|\Delta L| = 2$ processes (i) $K^+ \rightarrow \pi^- \mu^+ \mu^+$, from an experiment at BNL, and (ii) $\mu^- \rightarrow e^+$ conversion, from an experiment at PSI. In particular, we address the issue of constraints on neutrino masses and mixing, and on supersymmetric models with $R$-parity violation. © 2000 Published by Elsevier Science B.V.

PACS: 13.20.Eb; 14.60.Pq; 14.60.St; 14.80.Ly

At present there are increasingly strong indications for neutrino oscillations and hence neutrino masses and lepton mixing from the solar neutrino deficiency and atmospheric neutrino anomaly [1]. The existence of lepton mixing means that lepton family number is not a good symmetry. Majorana neutrino masses occur generically, and violate total lepton number $L$ by $|\Delta L| = 2$ units. However, so far, in contrast to the data suggesting lepton mixing, experimental searches for the violation of total lepton number have only set limits. Among these are searches for the $|\Delta L| = 2$ processes (i) neutrinoless double beta ($0\nu\beta\beta$) decay of nuclei and (ii) $\mu^- \rightarrow e^+$ conversion in the field of a nucleus. A third class of $|\Delta L| = 2$ processes includes the decays $K^+ \rightarrow \pi^- \ell^+ \ell^+$, where $\ell^+ \ell^- = e^+e^-, \mu^+\mu^-$, or $\mu^+\mu^- [2,3]$. In a previous work we considered these decays and, from a retroactive data analysis, set the first upper limit on one of them, namely [4,5],

$$BR(K^+ \rightarrow \pi^- \mu^+ \mu^+) < 1.5 \times 10^{-4} \quad (90\% \, CL).$$  \(1)$$

In [4] we also noted that rare $K$ decay experiments at BNL could greatly improve this limit and proposed a search for $K^+ \rightarrow \pi^- \mu^+ \mu^+$ [6]. Among these experiments was BNL E865, which was searching for, and has now set a stringent upper limit on, the decay $K^+ \rightarrow \pi^- \mu^+ e^-$ [7]. This experiment has also recently obtained the 90% CL upper limit [8]

$$BR(K^+ \rightarrow \pi^- \mu^+ e^-) < 3.0 \times 10^{-9}.$$  \(2)$$

In the present paper we discuss the implications of this limit. In the context of current bounds on neutrinoless double beta decay and $\mu^- \rightarrow e^+$ conversion, we shall also consider the implications of two other 90% CL limits on $|\Delta L| = 2$ decays from E865 [8]:

$$BR(K^+ \rightarrow \pi^- e^+ e^+) < 6.4 \times 10^{-10}$$  \(3)$$

and

$$BR(K^+ \rightarrow \pi^- \mu^+ e^+) < 5.0 \times 10^{-10}.$$  \(4)$$
We first discuss physics sources for these decays, concentrating on massive Majorana neutrinos and $R$-parity-violating supersymmetric (SUSY) theories. In a modern theoretical context, one generally expects nonzero neutrino mass terms, of both Dirac and Majorana type. Let us denote the left-handed flavor vector of $\text{SU}(2) \times \text{U}(1)$ doublet neutrinos as $v_L = (v_e, v_\mu, v_\tau)_L$ and the right-handed vector of electroweak-singlet neutrinos as $N_R = (N_1, \ldots, N_n)_R$. The Dirac and Majorana neutrino mass terms can then be written compactly as

$$-\mathcal{L}_m = \frac{1}{2} \left( \overline{v_L} \begin{pmatrix} m_L & m_D \\ (m_D)^T & M_R \end{pmatrix} \begin{pmatrix} v_R^c \\ N_R \end{pmatrix} + \text{h.c.} \right),$$

where $M_L$ is the $3 \times 3$ left-handed Majorana mass matrix, $M_R$ is a $n_e \times n_\nu$ right-handed Majorana mass matrix, and $M_D$ is the $3$-row by $n_\nu$-column Dirac mass matrix. In general, all of these are complex, and $(M_L)^T = M_L$, $(M_R)^T = M_R$. The diagonalization of the matrix in Eq. (5) then yields $3 + n_\nu$ mass eigenstates, which are generically nondegenerate Majorana neutrinos (degeneracies in magnitudes of eigenvalues can yield Dirac neutrinos). Writing the charged current in terms of mass eigenstates as $J_\nu = \bar{\ell}_L \gamma^\nu \ell_L$, one has, in particular, $v_L = \sum_{j=1}^{3+n_\nu} U_{Lj} v_j$. The seesaw mechanism naturally yields a set of three light masses for the three known neutrinos, generically of order $m_\nu \sim m_D^2/M_R$, and $n_\nu$ very large masses generically of order $m_R \sim M_{\text{EW}}$ for the electroweak-singlet neutrinos, where $m_D \sim M_{\text{EW}}$ and $m_R \gg M_{\text{EW}}$ denote typical elements of the matrices $M_D$ and $M_R$, and $M_{\text{EW}} \simeq 250$ GeV is the electroweak symmetry breaking scale. However, if one tries to fit all current neutrino experiments, including not only the solar and atmospheric, but also the LSND data, then it is necessary to include electroweak-singlet (“sterile”) neutrinos with masses $\ll M_R$ in order to achieve an acceptable fit. In this case, the weak eigenstate $v_\mu$ may contain significant components of mass eigenstates beyond the usual three light ones $v_j$, $j = 1, 2, 3$, and, it is a priori possible that some of these mass eigenstates might have masses lying in the theoretically “disfavored” intermediate range $m_D/m_R^2 \ll m_{v_j} \ll m_R$ (subject to both particle physics and astrophysical/cosmological constraints). Whether or not such intermediate-mass neutrinos exist is ultimately an empirical question that must be settled by experiment. It is therefore of continuing interest to address constraints from data on neutrino masses in this intermediate mass region. Since current indications from solar and atmospheric data suggest neutrino masses in the seesaw-favored region and since, independent of this, neutrino masses in the range of a few to several hundred MeV can suppress large-scale structure formation and hence may be disfavored by cosmological constraints [9], we concentrate mainly on neutrino masses smaller or larger than this range here.

There are two types of lowest-order graphs involving massive neutrinos that contribute to the decay $K^+ \rightarrow \pi^+ \ell^+ \ell^-$, where $\ell^+\ell^- = \mu^+\mu^-, \mu^+e^-, \text{ or } e^+e^-$. In the case of identical $\ell^+$ and $\ell^-$, it is understood that the contributions are from the diagrams minus the same diagrams with the outgoing antilepton lines crossed.

![Graphs involving massive Majorana neutrinos](image)

Fig. 1. Graphs involving massive Majorana neutrinos that contribute to the decay $K^+ \rightarrow \pi^+ \ell^+ \ell^-$, where $\ell^+\ell^- = \mu^+\mu^-, \mu^+e^-, \text{ or } e^+e^-$. In the case of identical $\ell^+$ and $\ell^-$, it is understood that the contributions are from the diagrams minus the same diagrams with the outgoing antilepton lines crossed.
jection operator. The t-channel graphs cannot be evaluated so easily, because the hadronic matrix element that occurs,
\[ \int d^4 x \, d^4 y \, e^{i(p_\mu - p_\nu) \cdot x} \times [\pi^-] \left[ \hat{D}^\mu_{\nu}(y) \gamma_i u_\nu(x) \right] \left[ \hat{\bar{s}} \gamma_i (x) \gamma_0 u_\mu(x) \right] K^+ \] (8)
cannot be directly expressed in terms of measured quantities, unlike the matrix elements \( \langle 0|\bar{s}_L \gamma_0 u_\mu|K^+\rangle \) and \( \langle 0|\bar{s}_L \gamma_0 u_\mu|0\rangle \) from the first graph. In the limits where \( m_{\nu j}^2 \) is much smaller or larger than the magnitude of the typical \( q^2 \), \( |q^2|_{\text{ave}} \sim O((10^2 \text{ MeV})^2) \), the propagator factor simplifies:
\[ \frac{m_{\nu j}}{q^2 - m_{\nu j}^2} \sim \left\{ \begin{array}{ll} m_{\nu j}/|q^2|_{\text{ave}} & \text{if } m_{\nu j} \ll |q^2|_{\text{ave}}, \\ -1/m_{\nu j} & \text{if } m_{\nu j} \gg |q^2|_{\text{ave}}. \end{array} \right. \] (9)

Since the contribution of each virtual \( \nu_j \) is accompanied by a complex factor \( (U_{\nu j}^*)^2 \), it is possible for these contributions to add constructively or destructively. Because of the possibility of such cancellations, one cannot put an upper limit on neutrino masses or lepton mixing matrix coefficients from an upper bound on the decay \( K^+ \to \pi^- \mu^+ \mu^+ \). (A similar remark applies to \( K^+ \to \pi^- \mu^- e^+ \) and \( K^+ \to \pi^- e^- e^+ \) since cancellations can also occur among contributions of various \( \nu_j \)’s to the respective amplitudes for those decays.) The same comment is well-known in the case of neutrinoless double beta decay; for example, in the light-mass region, the lower limit on half-lives for 0ν2β transitions places an upper limit on \( \sum_j U_{\nu j}^2 m_{\nu j} \), not on \( |U_{\nu j}| \) or \( m_{\nu j} \) themselves.

With these inputs, we estimated [4]
\[ \text{BR}(K^+ \to \pi^- \mu^+ \mu^+) \sim 10^{-13 \pm 2} r_{\mu \mu} \left( \sum_j U_{\nu j}^2 f(m_{\nu j}/100 \text{ MeV}) \right)^2, \] (10)
where
\[ f(z) = \left\{ \begin{array}{ll} z & \text{if } z \ll 1, \\ 1/z & \text{if } z \gg 1, \end{array} \right. \] (11)
and the factor \( r_{\mu \mu} \simeq 0.2 \) is a relative phase space factor (normalized relative to the decay \( K^+ \to \pi^- e^+ e^+ \)). If \( z \sim O(1) \), one must, of course, retain the exact propagator. In Fig. 1(a), the range of (timelike) \( q^2 \) is \( (m_{\pi^+} + m_\mu)^2 \leq q^2 \leq (m_{K^+} - m_\mu)^2 \), i.e., 245 \( \leq \sqrt{q^2} \leq 388 \text{ MeV} \). Hence, if there exists a neutrino with \( m_{\nu j} \) in this range, 245 \( \leq m_{\nu j} \leq 388 \text{ MeV} \), then \( q^2 - m_{\nu j}^2 \) can vanish, leading to a resonant enhancement of the amplitude for this graph [10].

In the following, we assume for simplicity that a single mass eigenstate \( \nu_j \) dominates the sum in (10) but recall our remark above concerning the possibility of cancellations and note that it is straightforward to generalize our discussion to the case where there are several comparable contributions. If \( m_{\nu j} \ll m_K \), then, using the new upper bound (2) and the most conservative choice for the numerical prefactor in Eq. (10) (i.e., taking \( 10^{-13 \pm 2} \to 10^{-15} \)), we obtain
\[ |U_{\nu j}|^2 \left( \frac{m_{\nu j}}{100 \text{ MeV}} \right) \leq 4 \times 10^2 \left( \frac{\text{BR}(K^+ \to \pi^- \mu^+ \mu^+)}{3 \times 10^{-9}} \right)^{1/2}. \] (12)
On the other hand, if \( m_{\nu j} \gg m_K \), we obtain
\[ |U_{\nu j}|^2 \left( \frac{m_{\nu j}}{100 \text{ MeV}} \right) \leq 4 \times 10^2 \left( \frac{\text{BR}(K^+ \to \pi^- \mu^- e^+)}{3 \times 10^{-9}} \right)^{1/2}. \] (13)
Since \( m_{\nu j} \ll m_K \) in order for (12) to hold, and since \( |U_{\nu j}| < 1 \) by unitarity, the bound (12) does not place a significant restriction on either of these quantities. Similarly, in the heavy neutrino mass region, since \( m_{\nu j} \gg m_K \) in order for the bound (13) to hold, it does not place any restriction on \( m_{\nu j} \) or \( |U_{\nu j}| \) in this mass region [14]. This is not to say, however, that it is not worthwhile to search further for the decay \( K^+ \to \pi^- \mu^+ \mu^+ \), since it constitutes a testing ground for violation of total lepton number that is quite different from the usual searches for neutrinoless double beta decay of nuclei.

We also give an update of the limit on the decay \( K^+ \to \pi^- \mu^+ e^+ \). In [4] we obtained an indirect upper limit on this decay by observing that the leptonic part of the amplitude for this decay is related by crossing to the leptonic part of the amplitude for the conversion process in the field of a nucleus (\( Z, A \)):
\( \mu^- + (Z, A) \to e^+ + (Z - 2, A) \). At the time of [4], the best upper limit on this conversion process was
\( \sigma(\mu^- + \text{Ti} \to e^+ + \text{Ca})/\sigma(\mu^- + \text{Ti} \to \nu_\mu + \text{Se}) < 1.7 \times 10^{-10} \) from a TRIUMF experiment [15], and we
The current best bound, from a PSI experiment, is \[ \sigma \left( \mu^- + \text{Ti} \to e^+ + \text{Ca} \right) / \sigma \left( \mu^- + \text{Ti} \to v_\mu + \text{Sc} \right) < 1.7 \times 10^{-12} \quad (90\% \text{ CL}). \] (14)

Using this new bound, we conservatively infer that
\[ \text{BR}(K^+ \to \pi^- \mu^+ e^+) \lesssim \text{few} \times 10^{-11}. \] (15)

As was noted in [4], this is an indirect limit since it requires a theoretical estimate of the hadronic matrix element as input. Our indirect bound (15) is more stringent than the E865 limit (4), but the latter limit is still valuable since it is direct.

One can obtain an indirect upper limit on the decay \( K^+ \to \pi^- e^+ e^+ \) from the existing upper limit on neutrinoless double beta decay, which, for light neutrinos, gives \( |\sum J_i U_{i\alpha}^2 m_{\nu_i}| \lesssim 0.4 \text{ eV} \) (depending on the input used for the nuclear matrix elements) [17]. Using the same method as above, we obtain an upper limit many orders of magnitude less than the direct limit (3).

One can also consider \( \Delta L = 2 \) decays of \( D \) and \( B \) mesons. Here only rather modest direct upper limits of order \( 10^{-3} \) to \( 10^{-4} \) have been set on the branching ratios [12]. A similar comment applies to \( |\Delta L| = 2 \) hyperon decays, on which we previously set upper limits [18].

We next proceed to discuss constraints on \( R \)-parity violating (RPV) SUSY models. \( R \)-parity may be defined as \( R = (-1)^{3B+L+2S} \), where \( B \), \( L \), and \( S \) refer to the baryon and lepton numbers and to the spin of the particle [19]. Although \( R \)-parity was originally hypothesized in order to prevent intolerably rapid proton decay in SUSY models, one can achieve the same end by imposing weaker global symmetries that forbid terms of the form \( U_i^j D_j^k D_k^l \) in the superpotential (where the subscripts \( i, j, k \) here are generation indices and we follow the usual convention of writing the holomorphic operator products in terms of left-handed chiral superfields) while still allowing the \( R \)-parity violating terms
\[ W_{\text{RPV}} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \kappa_i L_i H_u. \] (16)

These terms violate total lepton number and, in general, also lepton family number. A recent review of \( R \)-parity violating SUSY models is [20]. The second term in (16) yields several contributions to \( K^+ \to \pi^- \mu^+ e^+ \), shown in Fig. 2(a), (b), where \( \tilde{\mu} \) and \( \tilde{\chi}^0 \) denote the scalar muon and neutralino.

Note that there may be neutrino-neutralino mixing in RPV theories, even if at some mass scale one rotates the terms \( \kappa_i L_i H_u \) to zero. A third type of diagram is shown in Fig. 2(c). As noted, for \( K^+ \to \pi^- \mu^+ e^+ \), each diagram is accompanied by minus the same diagram with the outgoing antilepton lines crossed.

The \( \tilde{\mu} \tilde{\chi}^0 \mu \) vertices are \( \propto \sqrt{g^2 + g'^2} \), where \( g \) and \( g' \) are the SU(2) and U(1) gauge couplings, while the \( ud\tilde{\mu} \) and \( \bar{s}\tilde{u}\tilde{\mu} \) vertices are \( \propto \lambda'_{211} \) and \( \lambda_{212} \), respectively, and the \( ud\tilde{e} \) and \( \bar{s}\tilde{u}\tilde{e} \) vertices are \( \propto \lambda_{121} \) and \( \lambda'_{121} \), respectively. In the third type of diagram, there can be a gluino on the internal line, with \( \tilde{d}\tilde{g}d \) and \( \tilde{u}\tilde{g}u \) vertices proportional to the strong coupling \( g_s \); or there can be a neutralino on the internal line, with vertices as given above. Bounds on RPV couplings are model-dependent, but typical current upper bounds on \( \lambda'_{211}, \lambda'_{212}, \lambda_{121}, \) and \( \lambda_{211} \) are \( \lesssim O(0.1) \) [20]. Using these inputs, we find that these \( R \)-parity violating contributions could be much larger than those from massive neutrinos and lepton mixing.

Fig. 2. Graphs that contribute to \( K^+ \to \pi^- \ell^+ \ell'^+, \) where \( \ell^+, \ell'^+ = \mu^+ \ell^+, \mu^+ e^+, \) or \( e^+ e^+, \) in supersymmetric theories with \( R \)-parity violation. In the case of identical \( \ell^+ \) and \( \ell'^+ \), it is understood that the contributions are from the diagrams minus the same diagrams with the outgoing antilepton lines crossed.
but are still expected to be small compared with the upper limit (2):

$$BR(K^+ \rightarrow \pi^- \mu^+ \mu^+)_{\text{RPV}} \lesssim 10^{-16} \left(\frac{\lambda_{211} \lambda_{212}^*}{m_{\text{SUSY}}}\right)^2 \left(\frac{200 \text{ GeV}}{m_{\text{SUSY}}}\right)^{10}. \quad (17)$$

For the purpose of this rough estimate, we have taken the masses of the various superpartners $\tilde{u}, \tilde{d}, \tilde{\mu}, \tilde{\chi}^0$, and $\tilde{g}$ to be comparable and denoted this mass scale as $m_{\text{SUSY}} \sim M_{\text{EW}}$. Note that, given the lower bounds on the masses of $\tilde{e}$ and $\tilde{\mu}$ or order 100 GeV, no resonance is possible in the amplitude of Fig. 2(a). One could, of course, take a particular SUSY parameter set and perform the calculation for this set; however, there is a large range of variation in possible superpartner masses as well as allowed values of other relevant parameters such as $\tan \beta$ (as illustrated, e.g., by the parameter sets used in [21]), so we deliberately keep our estimate general. Hence, it appears that the limit (2) does not strongly constrain possible RPV SUSY theories. The constraints on $R$-parity violating models from $\mu^- \rightarrow e^+ \nu \bar{\nu}$ conversion and neutrinoless double beta decay are also more stringent than those from the limits (4) and (3).

Thus, while neutrinoless double beta decay and $\mu^- \rightarrow e^+$ conversion are the most sensitive ways to search for $|\Delta L| = 2$ transitions with $|\Delta L_{\mu\nu}| = 2$ and $\Delta L_{\mu\tau} = \Delta L_{\tau\nu} = \pm 1$, respectively, the decay $K^+ \rightarrow \pi^- \mu^+ \mu^+$ is, at present, the best way to search for $|\Delta L| = 2$ transitions with $|\Delta L_{\mu\nu}| = 2$. It is, therefore, worthwhile to estimate the potential of future $K^+$ decay experiments to probe for this decay to lower values of branching ratio. In particular this might be undertaken by the CKM experiment planned at Fermilab [22]. The proposal for this experiment anticipates a statistical sensitivity of $\sim 10^{-12}$/event for the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. While it would require a substantial, and as yet unplanned, effort to design and build a trigger to search for $K^+ \rightarrow \pi^- \mu^+ \mu^+$, it appears [22] that this experiment might be able to reach a level near to $10^{-12}$ in branching ratio and thus improve substantially on the already impressive upper limit on $BR(K^+ \rightarrow \pi^- \mu^+ \mu^+)$ from E865 at Brookhaven. We also suggest that the planned experiment MECO at BNL [23], which anticipates searching for $\mu^- \rightarrow e^+$ conversion below the level $\sim 10^{-16}$ relative to $\mu^- \rightarrow \mu$ capture, should also undertake a search for $\mu^- \rightarrow e^+$ conversion.

Acknowledgements

We thank H. Ma for discussions. The research of L.S.L. was supported by DOE contract DE-AC02-98CH10886. The research of R.E.S. was supported at BNL by the DOE contract DE-AC02-98CH10886 and at Stony Brook by the NSF grant PHY-97-22101. The US government retains a non-exclusive royalty-free license to publish or reproduce the published form of this contribution or to allow others to do so for US government purposes.

References

[1] The solar neutrino experiments include Homestake, Kamiokande, GALLEX, SAGE, and SuperKamiokande. The atmospheric neutrino data is from Kamiokande, IMB, Soudan, SuperKamiokande with highest statistics, and MACRO. The LSND experiment also reports evidence for neutrino oscillations, although this has not been confirmed (or completely excluded) by the KARMEN experiment. The K2K and SNO experiments are in progress. Recent experimental reviews include: L. DiLella, hep-ex/9912010; H. Robertson, hep-ex/0001034, and talks at the Workshop on the Next Generation Nucleon Decay and Neutrino Detector NN999, Stony Brook (September 1999).


[3] We focus here on $|\Delta L| = 2$ processes $|\Delta L_{\mu\nu}| = 1$ processes are also of interest, e.g., in the context of searches for nucleon decay modes such as $\rho \rightarrow e^+ e^- \mu^+ \mu^-$, etc.


[5] In Ref. [4] and here we neglect very small $CP$ violating effects that could make $I'(K^+ \rightarrow \pi^- \ell^+ \ell'^-)$ differ slightly from $I'(K^0 \rightarrow \pi^+ \ell^- \ell'^+).$

[6] The BNL-Princeton-TRIUMF experiment E787, which has now observed the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$; E787 Collaboration, S. Adler et al., Phys. Rev. Lett. 79 (1997) 2204; E787 Collaboration, S. Adler et al., Phys. Rev. Lett. 84 (2000) 3768, and measured $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.5^{+3.4}_{-1.5} \times 10^{-10}$, also has some sensitivity for searching for $K^+ \rightarrow \pi^- \mu^+ \mu^-$. However, an estimate of the upper limit that can be derived from E787 data is less stringent than the limit (2) that has now been obtained by E865.

After the submission of this paper (hep-ph/0005285), another appeared by Dib et al. (hep-ph/0006277) in which it was claimed that the current upper limit on $K^+ \rightarrow \pi^- \mu^+ \mu^+$ sets a new limit on $|U_{\mu j}|^2$ in this mass region, e.g., $|U_{\mu j}|^2 < (5.6 \pm 1) \times 10^{-9}$. As noted in the text, because of the possibility of cancellations, one cannot set such a limit. Even if one assumes that there is no cancellation, this would not be a new limit, since it is less stringent than the 1988 limits from a CERN experiment [11,12], e.g., $|U_{\mu j}|^2 < 2 \times 10^{-9}$ for $m(\nu_j) = 300$ MeV. Furthermore, inserting the upper limits on $|U_{\mu j}|^2$ and $|U_{\tau j}|^2$ from [11] and calculating the resultant lower bound on the lifetime of such a neutrino (using, e.g., [13]), one finds a possible conflict with constraints from cosmology resulting from the fact that decays of such a neutrino would suppress large-scale structure formation in the early universe [9].

After the submission of this paper (hep-ph/0005285), another appeared by Dib et al. (hep-ph/0006277) in which it was claimed that the current upper limit on $K^+ \rightarrow \pi^- \mu^+ \mu^+$ sets a new limit on $|U_{\mu j}|^2$ in this mass region, e.g., $|U_{\mu j}|^2 < (5.6 \pm 1) \times 10^{-9}$. As noted in the text, because of the possibility of cancellations, one cannot set such a limit. Even if one assumes that there is no cancellation, this would not be a new limit, since it is less stringent than the 1988 limits from a CERN experiment [11,12], e.g., $|U_{\mu j}|^2 < 2 \times 10^{-9}$ for $m(\nu_j) = 300$ MeV. Furthermore, inserting the upper limits on $|U_{\mu j}|^2$ and $|U_{\tau j}|^2$ from [11] and calculating the resultant lower bound on the lifetime of such a neutrino (using, e.g., [13]), one finds a possible conflict with constraints from cosmology resulting from the fact that decays of such a neutrino would suppress large-scale structure formation in the early universe [9].
Maximal $\nu_\mu - \nu_\tau$ oscillations, the see-saw mechanism and the Exact Parity Model

T.L. Yoon *, R. Foot

School of Physics, Research Centre for High Energy Physics, The University of Melbourne, Victoria 3010, Australia

Received 11 August 2000; accepted 15 September 2000
Editor: H. Georgi

Abstract

We examine one simple mechanism which leads to approximate maximal $\nu_\mu - \nu_\tau$ oscillations in the standard see-saw model. In particular, we show that this scheme could be implemented in the Exact Parity Model (also known as the mirror matter model). Within this framework the solar neutrino problem is solved by maximal $\nu_e$ oscillations (with $\nu'_e$ is the essentially sterile mirror partner of $\nu_e$) and the LSND evidence for $\nu_e \rightarrow \nu_\mu$ oscillations can also be explained.

Neutrino physics continues to provide the most promising window on physics beyond the standard model. The evidence comes from three different classes of experiments, namely the atmospheric [1,2], solar [3] and LSND [4] experiments. Of these, the most convincing evidence of neutrino oscillation comes from the atmospheric neutrino anomaly as confirmed by SuperKamiokande [2]. At the present time the only two flavour oscillation solutions consistent with the superKamiokande data are $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_{\text{sterile}}$ oscillations [5]. Recently the superKamiokande collaboration have argued that the $\nu_\mu \rightarrow \nu_{\text{sterile}}$ solution is disfavoured at more than 99% C.L. [6]. However this conclusion depends on how the data is analysed and should not be considered as conclusive [7]. Furthermore, a global analysis of the $\nu_\mu \rightarrow \nu_{\text{sterile}}$ solution to the atmospheric neutrino data provides a reasonable fit [8]. Thus, the $\nu_\mu \rightarrow \nu_{\text{sterile}}$ oscillations remain a viable explanation of the atmospheric neutrino data. However, for the purposes of this paper we will assume that nature chooses the $\nu_\mu \rightarrow \nu_\tau$ oscillation solution. The main point of our paper is to point out a rather simple way of achieving approximately maximal $\nu_\mu \rightarrow \nu_\tau$ oscillations in the context of the usual see-saw model in a natural way. We also point out how the solar neutrino and LSND indications for neutrino oscillations could also be understood in this scheme.

In the context of the standard see-saw model, it is well known that the effective mass matrix of the light left-handed neutrinos $m_L$ can be obtained from diagonalising the see-saw mass matrix

\[ M = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}, \tag{1} \]

leading to

\[ m_L = -V_L d_t W_R D^{-1} W_R^T d_t V_L^T. \tag{2} \]

In Eq. (1) and Eq. (2), $M_D$ and $M_R$ are the $3 \times 3$ Dirac and Majorana matrix for $\nu_L$ and $\nu_R$, respectively.

* Corresponding author.
E-mail addresses: tyoon@physics.unimelb.edu.au (T.L. Yoon), foot@physics.unimelb.edu.au (R. Foot).
$W_R = V_R^T U_R^*$, where $V_{R,L}$ and $U_R$ are unitary matrices that diagonalise $M_D$ and $M_R$ as

\[ M_D = V_L d_e V_R^T, \quad M_R = U_R D U_R^T, \]  

(3)

where $d_e$ and $D$ are diagonal matrices containing real eigenvalues of $M_D$ and $M_R$, i.e.,

\[ d_e = \text{diag}(m_1^D, m_2^D, m_3^D), \]

\[ D = \text{diag}(M_1, M_2, M_3). \]  

(4)

From general considerations, it seems most natural to assume that $M_D$ is hierarchical.\(^1\) In addition, it is also most natural to expect that $M_D$ is approximately aligned with the charged lepton mass matrix, as a parallel to the charged quark sector. In fact, these features arise in simple GUT models such as the so-called GUT models such as $SO(10)$ \(^9\) as well as in models with quark lepton symmetry \(^{10, 11}\). For $M_R$, however, there is no natural expectation as such because the charged leptons and quarks have no Majorana mass due to electric charge conservation. In contrast to the Dirac mass terms in the Standard Model, $M_R$ is expected to arise from quite different physics, e.g., from coupling with a distinct Higgs field at a much higher scale. We can then ask the question: Is there any reasonable choice for $M_R$ that could lead to approximately maximal $v_{\mu L} - v_{\tau L}$ oscillations compatible with the atmospheric neutrino anomaly?

Let’s consider the 2–3 sector of the mass matrices. The symmetric Majorana mass matrix of $\nu_R$ can be generally written as\(^2\)

\[ M_R = \begin{pmatrix} \lambda_1 & \lambda_2 \\ \lambda_2 & \lambda_3 \end{pmatrix}, \]  

(5)

and is diagonalised by

\[ U_R = \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix}. \]  

(6)

The mixing angle among $v_{\mu R} - v_{\tau R}$ is parametrised by

\[ \tan 2\psi = \frac{2\lambda_3}{\lambda_1 - \lambda_2}, \]  

(7)

and the eigenvalues are given by

\[ M_{2,3} = \frac{1}{2} (\lambda_1 + \lambda_2) \mp \frac{1}{2} \sqrt{(\lambda_2 - \lambda_1)^2 + 4\lambda_3^2}. \]  

(8)

\(^1\) Although it is not a necessary condition in the scheme we shall propose here.

\(^2\) We use the basis with $V_R = 1$, which we can choose without loss of generality.

The mixing angle among $v_{\mu L} - v_{\tau L}$ implied by Eq. (2) is easily worked out to be

\[ \tan 2\theta_L \]  

\[ = \frac{2m_2^D m_3^D \sin \psi \cos \psi \left( \frac{1}{M_2} - \frac{1}{M_3} \right)}{(m_3^D)^2 \left( \frac{\sin^2 \psi}{M_2} + \frac{\cos^2 \psi}{M_3} \right) - (m_2^D)^2 \left( \frac{\cos^2 \psi}{M_2} + \frac{\sin^2 \psi}{M_3} \right)}. \]  

(9)

Approximate maximal mixing requires the condition

\[ \tan^2 2\theta_L \gg 1. \]  

(10)

As already mentioned, we will assume that $M_D$ is diagonal (i.e., $V_L = 1$) (needless to say, our conclusion requires only that $V_L$ is approximately diagonal as in the case of the CKM matrix). The issue is to find the form of $M_R$ which then leads to maximal oscillations for the light $v_{\mu L} \rightarrow v_{\tau L}$ states. One possibility already discussed in the literature \(^{12}\) is that

\[ M_R^{-1} \propto \begin{pmatrix} 1 & p \\ p & p^2 \end{pmatrix}, \]  

(11)

where $p = m_2^D / m_3^D$. This case has been studied in Ref. \(^{12}\) in detail. In this case the light neutrinos are hierarchical. While this is certainly an interesting possibility, it doesn’t seem to be the simplest possibility in our opinion. In particular, as far as we can see there is no obvious approximate symmetry corresponding to the form Eq. (11) (although the authors of Ref. \(^{12}\) argue that it is more natural than it looks).

It seems to us that the most natural choice of $M_R$ which does the trick is where $M_R$ is approximately maximally mixed, with

\[ |\lambda_3| \gg |\lambda_2|, |\lambda_1|. \]  

(12)

In this limit, the heavy Majorana mass eigenvalues are [from Eq. (8)],

\[ M_{2,3} \simeq \mp \lambda_3 + \frac{\lambda_1 + \lambda_2}{2}, \]  

(13)

which are approximately degenerate. It is easy to show that $M_R$ is diagonalised by the unitary transformation, Eq. (6) with

\[ \tan^2 \psi = 1 - \frac{(\lambda_1 - \lambda_2)}{\lambda_3} + O \left( \frac{\lambda_{1,2}}{\lambda_3} \right)^2. \]  

(14)

It is easily checked that indeed, in the limit of Eq. (12),

\[ \tan^2 2\theta_L \simeq \left( \frac{-2\lambda_3 \sin^2 \psi m_3^D}{(m_3^D)^2 \lambda_2 - (m_2^D)^2 \lambda_1} \right)^2 \gg 1, \]  

(15)
irrespective of whether there is hierarchy in $m_{1,2,3}^D$. Hence maximal mixing for the light $\nu_\mu, \tau$ states follows in the limit $|\lambda_3| \gg |\lambda_1|, |\lambda_2|$. Meanwhile, the effective $\nu_L$ light Majorana mass matrix has the form:

$$m_L \approx \begin{pmatrix} (m_{10}^D)^2 \frac{-\lambda_1}{\lambda_2} & \frac{m_{20}^D m_{10}^D}{\lambda_2} \\ \frac{m_{20}^D m_{10}^D}{\lambda_2} & (m_{30}^D)^2 \frac{-\lambda_3}{\lambda_2} \end{pmatrix} ,$$

resulting in an approximate degenerate pair of light flavour neutrino masses (where we have made the usual phase transformation to make them positive) of

$$m_L \approx \frac{m_{10}^D m_{30}^D}{\lambda_3} \pm \frac{1}{2} \left( (m_{10}^D)^2 \frac{-\lambda_1}{\lambda_2} + (m_{30}^D)^2 \frac{-\lambda_3}{\lambda_2} \right).$$

Observe that in this scenario there is an approximate $U(1)_{L\mu - L\tau}$ global symmetry, which may simply be an accidental approximate symmetry of the theory. Nevertheless, this approximate global symmetry could play an important role in preventing radiative corrections from spoiling the form of $m_L$.

Of course, this see-saw model with an approximately diagonal $M_D$ but with the “lop sided” $M_R$ is not the only way of achieving approximately maximal $\nu_\mu \to \nu_\tau$ mixing. However, it does seem rather nice to us. As mentioned already, this scheme could easily be implemented in standard GUT models as well as models with quark–lepton symmetry. Moreover, as we will show, it fits in nicely with models with unbroken parity symmetry (Exact Parity model). While we arrived at this scheme independently, we have searched the literature and have discovered that similar ideas are contained in Ref. [13] in the context of models with $U(1)_F$ family gauge symmetry. Let us now turn to the other neutrino anomalies: The solar and the LSND experiments.

The LSND experiment suggests $\nu_e \to \nu_\mu$ oscillations with $\sin^2 \theta_{\mu e} \sim 10^{-2}$ and $\delta m^2 \sim 1 \text{ eV}^2$ [4]. This can easily be incorporated into this scheme. We need only assume that $M_R$ has the approximate form:

$$M_R \approx \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{pmatrix} ,$$

where the zero entries are in general non-zero but small (relatively to the other entries).

Within the context of our scheme, the solar neutrino anomaly may be solved by oscillations into a light sterile neutrino (and elegance suggests three such species, which we denote by $\nu'_{e,\mu,\tau}$). There are essentially two possibilities: either small angle $\nu_e \to \nu'_{e}$ MSW solution is invoked [14] or approximately maximal $\nu_\mu \to \nu'_{\mu}$ oscillations may be responsible [15]. The latter can explain (and in fact predicted [15, 16]) the approximate 50% reduction of solar neutrinos including the observed energy independent superKamiokande flux suppression [17] (although Homestake is a little on the low side). Furthermore, observations from Borexino and Kamland must find at least one “smoking gun” signature if this is the physics responsible for the solar neutrino deficit (and SNO should see no anomalous Neutral current/charged current ratio) [15].

On the theoretical front, an elegant motivation for maximal $\nu_\mu \to \nu'_{\mu}$ oscillations comes from the observation that gauge models with unbroken parity symmetry predict that each of the three ordinary neutrinos are approximately maximally mixed with a sterile partner if neutrinos have mass [16,18]. Obviously this would suggest that the most natural explanation for the atmospheric neutrino anomaly is with maximal $\nu_\mu \to \nu'_{\mu}$ oscillations. Nevertheless it is still possible that the atmospheric neutrino anomaly may be predominantly due to $\nu_\mu \to \nu_\tau$ oscillations if the the oscillation length for $\nu_\mu \to \nu'_{\mu}$ oscillations is much longer than the diameter of the earth for typical atmospheric neutrino energies [19]. If this is the case then the simple scheme discussed in this paper may easily be invoked to give approximately maximal $\nu_\mu \to \nu_\tau$ oscillation solution for the atmospheric neutrino anomaly in such models.

To be more concrete, let’s consider the Exact Parity Model (EPM) extended to include a heavy see-saw gauge singlet $\nu_R$ (together with it’s parity partner $\nu'_R$) [18]. In each generation there are 4 Weyl neutrino fields, namely, $\nu_L, \nu_R$ and their parity partners $\nu'_R$ and $\nu'_L$. With the minimal Higgs and it’s mirror doublet, the mass matrix has the form [18]

$$\mathcal{L}_{mass} = \bar{\nu}_L \mathcal{M} (\Psi_L)^T + \text{H.c.},$$

where

$$\Psi_L = \begin{pmatrix} (\nu_L)^c, \nu'_R, \nu_R, (\nu'_L)^c \end{pmatrix}^T,$$

and

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & m_a & m_b \\ 0 & 0 & m_b & M_a \\ m_a & m_b & M_d & M_b \\ m_a & m_b & M_b & M_d \end{pmatrix}.$$
Note that \( m_{a,b} \) are mass terms that arise from spontaneous symmetry breaking, while \( M_{a,b} \) are bare mass terms, all being free parameters of the theory. The masses are assumed to be real without loss of generality. In the parity diagonal basis

\[
v_{L}^\pm = \frac{v_{L} \pm (v_{R}^\prime)^{c}}{\sqrt{2}} \quad \text{and} \quad v_{R}^\pm = \frac{v_{R} \pm (v_{L}^\prime)^{c}}{\sqrt{2}},
\]

the mass matrix of Eq. (21) is diagonalised to give eigenvalues \( m_{\pm}, m_{-} \), \( M_{\pm}, M_{-} \), where \( m_{\pm}, M_{\pm} \) are functions of \( m_{a,b} \) and \( M_{a,b} \) [18]. Following the usual nomenclature, \( m_{+}, m_{-} \) refer to the masses of light neutrino parity states of \( v_{L}^{+} \), whereas \( M_{+}, M_{-} \) are the masses of the heavy parity states \( v_{R}^{+} \). In the limit \( M_{\pm} \gg m_{\pm} \), the singlet states decouple from the SU(2) doublet states. It is straightforward to generalise the 1 generation case to describe cross generation mixing among the 2–3 sector. Introducing 2 arbitrary intergenerational mixing angles \( \theta \) and \( \phi \) to parametrise the generation mixing between the \((+)(-)\) parity states \( v_{L}^{+} \) and \( v_{L}^{-} \) (\( v_{R}^{+} \) and \( v_{R}^{-} \)), the active flavour states will have the form

\[
v_{\mu L} = \frac{\cos \theta v_{3L}^{+}}{\sqrt{2}} + \frac{\sin \theta v_{3L}^{-}}{\sqrt{2}} + \frac{\cos \phi v_{3L}}{\sqrt{2}} + \frac{\sin \phi v_{2L}}{\sqrt{2}},
\]

\[
v_{\tau L} = -\frac{\sin \theta v_{3L}^{+}}{\sqrt{2}} + \frac{\cos \theta v_{3L}^{-}}{\sqrt{2}} - \frac{\sin \phi v_{2L}^{-}}{\sqrt{2}} - \frac{\cos \phi v_{2L}}{\sqrt{2}}.
\]

From Eq. (22), the transition probability from \( v_{\mu L} \) to \( v_{\tau L} \) in the relativistic limit is

\[
P(v_{\mu L} \rightarrow v_{\tau L}) = \left| \langle v_{\tau L}|v_{\mu L}(t) \rangle \right|^{2}
\]

\[
= \frac{1}{4} \sin^{2}2\theta \sin^{2}2\phi \sin^{2}(\pi L/L_{+})
\]

\[
+ \frac{1}{4} \sin^{2}2\phi \sin^{2}(\pi L/L_{-})
\]

\[
+ \frac{1}{2} \sin 2\theta \sin 2\phi \sin(\pi L/L_{+}) \cos(\pi L/L_{int})
\]

\[
\times \sin(\pi L/L_{+}) \cos(\pi L/L_{int}),
\]

where the oscillation lengths are

\[
L_{\pm} = \frac{4\pi E}{\delta m_{32}^{\pm}}, \quad L_{int} = \frac{4\pi E}{\delta m_{32}^{\pm}}.
\]

In the above equations, \( E \) denotes the energy of \( v_{\mu L} \), \( L \) is the distance from where it is produced, \( \delta m_{32}^{\pm} \equiv (m_{3}^{\pm})^{2} - (m_{2}^{\pm})^{2} \), and \( \delta m_{32}^{\pm} \) is the mass square difference.

The limit of the (accidental?) \( U(1)_{L_{\mu} - L_{\tau}} \) global symmetry would mean (i) \( \phi \approx \theta \approx \pi/4 \) and (ii) the eigen masses of the two generations will be approximately degenerate, i.e., \( (m_{3}^{\pm})^{2} \approx (m_{2}^{\pm})^{2} \). To produce \( v_{\mu} - v_{\tau} \), maximal oscillations in the EPM, we also need to render approximate equality in the oscillation lengths, \( L_{+} \approx L_{-} \) and \( L_{int} \gg L_{-} \). This happens when the coupling between the ordinary and mirror neutrinos is small (so that \( m_{3}^{\pm} \rightarrow m_{2}^{\pm} \), \( m_{3}^{\pm} \rightarrow m_{2}^{\pm} \)). In this limit, the standard two-neutrino oscillation formula

\[
P(v_{\mu L} \rightarrow v_{\tau L}) = \sin^{2}2\theta \sin^{2}(\pi L/L_{+})
\]

is then recovered. Thus for atmospheric neutrinos, which have typical energy \( E \sim \text{GeV} \), we require \( L_{int} \gg 10^{000} \text{ km} \). In this limit, the atmospheric \( v_{\mu} \) oscillations approximately reduce to pure \( v_{\mu} \rightarrow v_{\tau} \) oscillations.

Thus we have shown that this “lop sided” \( M_{R} \) scenario does fit in nicely with the EPM. Hence, if experiments do end up proving that the atmospheric neutrino anomaly is due to \( v_{\mu} \rightarrow v_{\tau} \) oscillations, and the solar problem is due to maximal \( v_{e} \rightarrow v_{\mu} \) oscillations then this is consistent with the EPM. Of course, within the framework of the EPM we would most naturally expect the atmospheric neutrino anomaly to be solved by maximal \( v_{\mu} \rightarrow v_{\mu}^{\prime} \) oscillations, since then we can solve the atmospheric neutrino anomaly and solar neutrino problem by the same mechanism. This would seem to be theoretically most satisfying.

The implications for the early Universe of such models with sterile/mirror neutrinos have been discussed in some detail in the literature (see, e.g., Ref. [20] and references there-in). Models with sterile neutrinos have the amusing feature that they typically lead to the dynamical generation of a large neutrino asymmetry [21,22]. This leads to a number of interesting effects for big bang nucleosynthesis (BBN), and the microwave background. For example, it was shown in Ref. [22] that the maximal \( v_{e} \rightarrow v_{\mu}^{\prime} \) oscillations do not necessarily lead to adverse implications for BBN even if \( \delta m^{2} \) is relatively large \( \sim 10^{-3} \text{ eV}^{2} \) because of
the suppression of the oscillations due to the dynamically generated $L_{\nu_\mu}, L_{\nu_e}$ asymmetry. Further implications for models with degenerate $\nu_\mu, \nu_e$ are discussed in Ref. [23].

In conclusion, we have suggested a simple scheme to achieve approximately maximal $\nu_\mu - \nu_\tau$ oscillations which is one of the possible solutions to the atmospheric neutrino anomaly. In the context of this scheme, we have shown that the Exact Parity model could also accommodate $\nu_\mu - \nu_\nu$ oscillations in place of the usual $\nu_\mu - \nu_\tau$ oscillations. This scheme is also compatible with the LSND oscillation signal, which leaves the solar neutrino problem to be solved by $\nu_e \rightarrow \nu_{\text{sterile}}$ oscillations.

Acknowledgements

R.F. is an Australian Research Fellow. T.L. Yoon is supported by OPRS and MRS

References

NUSEX Collaboration, M. Aglietta et al., Europhys. Lett. 8 (1989) 611;


For some 4 neutrino models with small angle MSW $\nu_e \rightarrow \nu_s$ solution, see, e.g.:


Relaxing $b \to s\gamma$ constraints on the supersymmetric particle mass spectrum

Ernest Ma $^a$, Martti Raidal $^{a,b}$

$^a$ Department of Physics, University of California, Riverside, CA 92521, USA
$^b$ National Institute of Chemical Physics and Biophysics, Rävala 10, 10143 Tallinn, Estonia

Received 21 June 2000; received in revised form 5 September 2000; accepted 6 September 2000

Editor: M. Cvetič

Abstract

We consider the radiative decay $b \to s\gamma$ in a supersymmetric extension of the standard model of particle interactions, where the $b$-quark mass is entirely radiative in origin. This is accomplished by the presence of nonholomorphic soft supersymmetry breaking terms in the Lagrangian. As a result, the contributions to the $b \to s\gamma$ amplitude from the charged Higgs-boson and the charginos/neutralinos are suppressed by $1/\tan^2\beta$ and $O(\alpha_s/\alpha)$, respectively, allowing these particles to be lighter than in the usual supersymmetric model. Their radiatively generated couplings differ from the usual tree-level ones and change the collider phenomenology drastically. We also study how this scenario may be embedded into a larger framework, such as supersymmetric SU(5) grand unification.

1. Introduction

The minimal supersymmetric standard model (MSSM) [1] is one of the most popular extensions of the standard model (SM). In recent years, both theorists and experimentalists have devoted enormous amounts of time to study its predictions. While superpartner masses are expected to be below 1 TeV in some scenarios, there is actually a lot of uncertainty regarding the soft supersymmetry (SUSY) breaking sector of the theory. In the most general case, the MSSM contains more than one hundred free parameters. Nevertheless, present collider data as well as low-energy experiments are starting to place nontrivial constraints on the supersymmetric particle mass spectrum.

One of the processes known to put stringent constraints on new physics is the radiative decay $b \to s\gamma$ with a branching ratio experimentally determined to be in the range [2]

$$2 \times 10^{-4} < \text{BR}(B \to X_s\gamma) < 4.5 \times 10^{-4}. \quad (1)$$

This result agrees with the SM prediction. On the other hand, in the MSSM framework, the decay $b \to s\gamma$ receives large additional contributions from charged Higgs-boson, chargino, neutralino, and gluino loops [3]. The leading-order quantum-chromodynamics (LO QCD) corrections to $b \to s\gamma$ are known in the MSSM for arbitrary flavour structures [3,4], while the next-to-leading-order (NLO) analyses have been performed only for specific scenarios [5]. The charged Higgs-boson contribution always adds constructively to that of the SM (i.e., the W-boson contribution). The magnitudes of chargino and neutralino contributions depend strongly on $\tan\beta$. For large values of $\tan\beta$, the chargino contribution becomes the dominant one. In that case, its sign is determined by that of the $\mu$ para-
There is a part of the parameter space with a radiatively driven mass spectrum, it adds constructively to the chargino contribution [6].

An important issue for understanding the $b \to s\gamma$ constraints on SUSY models, recently reemphasized in Ref. [7], is the proper inclusion of loop corrections to the $b \to s\gamma$ rate. These are enhanced for large $\tan\beta$ and their sign is also determined by the $\mu$ parameter. This implies a strong correlation between the values of the $b\to s\gamma$ Yukawa coupling, SUSY model parameters, and the $b \to s\gamma$ constraints.

The gauge-coupling unification in the MSSM strongly suggests that there is a grand unified theory (GUT) above the unification scale $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV, such as SU(5) or SO(10) which allows tau-bottom or tau-bottom-top Yukawa coupling unification, respectively. However, successful Yukawa unification [9] is achieved only for one sign of the $\mu$ parameter, $\text{sign}(\mu) = -$ in our convention, and for very large values of $\tan\beta$, say $\sim 30–50$ for SU(5) and $\sim 50$ for SO(10) [10]. For $\text{sign}(\mu) = +$, all dominant contributions to $b \to s\gamma$ add constructively, implying thus very strong constraints on these SUSY scenarios. Typically, the SUSY mass scale must exceed 1 TeV to be consistent with the bound given by Eq. (1) [10].

The charged Higgs-boson mass is forced to be large in this case, typically above the reach of the tevatron at Fermilab as well as that of the large hadron collider (LHC) at CERN. In more general scenarios, $\text{sign}(\mu) = +$ allows cancellations between different terms. However for large $\tan\beta$, the cancellation can happen only for a restricted part of the parameter space and stringent constraints may still be in force. In conclusion, the $b \to s\gamma$ bound of Eq. (1) implies strong constraints on MSSM parameters in general, and on GUT scenarios in particular.

There are some proposals to satisfy the $b \to s\gamma$ constraints which still allow light sparticle masses accessible at future colliders. In Ref. [11], it has been pointed out in the context of SO(10) GUT that with $m, A \gg M_{1/2}$, where $m$ and $A$ denote generically common masses of matter multiplets and soft $A$ terms, respectively, and $M_{1/2}$ is the common gaugino mass, there is a part of the parameter space for which the $b \to s\gamma$ rate is in the range given by Eq. (1). In that case, the scalar superpartner masses are very large and suppress the new SUSY contributions to $b \to s\gamma$ whereas the wino masses can still be light. This scenario implies that the only discoverable SUSY particles are light charginos and neutralinos. The new Higgs-bosons are heavy and the collider phenomenology discussed in Ref. [12] is not allowed.

The authors of Refs. [4,7] argue instead that there might be new flavour violation present in the squark sector of the model which modifies and enhances the gluino contribution which then cancels the other large SUSY contributions. This scenario requires the introduction of new unknown flavour physics in general SUSY models and is not realized in GUTs [6]. Also, the cancellation of two large terms cannot be considered a natural solution.

The purpose of this letter is to show that the dominant SUSY contributions to $b \to s\gamma$ may be naturally suppressed if the SUSY radiative corrections to the $b$-quark Yukawa coupling are large. We consider the limit of vanishingly small down-quark Yukawa couplings so that the corresponding quark masses are generated radiatively [13,14]. Making the usual assumption that the trilinear $A$ terms are proportional to Yukawa couplings, we are forced to introduce the nonholomorphic $A'$ terms [14–17] to give a correct mass to the $b$-quark. After all, the soft $A'$ terms should be included in the complete SUSY Lagrangian on general grounds. We find that in this scenario, the charged Higgs-boson and the dominant chargino contributions to $b \to s\gamma$ rate are suppressed by $1/\tan^2\beta$ and $\mathcal{O}(a/\alpha_s)$, respectively. Therefore, the soft, radiatively induced couplings of these particles change the Tevatron and LHC phenomenology considerably by reducing their production rates. This scenario can also be embedded into a GUT framework, such as SUSY SU(5), removing the stringent constraint on the MSSM parameter space coming from Yukawa unification. We discuss the sparticle mass spectrum in that case.

2. Proposed model and the radiative decay $b \to s\gamma$

In the following we work with the usual particle content of the MSSM [11]. However, we assume that the Yukawa coupling matrix in the $(d, s, b)$ quark
sector is vanishing, i.e., $f_{d_{ij}} = 0$. In this case, the relevant MSSM superpotential $W$ for quark and Higgs left-chiral superfields is

$$W = Q_i (f_{u_{ij}}) U_{ij}^c H_2 - \mu H_1 H_2. \quad (2)$$

Further we make the usual assumption that the trilinear soft SUSY breaking terms $a_{ij}$ have the same structure as the Yukawa coupling matrices: $a_{ij} = A \cdot f_{ij}$. Thus, neglecting leptons, the most general soft SUSY breaking terms in the MSSM are given by

$$-\mathcal{L}_{\text{soft}} = \tilde{Q}_i^c (m^2 \tilde{Q}_{ij})_{ij} \tilde{Q}_j + \tilde{U}_i^c (m^2 \tilde{U}_{ij})_{ij} \tilde{U}_j ^c + \tilde{D}_i^c (m^2 \tilde{D}_{ij})_{ij} \tilde{D}_j + m^2 H_1 H_1 + m^2 H_2 H_2
$$

$$+ (\tilde{Q}_i (A_u + f_{u_{ij}}) \tilde{U}_{ij}^c H_2
$$

$$- \tilde{Q}_i (A_d + f_{d_{ij}}) \tilde{D}_{ij} H_1^c + B_H H_1 H_2 + \frac{1}{2} M_1 \tilde{B} \tilde{B} ^c + \frac{1}{2} M_2 \tilde{W}_{\alpha} \tilde{W}^\alpha + \frac{1}{2} M_3 \tilde{W}_{\alpha} \tilde{W}^\alpha + \text{h.c.}, \quad (3)$$

where the $A'_{ij}$ term is the nonholomorphic soft term and it does not cause quadratic divergences. Therefore, it has been emphasized in Ref. [15], and more recently in Ref. [16], that these terms should be included in the MSSM to study its low-energy phenomenology; their omission cannot be justified in the general context. In the framework of high-energy physics, the nonholomorphic terms are generated, and not necessarily suppressed, in scenarios of spontaneous SUSY breaking such as those being mediated by supergravity [17]. In strongly coupled supersymmetric gauge theories, the nonholomorphic soft terms occur naturally [18]. Without understanding the real origin of SUSY breaking, we should keep the $A'$ terms in the general soft Lagrangian of the MSSM, such as Eq. (3).

Radiatively induced quark masses from the $A'$ terms have been recently studied in Ref. [14]. The $(u, c, t)$ quark masses come directly at tree level from the hard Yukawa interaction in the superpotential Eq. (2). However, the $(d, s, b)$ quark masses must be generated radiatively [13,14]. Taking only the dominant gluino-mediated contribution and neglecting intergenerational mixings, the one-loop soft bottom quark mass is given by

$$m_b = -\frac{\alpha_y}{3\pi} m_{\tilde{g}} m_t A_b' I (m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}}). \quad (4)$$

Here the necessary chirality violation is due to the gluino mass insertion and the term $m_t A_b'$ is the off-diagonal entry of the sbottom mass matrix. The loop function $I (m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})$ is given by [8]

$$I (m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}}) = -\frac{m^2_{\tilde{b}_1} m^2_{\tilde{b}_2} \ln (m^2_{\tilde{b}_1}/m^2_{\tilde{b}_2})}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})}
$$

$$+ \frac{m^2_{\tilde{b}_1} m^2_{\tilde{b}_2} \ln (m^2_{\tilde{b}_2}/m^2_{\tilde{b}_1}) + m^2_{\tilde{b}_1} m^2_{\tilde{b}_2} \ln (m^2_{\tilde{b}_2}/m^2_{\tilde{g}})}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})(m^2_{\tilde{b}_2} - m^2_{\tilde{g}})}. \quad (5)$$

Obtaining the correct bottom quark mass via Eq. (4) implies stringent constraints on the model parameters.

As the tree-level bottom Yukawa term is missing in the superpotential Eq. (2), the $b_R b_R H^+$ coupling is induced radiatively by the diagram depicted in Fig. 1. The induced soft Lagrangian at the one-loop level can be expressed as

$$\mathcal{L}_{\text{Yukawa}}^\text{rad} = -\frac{m_b}{\tan \beta} \left( r_1 P_L + r_2 P_R b b H^+ + \text{h.c.}, \right. \quad (6)$$

where the form factors $r_1$ and $r_2$ are given by

$$r_1 = \frac{\sin 2\theta_t \sin 2\theta_b}{4 I (m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})} \times \left[ C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_1}) + C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_2}) - C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_1}) - C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_1}) \right], \quad (7)$$

$$r_2 = \frac{1}{I (m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})} \times \left[ \cos^2 \theta_t \sin^2 \theta_b C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_1}) + \sin^2 \theta_t \cos^2 \theta_b C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_2}) + \cos^2 \theta_t \cos^2 \theta_b C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_1}) + \sin^2 \theta_t \sin^2 \theta_b C_0 (0, 0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_2}) \right]. \quad (8)$$

Fig. 1. Feynman diagram giving rise to the radiative $b_R b_H H^+$ coupling.
Here the three-point functions \( C_0(0,0,m_H^2; m_Q^2, m_H^2) \) are defined in [14] and we do not present them here. In the limit \( m_H \to 0 \), which is a good approximation if \( m_H \ll m_t, m_b, m \) the three-point functions become \( C_0(0,0; m_Q^2, m_Q^2) \to I(m_Q^2) \).

As with the usual hard tree-level coupling which we have omitted, the Lagrangian Eq. (6) is also proportional to \( m_b \). However, the most important feature to notice is that the coupling is now suppressed by \( \tan \beta \) and not enhanced by it. This is because the nonholomorphic \( A' \) term couples to \( H_2 \) rather than to \( H_1 \) as in the usual case. The induced couplings are momentum-dependent and are characterized by the form factors \( r_1 \) and \( r_2 \). If the couplings arise from the tree-level superpotential but is suppressed by \( 1/\tan^2 \beta \) compared to it.

For quarks which obtain their masses radiatively, their higgsino couplings are also generated radiatively. The details for the neutral higgsinos are given in Ref. [14]; similar arguments apply also for the charged higgsinos. Without going into details, the most important feature of the radiatively induced couplings of the right-handed \( b \)-quark to the higgsino \( \tilde{H} \) and squark is that it is induced by loops involving binos. Thus the radiatively induced couplings for \( b_R \) are always suppressed by \( O(\alpha/\alpha_s) \) compared to the hard couplings.

The effective Hamiltonian for \( b \to s \gamma \) in SUSY models can be expressed in two terms:

\[
\mathcal{H}_\text{eff} = \mathcal{H}_\text{eff}^{\text{CKM}} + \mathcal{H}_\text{eff}^{\tilde{b}} \tag{9}
\]

where

\[
\mathcal{H}_\text{eff}^{\text{CKM}} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_j C_j(\mu) \mathcal{O}_j(\mu) \tag{10}
\]

contains the SM as well as the charged Higgs-boson, chargino, and neutralino contributions with the same flavour structure as in the SM. The explicit formulas including LO QCD corrections can be found in Ref. [3]. The gluino contribution \( \mathcal{H}_\text{eff}^{\tilde{b}} \) may exhibit in addition the new flavour violation present in general SUSY models but absent in our scenario; details including LO QCD corrections can be found in Ref. [4]. The decay width of \( b \to s \gamma \) can be written as

\[
\Gamma(b \to s \gamma) = \frac{m_b^5 G_F^2 |V_{tb} V_{ts}^*|^2 \alpha}{32 \pi^4} |C_7^{\text{eff}}|^2, \tag{11}
\]

where \( |C_7^{\text{eff}}|^2 = |C_7 + C_4^2|^2 + |C_4^2|^2 \). Here \( C_7 \) stands for the total contribution from the effective Hamiltonian Eq. (10), while \( C_4^2 \) and \( C_4^2 \) arise from gluino loops. The present experimental result Eq. (1) implies the allowed range:

\[
0.25 < |C_7^{\text{eff}}| < 0.375. \tag{12}
\]
Let us now consider different SUSY contributions to \( b \to s \gamma \) in our scenario. The charged Higgs-boson contribution is induced by two different chiral structures: the SM-like one with the chirality flip in the external \( b \)-quark line which is induced by the hard top Yukawa interactions, and the one with the chirality flip in the internal \( t \)-quark line which is induced by the radiative coupling Eq. (6). The latter diagram, presented in Fig. 3, is actually a two-loop diagram. A rigorous calculation would require that it be performed in two loops. However, since the coloured sparticles are expected to be much heavier than the Higgs-bosons, we can integrate them out at their mass scale and assume with a good accuracy that the \( H^C \) contribution is given by the one-loop diagram in Fig. 3 with the radiative coupling Eq. (6).

In that case, the latter contribution to \( C_7 \), which is not suppressed by \( \tan^2 \beta \) if the bottom Yukawa couplings are hard, becomes suppressed by \( 1/\tan^2 \beta \) implying that the full \( H^+ \) contribution to \( C_7 \) is suppressed by the same factor. Explicitly,

\[
C_7^{H^+}(m_W) = \frac{1}{2} \frac{x_{tH}}{\tan^2 \beta} \left[ f_1(x_{tH}) + f_2(x_{tH}) 
+ r_2 \left( \frac{1}{2} f_3(x_{tH}) + f_4(x_{tH}) \right) \right],
\]

where \( x_{tH} = M_{H^+}^2/m_t^2 \) and the Inami–Lim type functions \( f_i(x_{tH}) \) can be found in Ref. [3]. This equation implies an important result: the mass bounds on the charged Higgs-boson coming from the measurement of \( b \to s \gamma \) can be relaxed and \( H^+ \) can be light.

The chargino and neutralino contributions to \( C_7 \) follow the same two chiral patterns discussed above. In this case the chirality flip may occur in the internal chargino/neutralino line implying the large enhancement factors \( m_{\tilde{g}}/m_b \). These enhanced terms, induced by the higgsinos, dominate the chargino/neutralino contributions. However, as we argued before, the relevant higgsino couplings are suppressed by \( O(\alpha/\alpha_s) \) compared to the case of hard bottom Yukawa couplings. The exact calculation of \( C_7^{\tilde{X}^+, \tilde{X}^0} \) involves two loops and is beyond the aim of this letter. In our numerical examples below, we take the known MSSM expressions for \( C_7^{\tilde{X}^+, \tilde{X}^0} \) from Ref. [3] and suppress the dominant chirality flipping terms by \( \alpha/\alpha_s \).

Because the \( b \)-quark mass is generated radiatively by the gluino loop, the gluino-mediated contribution to \( b \to s \gamma \) is expected to be sizable. However, in the absence of new large flavour violation beyond the Cabibbo–Kobayashi–Maskawa (CKM) matrix, the gluino contribution is always subdominant compared with that of the SM or the charged Higgs-boson and chargino ones. Nevertheless, in our scenario, the gluino contribution may become the largest SUSY contribution to \( b \to s \gamma \).

To conclude this section, in the scenario of radiatively induced \((d, s, b) \) quark masses in the MSSM, the \( b \to s \gamma \) rate does not impose serious constraints on the charged Higgs-boson, chargino, and neutralino masses. This allows the possibility of their production at colliders but with drastically modified couplings, implying new phenomenology at experiments.

### 3. Unification framework

Embedding the MSSM with the radiative \((d, s, b) \) quark masses into a unification framework is a well-motivated and appealing possibility. This can naturally happen in SUSY SU(5) GUT because the up and down Yukawa couplings are not unified in this model. This automatically solves all the constraints on the model parameters (see, e.g., Ref. [10]) coming from the prediction of \( b-\tau \) Yukawa unification; they are simply vanishing. In addition, the constraints from \( b \to s \gamma \) are practically removed, as shown in the previous section.

Nevertheless the mass spectrum in such a version of the MSSM is stringently constrained by the renormalization-group running of the model parameters, the requirement of radiative electroweak symmetry breaking, and most importantly, the requirement of generating a correct mass to the \( b \) quark via Eq. (4). We start the running of gauge and top Yukawa cou-
plings at $m_t$ using the two-loop MSSM renormalization group equations [19]. The bottom and tau Yukawa couplings are taken to be vanishing. We identify the unification scale $M_{\text{GUT}} = 2 \times 10^{16}$ GeV by the meeting of $g_1$ and $g_2$. At that scale we generate randomly the free parameters of the model: the common gaugino mass $M_{1/2}$, common squark mass $m_0$, common Higgs mass $M_{H_0}$, common $A$ parameter $A_0$, common $A'$ parameter $A'_0$, $\tan \beta$, and $\text{sign}(\mu)$ in the following ranges:

$$100 \text{ GeV} < M_{1/2} < 1000 \text{ GeV},$$
$$100 \text{ GeV} < m_0 < 1000 \text{ GeV},$$
$$100 \text{ GeV} < M_{H_0} < 1000 \text{ GeV},$$
$$-2000 \text{ GeV} < A_0 < 1000 \text{ GeV},$$
$$-2000 \text{ GeV} < A'_0 < 1000 \text{ GeV},$$
$$3 < \tan \beta < 60. \tag{14}$$

Note that $\tan \beta$ is now a free parameter and is not constrained by $b-\tau$ Yukawa unification. With these initial values we run the model parameters to the weak scale, assume radiative symmetry-breaking conditions and calculate the sparticle and Higgs-boson mass matrices there. The renormalization-group equations for nonholomorphic terms can be found in [16]. We require that the branching ratio of $b \rightarrow s \gamma$ is in the allowed range (1) and that the radiative $b$-quark mass Eq. (4) is in the range $2.8 < m_b(m_Z) < 3.2$. The scatter plots are almost independent of the sign of the $\mu$ parameter, our results are presented for $\text{sign}(\mu) = -$.

In Fig. 4 we present the scatter plots of the allowed values of the charged Higgs-boson mass $M_{H^+}$ against the lightest chargino mass $M_{\tilde{\chi}^-}$, and against the lightest bottom squark mass $m_{b_1}$. Notice that in this scenario $H^+$ is bounded to be rather heavy, $M_{H^+} \gtrsim 400$ GeV. This comes from the requirement of radiatively induced electroweak symmetry breaking. Because the $b$-quark Yukawa coupling is vanishing, the difference between the Higgs-boson mass parameters $M_{H_1}$ and $M_{H_2}$ is maximized. As their difference determines $m_A$, the charged Higgs-boson is naturally quite heavy. However, the charginos can be light and be discovered at future colliders.

It is interesting to see the SUSY contribution to $b \rightarrow s \gamma$ in this scenario. In Fig. 5 we plot the total value of $|C_7^{\text{eff}}|$ and the gluino contribution $C_7^g$ (see Eq. (11) for explanation) against the charged Higgs-boson mass $M_{H^+}$. The deviation from the LO SM value $C_7^{\text{SM}} = -0.29$ is small and, as follows from the figures, dominated by the gluino contribution. Thus in this scenario, the MSSM mass spectrum is not constrained by $b \rightarrow s \gamma$. 

![Fig. 4. Scatter plots of the allowed values of the charged Higgs-boson mass $M_{H^+}$ against the lightest chargino mass $M_{\tilde{\chi}^-}$, and against the lightest bottom squark mass $m_{b_1}$.](image)}
Fig. 5. Scatter plots of the total $|C^7_{Ceff}|$ and the dominant gluino contribution $C^7_Q$ against the charged Higgs-boson mass $M_{H^+}$.

4. Conclusions

We have studied the decay $b \rightarrow s \gamma$ in the MSSM in the case the $(d, s, b)$ quark masses are generated radiatively. The soft radiatively generated $b_R$ couplings to the charged Higgs-boson and higgsino are suppressed by $1/\tan^2 \beta$ and $O(\alpha/\alpha_s)$, respectively. The dominant contributions of these particles to $b \rightarrow s \gamma$ are suppressed by the same factors allowing the existence of light $H^+$ and $\tilde{\chi}^+$. Their production and decay processes at future colliders are changed drastically.

If this scenario is realized in the framework of GUTs, then the constraints from $b \rightarrow \tau \nu$ Yukawa unification as well as from $b \rightarrow s \gamma$ are removed. Nevertheless, the lightest sparticles in that case are binos and winos.

Acknowledgement

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-94ER40837.

References


On chiral symmetry breaking in a constant magnetic field in higher dimension

E.V. Gorbar

Instituto de Física Teórica, 01405-900 São Paulo, Brazil

Received 2 June 2000; received in revised form 17 August 2000; accepted 31 August 2000

Abstract

Chiral symmetry breaking in the Nambu–Jona-Lasinio model in a constant magnetic field is studied in spacetimes of dimension \( D > 4 \). It is shown that a constant magnetic field can be characterized by \( \frac{D-1}{2} \) parameters. For the maximal number of nonzero field parameters, we show that there is an effective reduction of the spacetime dimension for fermions in the infrared region \( D \to 1 + 1 \) for even-dimensional spacetimes and \( D \to 0 + 1 \) for odd-dimensional spacetimes. Explicit solutions of the gap equation confirm our conclusions. © 2000 Published by Elsevier Science B.V.

PACS: 11.10.Kk; 11.30.Qc; 11.30.Rd

Keywords: Chiral symmetry; Magnetic field; Higher dimension

1. Introduction

It was discovered recently [1,2] that a constant magnetic field in \( 3 + 1 \) and \( 2 + 1 \) dimensions is a strong catalyst of dynamical chiral symmetry breaking leading to the generation of a fermion dynamical mass even at the weakest attractive interaction between fermions. The essence of this effect is the dimensional reduction of spacetime for fermions in the infrared region, which is \( 3 + 1 \to 1 + 1 \) for \( D = 3 + 1 \) and \( 2 + 1 \to 0 + 1 \) for \( D = 2 + 1 \). The dimensional reduction can be understood as follows. The motion of charged fermions along the direction of magnetic field is free, therefore, the spectrum is continuous. On the other hand, the motion in directions perpendicular to the magnetic field is restricted and the spectrum is discrete (fermions fill the Landau levels). Thus, the dynamics of fermions in a constant magnetic field effectively corresponds to the dynamics of fermions in \( (1 + 1) \)- and \( (0 + 1) \)-dimensional spacetimes in the cases of \( (3 + 1) \)- and \( (2 + 1) \)-dimensional spacetimes, respectively. In this paper we consider chiral symmetry breaking in flat spacetimes of higher dimension \( D > 4 \) with trivial topology (if topology of spacetime is not trivial, then there can be an additional reduction of the spacetime dimension [3,4]). To study this problem, we are motivated in addition to purely academic interest also by recent activity in studying models with extra dimensions [5,6] and the availability of string solutions with constant magnetic field (see, e.g., [7]).

In Section 2 we specify the Nambu–Jona-Lasinio (NJL) model [8] in \( D > 4 \) and discuss the number of parameters which characterize a constant magnetic field in \( D > 4 \). We show in Section 3 that, for the
maximal number of field parameters, the effective reduction of the spacetime dimension for fermions in the infrared region is $D \to 1 + 1$ for even-dimensional spacetimes and $D \to 0 + 1$ for odd-dimensional spacetimes. We find the corresponding solutions of the gap equation in the NJL model. Our conclusions are given in Section 4.

2. The NJL model in a constant magnetic field

To study dynamical chiral symmetry breaking in a constant magnetic field, we first need to classify constant magnetic fields in $D > 4$, i.e., to define the number of independent parameters which specify a constant magnetic field [9]. Mathematically, the problem is the following: a constant electromagnetic field is completely characterized by the field strength $F_{\mu\nu}$. Elements $F_{0i}$ and $F_{i0}$ characterize the electric field. Elements $F_{ij}$, where $i$ and $j$ take values $1, \ldots, n$ ($D = n + 1$), define a constant magnetic field. By using orthogonal rotations, one can set some elements of $F_{ij}$ to zero. It is a well-known fact of linear algebra that the number of independent parameters, which define an arbitrary $F_{ij}$ up to orthogonal rotations, is $[n/2]$.

Let us present a simple inductive proof of this fact. Since $F_{ij}$ is antisymmetric, its diagonal elements are zero. Obviously, $F_{ij}$ is characterized in general by $n(n - 1)/2$ parameters, which we choose to be, e.g., the elements above the diagonal. On the other hand, the orthogonal group in $n$-dimensional space has also $n(n - 1)/2$ independent parameters because there are $n(n - 1)/2$ independent rotations in $n$-dimensional space. Does it mean that we can set to zero all elements of $F_{ij}$ by using appropriate orthogonal rotations? Of course, not. Explicit calculations show that an orthogonal rotation in the plane $mn$ leaves unchanged the $F_{mn}$ element. Another fact is that if we perform a rotation in the plane $kl$, where $kl$ takes value on the antidiagonal, then it leaves also all other antidiagonal elements unchanged. It can be shown inductively for any $n \geq 3$ that one can set all elements of $F_{ij}$ to zero (except the elements on the antidiagonal) by using orthogonal rotations in planes $pq$, where $pq$ take all values except those on the antidiagonal. Since the remaining rotations in planes $kl$, where $kl$ takes values on the antidiagonal, do not change antidiagonal elements, the number of independent elements of $F_{ij}$ is exactly the number of elements on the antidiagonal, which is obviously $[n/2]$.

We can assume without loss of generality that the magnetic part of the field strength $F_{\mu\nu}$ in a convenient reference frame is given by

$$F_{ij} = \sum_{k=1}^{[n/2]} H_k \left( \delta_{ik} \delta_{jn}^{n+1-k} - \delta_{jk} \delta_{in}^{n+1-k} \right)$$

and the corresponding vector potential is

$$A_i = -H_i x_{n+1-i}.$$  

Let us now consider dynamical chiral symmetry breaking in the NJL model in a constant magnetic field in $D > 4$. We first discuss what we mean by chiral symmetry in spacetimes of arbitrary dimension. As well known, the notion of chiral symmetry is connected with properties of representations of the Clifford algebra (for a very clear discussion see, e.g., [10]). The Clifford algebra for spacetimes of even dimension has only one complex irreducible representation in the $2^{[D/2]}$-dimensional spinor space. These spinors are reducible with respect to the even subalgebra (generated by products of an even number of Dirac matrices) and split in a pair of $2^{[D/2]-1}$-component irreducible Weyl spinors ($\gamma_D = \gamma_0 \cdots \gamma_{D-1}$ is an analog of the $\gamma_5$ matrix in $D$-dimensional spacetime and $(1 \pm \gamma_D)/2$ are the corresponding chiral projectors). In odd-dimensional spacetimes, there are two different representations of the Clifford algebra (they differ by the sign of the $\gamma$-matrices) and chiral symmetry is not defined because $\gamma_D$ is proportional to the unity. In order to define an analog of chiral symmetry in odd-dimensional spacetimes, it is the usual practice to assume that fermion fields are in a reducible representation of the Clifford algebra so that we can define an analog of chiral symmetry (for an explicit example in $(2 + 1)$-dimensional spacetime see, e.g., [11]). In what follows we understand chiral symmetry in odd-dimensional spacetimes in this sense.

For our aims it is enough to consider the following generalization of the NJL model with $U_L(1) \times U_R(1)$ chiral symmetry to $D > 4$:

$$\mathcal{L} = \bar{\psi} i \gamma^\mu D_\mu \psi + \frac{G}{2} (\bar{\psi} \psi)^2 + (\bar{\psi} i D_D \psi)^2,$$

where $D_\mu = \partial_\mu + ie A_\mu$ is the covariant derivative and fermion fields carry an additional ‘flavor’ index $i = 1, \ldots, N$. 


3. Dynamical chiral symmetry breaking

By introducing auxiliary fields, we can rewrite Lagrangian (3) in the following way:

\[
\mathcal{L} = \bar{\psi} i \gamma^\mu D_\mu \psi - \bar{\psi} (\sigma + i \gamma_D \pi) \psi = -\frac{1}{2G} (\sigma^2 + \pi^2).
\]

Indeed, the Euler–Lagrange equations for the auxiliary fields \(\sigma\) and \(\pi\) are

\[
\sigma = -G (\bar{\psi} \psi), \quad \pi = -G (\bar{\psi} i \gamma_D \psi),
\]

and Lagrangian (4) gives Lagrangian (3) if we use the equation of motion (5).

By using the method of proper time, we have

\[
\text{Tr} \ln (i \hat{D} - (\sigma + i \gamma_D \pi)) = \text{Det} (-i \hat{D} - \sigma),
\]

we find that

\[
\text{Tr} \ln (i \hat{D} - \sigma) = -\frac{1}{2} \text{Tr} \ln (\hat{D}^2 + \sigma^2).
\]

By using the method of proper time, we have

\[
\frac{i}{2} \int d^D x \int_0^\infty ds \frac{ds}{s} \text{tr} \langle x | e^{-is\hat{D}^2 + \sigma^2} | \chi \rangle,
\]

where

\[
\hat{D}^2 = D_\mu D^\mu - \frac{i}{2} \gamma^\mu \gamma^\nu F_{\mu\nu}.
\]

For \(A_\mu\) given by Eq. (2), it is obvious that the problem of calculation of the matrix element \(\langle x | e^{-i\theta (\hat{D}^2 + \sigma^2)} | x \rangle\) is reduced to the calculation of the corresponding matrix element for every \(H_k\), i.e., for \(x_k\) and \(x_{n+1-k}\) components. By using [12], we obtain the following effective potential:

\[
V(\rho) = \frac{\rho^2}{2G} + \frac{2 \delta / 16}{2 (4\pi)^D/2} \int_0^\infty \frac{ds}{s^{D/2 - (D-1)/2}} e^{-s\rho^2}
\]

\[
\times \prod_{k=1}^{[\frac{(D-1)/2}]} e^{-k H_k\coth(\epsilon H_k s)},
\]

where \(\Lambda\) is a ultraviolet cutoff. The gap equation \(dV/d\rho = 0\) takes the form

\[
\frac{1}{G} = \frac{2 \delta / 16}{(4\pi)^D/2} \int_0^\infty \frac{ds}{s^{D/2 - (D-1)/2}} e^{-s\rho^2}
\]

\[
\times \prod_{k=1}^{[\frac{(D-1)/2}]} e^{-k H_k\coth(\epsilon H_k s)}.
\]

If the magnetic field is absent, then the right-hand side of the gap equation is

\[
\frac{2 \delta / 16}{(4\pi)^D/2} \int_0^\infty \frac{ds}{s^{D/2}} e^{-s\rho^2},
\]

where the integrand is exactly the heat kernel of the Dirac operator squared in \(D\)-dimensional spacetime. Since coth \(x\) \(\to 1\) as \(x\) \(\to \infty\), it follows from Eq. (12) that every independent parameter of magnetic field \(H_k\), which is not equal to zero, effectively reduces the spacetime dimension by 2 units in the infrared region (for \(s \to \infty\) only the lowest part of the spectrum of the Dirac operator squared gives contribution). Consequently, for the maximal number of field parameters \(\frac{[D-1]}{2}\), we obtain that the effective reduction of the spacetime dimension in the infrared region for fermions is \(D \to 1 + 1\) for even-dimensional spacetimes and \(D \to 0 + 1\) for odd-dimensional spacetimes. Thus, we expect that for the maximal number of field parameters the critical coupling constant is zero for even-dimensional spacetimes and the gap analytically depends on coupling constant in odd-dimensional spacetimes, which are the characteristic features of solu-
tions of the gap equation in two- and one-dimensional spacetimes, respectively (see [1,2]).

To analyze the gap equation, we assume for simplicity that all $H_k$ are equal, i.e., $H_1 = H_2 = \cdots = H_{[(D-1)/2]} = H$. Since we are mainly interested only in qualitative results, we split the interval of integration in two parts from $1/\Lambda^2$ to $1/eH$ and from $1/eH$ to $\infty$ and approximate $\coth x$ by $1/x$ on the first interval and by 1 on the second (we assume also that $\rho^2 \ll eH$ and approximate $e^{-\rho^2}$ by 1 on the first interval). One can check that this approximation for $D = 3$ and $D = 4$ gives the same result for the gap (the same dependence on $eH$, $\Lambda^2$, and $G$) as the exact result [1,2] up to a numerical constant of order $O(1)$. For even $D$, we obtain the following gap equation:

$$
\frac{(2\pi)^{D/2}}{G N A^{D-2}} = \frac{1 - (eH/\Lambda^2)^{D/2-1}}{D/2 - 1} + (eH/\Lambda^2)^{D/2-1} \int_0^\infty \frac{ds}{s} e^{-s}. \tag{14}
$$

By using [13]

$$
\Gamma(\alpha, x) = \int_0^\infty e^{-\rho^2} \eta^{\alpha-1}
$$

and an expansion of the incomplete Gamma-function for small $x$

$$
\Gamma(0, x) = -C - \ln x - \sum_{k=1}^\infty \frac{(-x)^k}{k \cdot k!}, \tag{15}
$$

where $C$ is the Euler constant, we obtain the solution

$$
\rho^2 = eH \exp\left(\frac{(2\pi)^{D/2}(1 - g)}{G N (eH)^{D/2-1}}\right), \tag{16}
$$

where

$$
g = \frac{G N A^{D-2}}{(D/2 - 1)(2\pi)^{D/2}}.
$$

For odd $D$, the gap equation is

$$
\frac{(2\pi)^{D/2}}{G N \sqrt{2}} = \frac{\Lambda^{D-2}(1 - (eH/\Lambda^2)^{D/2-1})}{D/2 - 1} + \frac{(eH)^{(D-1)/2}}{\rho} \int_0^\infty \frac{ds}{s^{D/2}} e^{-s}. \tag{17}
$$

By using [13]

$$
\Gamma(\alpha, x) = \Gamma(\alpha) - \sum_{k=0}^{\infty} \frac{(-1)^k x^{\alpha+k}}{k!(\alpha+k)}
$$

we obtain the solution

$$
\rho = \frac{(eH)^{D-1}/2 GN}{(2\pi)^{D/2}}. \tag{19}
$$

(Note that $\rho^2 \ll eH$ for sufficiently small $G$ and the approximation of $e^{-\rho^2}$ by 1 on the interval $[1/\Lambda^2, 1/eH]$ is consistent.) Thus, for the maximal number of field parameters, the critical coupling constant is zero in even-dimensional spacetimes and the gap has an essential singularity at zero value of coupling constant. In odd-dimensional spacetimes the gap depends analytically on coupling constant. These results confirm our conclusions about the effective reduction of the spacetime dimension for fermions in the infrared region because our solutions are characteristic for solutions of the gap equation in (1 + 1)- and (0 + 1)-dimensional spacetimes, respectively, [1,2].

Finally, we would like to give a physical explanation why there is a maximal effective dimensional reduction for the maximal number of nonzero field parameters. The effective dimensional reduction in the case of $D = 3$ and $D = 4$ can be easily understood because of restricted motion of charged particles in directions perpendicular to the magnetic field as follows from solutions of the Dirac equation in an external constant magnetic field in $D = 3$ and 4. What are solutions of the Dirac equation for $D > 4$? These solutions were discussed in [9]. The $D$-dimensional Dirac equation in an external electromagnetic field has the form

$$
(P_{\mu} \gamma^\mu - m) \psi(x) = 0, \tag{20}
$$

where $P_{\mu} = i \partial_{\mu} - e A_{\mu}(x)$. As usual, it is convenient to present $\psi$ in the form

$$
\psi(x) = (P_{\mu} \gamma^\mu + m) \phi(x). \tag{21}
$$

Then the $\phi$ obeys the following "squared Dirac equation" in $D$ dimensions:

$$
\left( P^2 - m^2 - \frac{e}{2} a^{\mu\nu} F_{\mu\nu} \right) \phi(x) = 0,
$$

$$
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad a^{\mu\nu} = \frac{i}{2} \left[ \gamma^\mu, \gamma^\nu \right]. \tag{22}
$$
Since $F_{\mu\nu}$ is constant for a constant magnetic field, one can always find solutions of Eq. (22) as eigenfunctions of $P^2$ and the $x$-independent term $\sigma^{\mu\nu} F_{\mu\nu}$, which describes the interaction of intrinsic magnetic moment of a fermion with external magnetic field. For the maximal number of nonzero field parameters, $\lambda_{\mu\nu}$ is given by Eq. (2) and it is obvious that $x$ variables separate into a set of 2-dimensional problems in planes $i(D - i) (i = 1, 2, \ldots, \lceil\frac{D-1}{2}\rceil$ for the maximal number of nonzero field parameters) and eigenfunctions for a 2-dimensional problem of motion in the plane $i(D - i)$ are exactly the same as in the case of motion in the plane perpendicular to the magnetic field for $D = 3$ and 4. Consequently, the motion of fermions for $D > 4$ is restricted in the plane $i(D - i)$ if the corresponding field parameter $H_i$ is not equal to zero. This immediately implies that every nonzero field parameter yields the effective dimensional reduction for fermions in the infrared region by 2 units.

Let us consider briefly the case where only $m < \lceil\frac{D-1}{2}\rceil$ field parameters are not equal to zero. By taking the limit $H_i \to 0$ in Eq. (12) for zero field parameters, we obtain the following gap equation for a nonmaximal number $m < \lceil\frac{D-1}{2}\rceil$ of nonzero field parameters:

$$
\frac{1}{G} = \frac{2^{\lceil\frac{D-1}{2}\rceil}N}{(4\pi)^D/2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^{D/2-m}} e^{-s\rho^2} \prod_{k=1}^{m} e^{H_k \coth(eH_k s)}.
$$

Since $m < \lceil\frac{D-1}{2}\rceil$, we have $\frac{D}{2} - m > 1$ and the integral for $\rho = 0$ is convergent on the upper limit of integration. Therefore, the critical coupling constant is not equal to zero for a nonmaximal number of nonzero field parameters. To find the critical coupling constant in the case of a nonmaximal number of field parameters, we again for simplicity assume that all $H_k$ are equal, i.e., $H_1 = H_2 = \cdots = H_m = H$. By approximating $\coth x$ by 1 in the interval $[1/eH, \infty)$ and $1/x + x/3$ in the interval $[1/\Lambda^2, 1/eH]$, we find that the critical coupling constant is equal to (due to the approximations made, the coefficients near terms $(eH/\Lambda^2)^2$, $(eH/\Lambda^2)^2$, and $(eH/\Lambda^2)^2\ln(\Lambda^2/eH)$ are actually defined up to a constant of order 1)

$$
g_{\text{cr}} = \frac{1}{1 + 2(eH/\Lambda^2)^2} \quad \text{for } D = 5,
$$

Thus, we obtain the following gap equation for the critical coupling constant:

$$
g_{\text{cr}} = \frac{1}{1 + \left(\frac{eH}{\Lambda^2}\right)^2 + 2\left(\frac{eH}{\Lambda^2}\right)^2 \ln \frac{1}{eH^2}} \quad \text{for } D = 6,
$$

and

$$
g_{\text{cr}} = \frac{1}{1 + \frac{m(\frac{D-1}{2})}{3(\frac{D-1}{2}-3)} \left(\frac{eH}{\Lambda^2}\right)^2} \quad \text{for } D > 6.
$$

(We would like to note that Eq. (26) is valid only in the case of a nonmaximal number of nonzero field parameters.) For $D = 5$ and 6, a constant magnetic field is characterized by only two independent field parameters, therefore, $m$ can be equal only to 1 in this case. As follows from Eq. (26) for $D > 6$ the more the number of field parameters $m$, the less the critical coupling constant in agreement with expectations and in all cases the more the magnetic field, the less the critical coupling constant. Note that Eqs. (24)–(26) imply that the critical coupling constant is very close to 1 in the realistic case $eH \ll \Lambda^2$.

4. Conclusions

We considered chiral symmetry breaking in the NJL model in a constant magnetic field in $D > 4$. We showed that for the maximal number of field parameters the effective reduction of the spacetime dimension for fermions in the infrared region is $D \to 1 + 1$ for even-dimensional spacetimes and $D \to 0 + 1$ for odd-dimensional spacetimes. We studied the gap equation of the NJL model and found that for the maximal number of field parameters the gap is analytic in coupling constant for odd-dimensional spacetimes and the gap has an essential singularity at zero value of coupling constant for even-dimensional spacetimes, which are characteristic features of solutions of the gap equation in $0 + 1$ and $1 + 1$ dimension, respectively, that confirms the dimensional reduction. We would like to note also that our results can be relevant in the context of certain string solutions [7] in the low-energy domain, where we can have a constant magnetic field in spacetimes with dimension $D > 4$.

The author thanks I.L. Buchbinder, S.P. Gavrilov, D.M. Gitman, A.A. Natale, and F. Toppan for useful discussions and valuable remarks. I am grateful to V.P. Gusynin for reading the manuscript and useful suggestions. This work was supported in part by...
FAPESP grant No. 98/06452-9, by Foundation of Fundamental Researches of Ministry of Education and Sciences of the Ukraine under grant No. 2.51/00003, and the Grant-in-Aid of Japan Society for the Promotion of Science No. 11695030.

References


No primordial magnetic field from domain walls

M.B. Voloshin

Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455, USA

Institute of Theoretical and Experimental Physics, Moscow, 117259 Russia

Received 26 July 2000; accepted 11 August 2000

Abstract

It is pointed out that, contrary to some claims in the literature, the domain walls cannot be a source of a correlated at large scales primordial magnetic field, even if the fermionic modes bound on the wall had ferromagnetic properties. In a particular model with massive $(2 + 1)$-dimensional fermions bound to a domain wall, previously claimed to exhibit a ferromagnetic behavior, it is explicitly shown that the fermionic system in fact has properties of a normal diamagnetic with the susceptibility vanishing at high temperature.

The existence of magnetic field correlated at a galactic scale [1] is believed to require a strong primordial field correlated at cosmological distances at some stage in the early universe. This phenomenon would find a natural explanation, if the primordial field was created by extended objects, having a cosmological size. One class of extended objects that might have existed in the early universe is provided by domain walls, assuming that a model containing these walls successfully avoids general constraints [2] on undesirable cosmological consequences of such extended defects. There has been several claims in the literature [3–6] that the modes of fermion field bound to a domain wall produce, in certain models, a magnetic field $B$, which could provide a much needed explanation of a primordial magnetic field in the early universe. The purpose of this note is to point out that, as enticing as this explanation could be, it cannot be physically correct. Namely, it is almost trivial to show that, irrespective of the details of the dynamics, a correlated over the entire area of a flat domain wall magnetic field should be greatly suppressed by an inverse power of a cosmological size, even if the wall exhibited a ferromagnetic behavior. Furthermore, in the models that are sufficiently well formulated [3,4], the walls are in fact diamagnetic, contrary to the previous claims. In this letter the model of Refs. [4,5] with massive fermionic modes with broken parity is considered, and it is shown that the fermion system on the wall is a normal diamagnetic, while the previous claim of ferromagnetism is based on an incorrect formula [8] for the $B$ dependent free energy of the fermionic vacuum.

In order to prove the statement about the suppression of the magnetic field that a domain wall might spontaneously create, let us introduce a large normalization box with the sides of length $L$, and consider a flat domain wall, spanning the box along the $x$, $y$ plane, thus $z$ being the coordinate axis perpendicular to the wall. From symmetry, a correlated magnetic field can have only the $z$ component $B_z$, which does

1. The claim of [3] was explicitly shown [7] to be based on an incorrect calculation of the energy of the gas of massless $(2 + 1)$-dimensional fermions in magnetic field.
not depend on the coordinates \( x \) and \( y \) parallel to the wall. Then the Maxwell’s equation \( \text{div} \mathbf{B} = 0 \) dictates that \( B_z \) also cannot depend on the coordinate \( z \), i.e., \( B_z \) has to be constant: \( B_z = B \). The energy of such constant magnetic field is proportional to the volume \( L^3 \) of the box, while, the energy, associated with dynamics on the wall is proportional to the area \( L^2 \) of the wall, so that the total energy of the system is written as

\[
E(B) = L^2 \frac{B^2}{2} + L^2 f(B) \tag{1}
\]

with \( f(B) \) being the surface energy density in the presence of the magnetic field, determined by specific dynamics on the wall. Clearly, for any finite function \( f(B) \) the value of \( B \) providing the minimum to the energy \( (1) \) goes to zero when the size \( L \) of the normalization box is taken to infinity. Thus one concludes that the magnetic field of an infinite flat domain wall has to vanish, independently of the specifics of the model.

The analyses of the papers [3–6] find a ferromagnetic behavior for the function \( f(B) \) at small field: \( f(B) - f(0) = -aB \), with the spontaneous magnetization \( a \) being determined by the ‘microscopic’ parameters of the wall. According to Eq. (1) the generated field then is \( B = a/L \). This consequence of Eq. (1) is acknowledged only in the final version of Ref. [3], and is ignored in Refs. [4–6]. In the latter papers the extent of the field away from the wall is assumed to be of the order of the thickness of the wall in direct contradiction with the Maxwell’s equation. Physically, a finite size \( L \) can arise either as the Hubble size [3], or as a domain size in a multi-domain structure. In either case the magnitude of the correlated field is greatly suppressed by \( L^{-1} \) and is unlikely to be sufficient for explaining the required primordial field. As an example, one can consider the estimates of the field in the mechanism of Ref. [6], where the primordial field is associated with the axion domain walls at the QCD phase transition, i.e., at the temperature \( T_{\text{QCD}} \sim A_{\text{QCD}} \sim 0.2 \text{ GeV} \). The size of the correlations in the picture presented there is the Hubble scale \( 1(T_{\text{QCD}}) \approx 30 \text{ km} \). Thus even if one adopts the estimate of Ref. [6] for the spontaneous magnetization: \( a \sim eA_{\text{QCD}}^2 \), the estimated magnitude of the generated field should be \( B \sim eA_{\text{QCD}}/l(T_{\text{QCD}}) \sim 10^{-2} \text{ G} \) (instead of the claimed \( 10^{17} \text{ G} \)). Furthermore, it will be shown below that the magnetization of the axion wall is proportional to the total baryon number density accumulated on the wall, so that the estimate [6] of the magnetization is rather on the maximalist side.

The existence of even a suppressed by \( L^{-1} \) magnetic field is contingent on ferromagnetism of the fermion modes on the wall. Such behavior was claimed in Refs. [4,5,8] within a model of massive \((2 + 1)\)-dimensional fermions. Moreover, it has been argued [5] that the ferromagnetic term \(-B\) survives in the free energy at all temperatures, in contrast with the known behavior in any other systems. In what follows a calculation of the free energy of a massive \((2 + 1)\)-dimensional fermion field is presented, in close analogy with a similar calculation [7] for a massless case, and it is shown that, contrary to previous claims, the system in fact exhibits a normal diamagnetism with the diamagnetic susceptibility vanishing at high temperature. The previous erroneous findings of the ferromagnetic behavior were due to an inaccurate manipulation with a divergent sum in Ref. [8].

The model of massive \((2 + 1)\)-dimensional fermions is relevant to the situation where the fermions have a non-zero mode bound on the wall with eigenvalue \( m \), and there is no mode corresponding to the eigenvalue \( -m \). (This un-pairing of the modes arises in a situation where the parity is broken.) The two-dimensional motion of charged fermions in this mode is described by a \((2 + 1)\)-dimensional Dirac equation

\[
(i\gamma^\mu(\partial_\mu - ieA_\mu) - m)\psi = 0, \tag{2}
\]

where \( e \) is the electric charge, \( A_\mu = (A_0, A_1, A_2) \) is the vector potential of an (external) electromagnetic field, and \( \gamma_\mu \) is the set of \( 2 \times 2 \) gamma matrices, which can be chosen, e.g., in terms of the Pauli matrices as \( \gamma^0 = \sigma_3, \gamma^1 = i\sigma_1, \) and \( \gamma^2 = i\sigma_2 \). It is known (see, e.g., in Ref. [8]) that in an external magnetic field \( B \), perpendicular to the wall, there is an asymmetry between positive and negative Landau energy levels, depending on the relative sign of \( eB \) and \( m \). Namely, assuming for definiteness that \( m \) is negative and \( eB \) is positive, the spectrum of positive energy levels is given by \( E_{n+} = \sqrt{2eBn + m^2} \) with \( n = 1, 2, \ldots \), while that...
of the negative levels is \( E_{n-} = -\sqrt{2eBn + m^2} \) with \( n = 0, 1, 2, \ldots \). In other words, the level with \( n = 0 \) is absent from the positive energy part of the spectrum, and is present in the negative part.

The free energy per unit area for the fermion system at a temperature \( T = 1/\beta \) is expressed through the spectrum in the standard way:

\[
F = F_- + F_+ + E_{\text{vac}},
\]

where

\[
F_+ = -\beta^{-1} \frac{eB}{2\pi} \sum_{n=1}^{\infty} \ln \left( 1 + e^{-\beta \sqrt{2eBn + m^2}} \right)
\]

and

\[
F_- = -\beta^{-1} \frac{eB}{2\pi} \sum_{n=0}^{\infty} \ln \left( 1 + e^{-\beta \sqrt{2eBn + m^2}} \right)
\]

are the thermal parts of the free energy associated with the real gas of fermions and antifermions at the Landau levels. The term \( E_{\text{vac}} \) in Eq. (3) is the energy of the vacuum state, i.e., with all negative energy levels filled and those with positive energy being vacant,

\[
E_{\text{vac}} = \frac{eB}{2\pi} \sum_{n=0}^{\infty} \sqrt{2eBn + m^2}.
\]

The sums for the temperature dependent parts \( F_{\pm} \) are finite, and do not cause controversy. The most interesting is the sum in Eq. (6), which is divergent and thus should be handled with some care. In order to make the latter sum tractable without ambiguity, it needs to be regularized. A gauge invariant regulator factor should depend on a gauge invariant quantity, which naturally can be chosen as the energy of the levels. The exact form of the regulator is a matter of convenience, and we use here an exponential form of this factor: \( \exp(-\epsilon E_n^2) \), thus making the sum for the regularized vacuum energy read as

\[
E_{\text{vac}}^{(r)}(B) = -\frac{eB}{2\pi} \sum_{n=0}^{\infty} \sqrt{2eBn + m^2} \times \exp(-\epsilon eBn - \epsilon m^2),
\]

where \( \epsilon \) is the regulator parameter. In the physically meaningful quantity \( E_{\text{vac}}(B) - E_{\text{vac}}(0) \), one should take \( \epsilon \to 0 \) in the final result.

The sum in Eq. (7) can be evaluated using Poisson’s method based on the identity:

\[
\sum_{n=0}^{\infty} f(n) = \int_{\delta}^{\infty} f(x) \sum_{n=-\infty}^{\infty} \delta(x - n) \, dx = \sum_{k=-\infty}^{\infty} \int_{\delta}^{\infty} f(x) \exp(2\pi ikx) \, dx,
\]

where \( \delta \) is an arbitrary number, such that \(-1 < \delta < 0\). Notice that the sum over \( n \) in the second expression goes from \(-\infty \) to \( +\infty \). The identity is still valid since the terms with \( n < 0 \) are identically zero. The summand in Eq. (7) is non singular at \( n = 0 \), so that one can in fact set \( \delta = 0 \), and write

\[
E_{\text{vac}}^{(r)} = -\frac{eB}{2\pi} \sum_{k=-\infty}^{\infty} \int_{0}^{\infty} \sqrt{2eBx + m^2} \times \exp(-\epsilon eBx - \epsilon m^2) \, dx.
\]

In this expression the only term in the sum over \( k \) that is singular in the limit \( \epsilon \to 0 \) is the one with \( k = 0 \). This term is given by

\[
-\frac{eB}{2\pi} \int_{0}^{\infty} \sqrt{2eBx + m^2} \exp(-\epsilon eBx - \epsilon m^2) \, dx = -\frac{1}{4\pi} \int_{0}^{\infty} \sqrt{z + m^2} \exp(-\epsilon z - \epsilon m^2) \, dz
\]

and does not depend on \( B \). In fact this term is identically equal to the vacuum energy, regularized in the same way:

\[
E_{\text{vac}}(0) = -\int \sqrt{p^2 + m^2} \times \exp(-\epsilon p^2 - \epsilon m^2) \frac{d^2p}{(2\pi)^2}.
\]

Thus the \( k = 0 \) term in the sum in Eq. (9) totally cancels in the difference \( E_{\text{vac}}(B) - E_{\text{vac}}(0) \), and the difference itself is given by the sum of all terms with \( k \neq 0 \). The latter sum is finite in the limit \( \epsilon \to 0 \). However an infinitesimal parameter \( \epsilon \) should be retained in calculation of the integrals in order to ensure convergence and proper phase definition of the oscillatory integrals. The sum over \( k \neq 0 \) is
analytically tractable in two limiting cases: large field, \( eB \gg m^2 \), and small field, \( eB \ll m^2 \). In the large field limit the mass can be neglected, and the result is given by the massless case \([7,8]:\)

\[
E_{\text{vac}}(B) - E_{\text{vac}}(0) = \frac{\xi(3/2)}{16\pi^2}(2eB)^{3/2},
\]

(12)
corresponding to diamagnetism with a singular diamagnetic susceptibility.

The most interesting here is the case of small field, since this is the limit where the difference \( E_{\text{vac}}(B) - E_{\text{vac}}(0) \) is claimed \([4,5,8]\) to have a linear dependence on \( B \). Using the representation in Eq. (9) we find however quite different result. Namely, using Taylor expansion in \( (2eB/m^2) \) for the square root in Eq. (9), and grouping together terms with \( +k \) and \(-k\), one finds in the limit \( \epsilon \to 0^+ \):

\[
E_{\text{vac}}(B) - E_{\text{vac}}(0) = \frac{eB|m|}{2\pi} \sum_{s=0}^{\infty} \sum_{k=1}^{\infty} \frac{\Gamma(3/2)}{\Gamma(1/2 - s)} \times \left((2\pi k)^{s-1} - (2\pi ik)^{s-1}\right) \left(\frac{2eB}{m^2}\right)^s.
\]

(13)

One can easily see that in the sum over \( s \) only the terms with odd \( s \) are non vanishing, thus leaving only the even powers of \( B \) in the expansion. Denoting in the non-zero terms \( s = 2p + 1 \), one finally finds the asymptotic expansion in powers of the field:

\[
E_{\text{vac}}(B) - E_{\text{vac}}(0) = \frac{e^2B^2}{2\pi^3|m|} \sum_{p=0}^{\infty} \frac{(-1)^p \xi(2p + 2)\Gamma(3/2)}{\Gamma(1/2 - 2p)} \left(\frac{eB}{\pi m^2}\right)^{2p}
\]

\[
= \frac{e^2B^2}{24\pi|m|} + \ldots
\]

(14)
The leading at small \( B \) term in the expansion (14) is positive and quadratic in \( B \), thus corresponding to a normal diamagnetism. It is satisfying to verify that, in compliance with the common physical intuition, the diamagnetism vanishes at high temperature. By “high” one should understand a temperature constrained by the condition \( T \gg m \), since at lower temperatures the gas of real fermions cannot produce substantial effect because of the thermal blocking factor \( e^{-\beta|m|} \).

Notice, however that the temperature has to be still lower than the mass gap for the fermions on the wall, since otherwise the problem would not be reduced to dynamics of a \((2 + 1)\)-dimensional fermion system, but rather one would have to consider the full \((3 + 1)\)-dimensional problem. In the latter situation however the effects of the wall are small, and no significant phenomena are to be expected.

In order to calculate the thermal effects under the condition \( T \gg m \gg eB \) one can apply the Poisson summation formula (8) for the sums \( F_+ \) and \( F_- \) in Eqs. (4) and (5). Writing separately the term with \( k = 0 \) and grouping together the terms with symmetric non-zero values of \( k \) results in the expression

\[
F_+ + F_- = -\frac{eB}{\pi\beta} \int_{\delta_1}^{\infty} \ln \left[1 + \exp \left(-\beta\sqrt{2eBx + m^2}\right)\right] dx
\]

\[
= \frac{eB}{2\pi\beta} \ln \left(1 + e^{-\beta|m|}\right)
\]

\[
+ \sum_{k=1}^{\infty} \int_{\delta_1}^{\infty} \ln \left[1 + e^{-\beta\sqrt{2eBx + m^2}}\right]
\]

\[
\times \left(e^{2\pi ikx} + e^{-2\pi ikx}\right) dx.
\]

(15)

Here the lower limit of integration, \( \delta_1 \), is such that \( 0 < \delta_1 < 1 \), corresponding to summation over the Landau levels from \( n = 1 \) to \( \infty \). (The contribution of the \( n = 0 \) level in the negative energy spectrum is the first term in parenthesis.) The first term on the right-hand side of Eq. (15) arises from the \( k = 0 \) harmonic in the Poisson formula. Taking in this term the limit \( \delta_1 \to +0 \), one sees that this term does not depend on \( B \) and describes the thermal part of the free energy of the free fermion gas at zero field:

\[
F_0(\beta, m) = -2\beta^{-1} \int \ln \left[1 + \exp \left(-\beta\sqrt{p^2 + m^2}\right)\right]
\]

\[
\times \frac{d^2p}{(2\pi)^2}.
\]

(16)
The expression in the parenthesis in Eq. (15) vanishes at $B = 0$, since the first term cancels against the sum due to the identity

$$\sum_{k=1}^{\infty} \int e^{2\pi ikx} + e^{-2\pi ikx} \, dx = -\frac{1}{2}.$$  \hspace{1cm} (17)

The rest of the terms can be found by Taylor expansion in $B$ of the integrand in the parenthesis. For the first non vanishing term one finds

$$F_+ + F_- = F_0(\beta, m) = \frac{eB}{\pi\beta} \left\{ \frac{\beta e^{-|m|}}{(1+e^{-|m|})} \frac{eB}{|m|} \right\} \times \sum_{k=1}^{\infty} \int x(e^{2\pi ikx} + e^{-2\pi ikx}) \, dx + O(B^4)$$

$$= \frac{2e^{-|m|}}{1 + e^{-|m|}} \frac{e^2 B^2}{24\pi |m|} + O(B^4).$$  \hspace{1cm} (18)

Combining this result with Eq. (14), one readily sees that the diamagnetic susceptibility indeed vanishes at $\beta|m| \ll 1$, i.e., at $T \gg |m|$.

The presented calculation of the free energy of the fermionic system leads us to the conclusion that in the considered model the fermionic mode bound to the domain wall gives rise to a normal diamagnetism. As usually, the diamagnetic behavior for a system of charged fermions arises because the diamagnetism of the 'orbital' motion of the electric charges in the magnetic field overcomes the paramagnetism, associated with the normal magnetic moment. In order to avoid the diamagnetism, it was suggested [6] that neutral fermions with an anomalous magnetic or electric dipole moment, bound to a domain wall, are fully polarized and provide a spontaneous magnetization. Specifically, the strongest effect of the model of Ref. [6] arises from a mode for neutrons, bound to an axion domain wall right after the QCD chiral transition. Here we present some remarks on this model.

In terms of a $(2+1)$ dynamics of the bound fermions the Hamiltonian for the anomalous interaction has the form

$$H_{an} = -\kappa \bar{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi,$$  \hspace{1cm} (19)

where $\kappa$ is the anomalous magnetic moment, and $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$. For a magnetic field $B$ in the $z$ direction, $B = F_{12}$, one has $\sigma^{12} = \gamma^0$, so that the Hamiltonian takes the form $H_{an} = -\kappa B \bar{\psi} \gamma^0 \psi$. Thus the spontaneous magnetization $a = -(\partial F/\partial B)_{B=0}$ coincides, up to the factor $\kappa$ with the density of the fermionic charge:

$$a = \kappa \langle \psi^\dagger \psi \rangle,$$  \hspace{1cm} (20)

where the averaging over the appropriate thermal state is implied. For the model of Ref. [6] the net surface density of neutrons $\nu_B = \langle \psi^\dagger \psi \rangle$, occupying the zero mode on the axion wall, is limited by at least two factors. One is the maximal occupation density for the mode set by the requirement that the Fermi energy does not exceed the energy gap $\Delta$ between the modes: $\nu_B \lesssim \Delta^2$. The value of the gap is model dependent, however, one can perhaps take $\Delta \sim \Lambda_{QCD}$ for an estimate. The other limit arises from a consideration of the diffusion of neutrons from the bulk to the wall. Because of low density of the baryon number in the bulk $n_B$ it takes time for the neutrons to be accumulated on the wall. Taking the mean free path for the neutrons at $T_{QCD} \approx \Lambda_{QCD}$ as $\sim 1/\Lambda_{QCD}$ one estimates the density of neutrons accumulated by the wall over the time $t$ as $\nu_B \sim n_B \sqrt{t/\Lambda_{QCD}}$. Under a maximalist assumption, that the available diffusion time $t$ is given by the Hubble time, $t \sim 1/(T_{QCD})$, one finds that at $n_B/T^2 \sim 10^{-10}$ this limit for $\nu_B$ numerically is also close to $\Lambda_{QCD}$.

Summarizing the discussion of this paper, we conclude that the gas of modes of charged Dirac fermions on a domain wall is always diamagnetic with susceptibility vanishing at high temperature. Therefore such systems cannot spontaneously produce any correlated magnetic field. The spontaneous magnetization of the modes for neutral fermions with anomalous magnetic moment is proportional to the surface density of the fermion number on the wall, and the magnetic field generated by a spontaneous magnetization of a wall is suppressed by the inverse of the wall’s dimension. These considerations lead us to the general conclusion that it is at least highly unlikely that domain walls can provide a physically viable source of the primordial magnetic field.

This work is supported in part by DOE under the grant number DE-FG02-94ER40823.
References

Conformal brane world and cosmological constant

Zurab Kakushadze

C.N. Yang Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794, USA

Received 7 August 2000; accepted 2 September 2000

Abstract

We consider a recently proposed setup where a codimension one brane is embedded in the background of a smooth domain wall interpolating between AdS and Minkowski minima. Since the volume of the transverse dimension is infinite, bulk supersymmetry is intact even if brane supersymmetry is completely broken. On the other hand, in this setup unbroken bulk supersymmetry is incompatible with non-zero brane cosmological constant, so the former appears to protect the latter. In this paper we point out that, to have a consistent coupling between matter localized on the brane and bulk gravity, in this setup generically it appears to be necessary that the brane world-volume theory be conformal. Thus, unbroken bulk supersymmetry appears to actually protect not only the cosmological constant but also conformal invariance on the brane. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

In the Brane World scenario the Standard Model gauge and matter fields are assumed to be localized on branes (or an intersection thereof), while gravity lives in a larger dimensional bulk of space–time [1 – 12]. The volume of dimensions transverse to the branes is automatically finite if these dimensions are compact. On the other hand, the volume of the transverse dimensions can be finite even if the latter are non-compact. In particular, this can be achieved by using [13] warped compactifications [14] which localize gravity on the brane. A concrete realization of this idea was given in [15].

Recently it was pointed out in [16,17] that, in theories where extra dimensions transverse to a brane have infinite volume [18 – 22], the cosmological constant on the brane might be under control even if brane supersymmetry is completely broken. The key point here is that even if supersymmetry breaking on the brane does take place, it will not be transmitted to the bulk as the volume of the extra dimensions is infinite [16,17]. Thus, at least in principle, one should be able to control some of the properties of the bulk with the unbroken bulk supersymmetry. One then can wonder whether bulk supersymmetry could also control the brane cosmological constant [16,17].

Controlling the brane cosmological constant with bulk supersymmetry, however, appears to be non-trivial. Thus, in [23] it was pointed out that in the Dvali–Gabadadze–Porrati model [22], where a 3-brane is embedded in the 5-dimensional Minkowski space, unbroken bulk supersymmetry is perfectly compatible with non-vanishing, in particular, positive brane cosmological constant. There is a simple reason for this. The bulk curvature in this

E-mail address: zurab@insti.physics.sunysb.edu (Z. Kakushadze).
The model is constant, in particular, it vanishes. The Minkowski space can be foliated by codimension one surfaces of vanishing or positive constant curvature. Both of these types of foliations are compatible with bulk supersymmetry, that is, with the existence of Killing spinors in the bulk—the latter is a (local) property of the corresponding space–time, and is independent of the foliation. Note that this also applies to another space of constant curvature which admits Killing spinors, namely, the AdS space. In this case we have foliations with vanishing, positive and negative constant curvature, all of which are perfectly consistent with bulk supersymmetry.

It is then natural to consider examples where the bulk is a space with non-constant curvature, which, nonetheless, admits Killing spinors. It is clear that (in the codimension one case) such a space would have half of the supersymmetries compared with the constant curvature cases. One way to parameterize such a space is to consider a bulk theory where gravity is coupled to a scalar field $\phi$ with a non-trivial scalar potential $V(\phi)$ (that is, $V(\phi)$ is not a constant). For an appropriately chosen scalar potential there exist BPS domain wall solutions which preserve half of the original supersymmetries (that is, half of the supersymmetries corresponding to Minkowski or AdS minima of the scalar potential). The corresponding foliation is then necessarily flat. The reason why this is so different from the cases where the bulk curvature is constant is that in the latter cases one only needs to ensure vanishing of the gravitino variation, while in the presence of a non-trivial bulk potential preserving bulk supersymmetry also requires vanishing of the variation of the superpartner of $\phi$.

This is precisely the proposal of [24]. More concretely, let a codimension one brane (which, for simplicity, is taken to be $\delta$-function-like) be embedded in the BPS domain wall background of the aforementioned type. To have an infinite volume extra dimension, the scalar potential is assumed to have one Minkowski minimum and one AdS minimum, and the domain wall interpolates between these minima. Note that such a domain wall (unlike a domain wall interpolating between two Minkowski minima [17,19]) does not have to violate the weak energy condition. In particular, if the domain wall is smooth, which is the case if the brane cosmological constant equals the brane tension, then the weak energy condition is not violated [24]. Now, since the volume of the extra dimension is infinite, supersymmetry breaking on the brane does not result in bulk supersymmetry breaking. However, the latter is not compatible with non-zero brane cosmological constant. Thus, even if supersymmetry breaking on the brane does occur, the brane cosmological constant appears to remain zero.

The purpose of this paper, which is essentially a follow-up of [24], is to point out that the theory living on the brane in the setup of [24] generically appears to be conformal. One way to see this is to consider small fluctuations around the background in the presence of matter localized on the brane. Then the system of equations for the small fluctuations of the scalar field and the metric is essentially overconstrained, and has consistent solutions if and only if the energy-momentum tensor of the matter localized on the brane is traceless and the coupling of the scalar field to the brane matter is vanishing. This appears to be due to the fact that a tensionless brane by itself does not explicitly break diffeomorphism invariance, so that the latter is broken spontaneously by the domain wall solution. Spontaneous breaking of translational invariance results in the gravitational Higgs mechanism discussed in [25], in the process of which the trace part $h_{\mu\nu}^T$ of the graviton roughly becomes a pure gauge. This then implies that in such a setup a consistent coupling of bulk gravity to the brane matter is possible only if the latter is conformal.

Thus, the approach of [24] might provide a realization of a setup where a conformal theory on the brane is gravitationally coupled to a non-conformal theory in the bulk. The conformal property of the brane world-volume theory is then preserved due to bulk supersymmetry, which is unbroken even if the brane theory is non-supersymmetric as the volume of the extra dimension is infinite. In particular, if gravitational interactions localized on the brane are generated via, say, loop diagrams, then the corresponding brane world gravity is expected to be conformal as well.

The remainder of this paper is organized as follows. In Section 2 we briefly review the setup of [24]. In Section 3 we study small fluctuations around the solution in the presence of brane matter sources, and discuss the requirement that the brane matter be conformal. Section 4 contains concluding remarks.
2. The setup

In this section we review the setup of [24]. Thus, consider the model with the following action (more precisely, here we give the part of the action relevant for the subsequent discussions):

\[
S = \tilde{M}_P^{D-3} \int_\Sigma d^{D-1}x \sqrt{-\tilde{G}} \left[ \tilde{R} - \tilde{\Lambda} \right] + M_P^{D-2} \int d^Dx \sqrt{-G} \left[ R - \frac{4}{D-2} (\nabla \phi)^2 - V(\phi) \right].
\]  (1)

For calculational convenience we will keep the number of space-time dimensions \( D \) unspecified. In (1) \( \tilde{M}_P \) is (up to a normalization factor—see below) the \((D - 1)\)-dimensional (reduced) Planck scale, while \( M_P \) is the \( D \)-dimensional one. The \((D - 1)\)-dimensional hypersurface \( \Sigma \), which we will refer to as the brane, is the \( y = y_0 \) slice of the \( D \)-dimensional space–time, where \( y \equiv x^0 \), and \( y_0 \) is a constant. Next,

\[
\tilde{G}_{\mu\nu} \equiv \delta_{\mu}^M \delta_{\nu}^N G_{MN} |_{y=y_0},
\]  (2)

where the capital Latin indices \( M, N, \ldots = 1, \ldots, D \), while the Greek indices \( \mu, \nu, \ldots = 1, \ldots, (D - 1) \). The quantity \( \tilde{\Lambda} \) is the brane tension. More precisely, there might be various (massless and/or massive) fields (such as scalars, fermions, gauge vector bosons, etc.), which we will collectively denote via \( \phi^I \), localized on the brane. Then \( \tilde{\Lambda} = \tilde{\Lambda}(\phi^I, \nabla_\mu \phi^I, \ldots) \) generally depends on the vacuum expectation values of these fields as well as their derivatives. In the following we will assume that the expectation values of the \( \phi^I \) fields are dynamically determined, independent of the coordinates \( x^\mu \), and consistent with \((D - 1)\)-dimensional general covariance. The quantity \( \tilde{\Lambda} \) is then a constant which we identify as the brane tension. The bulk fields are given by the metric \( G_{MN} \), a single real scalar field \( \phi \), as well as other fields (whose expectation values we assume to be vanishing) which would appear in a concrete supergravity model (for the standard values of \( D \)).

Let us briefly comment on the \( \sqrt{-\tilde{G} \tilde{R}} \) term in the brane world-volume action. Typically such a term is not included in discussions of various brane world scenarios (albeit usually the \(-\sqrt{-G \tilde{\Lambda}} \) term is). However, as was pointed out in [22], even if such a term is absent at the tree level, as long as the brane world-volume theory is not conformal, it will typically be generated by quantum loops of other fields localized on the brane (albeit not necessarily with the desired sign, which, nonetheless, appears to be as generic as the opposite one). This is an important observation, which allows to reproduce the \((D - 1)\)-dimensional Newton’s law on, say, a non-conformal brane embedded in \( D \)-dimensional Minkowski space–time [22]. However, as we will see in the following, in the above setup the brane world-volume theory is actually conformal, and \( \tilde{M}_P = 0 \).

To proceed further, we will need equations of motion following from the action (1). Here we are interested in studying possible solutions to these equations which are consistent with \((D - 1)\)-dimensional general covariance. That is, we will be looking for solutions with the warped metric of the following form:

\[
\frac{1}{8} \left[ \phi'' + (D - 1)A' \phi' \right] - V_\phi - Lf_\phi \delta(y - y_0) = 0,
\]  (4)

\[
(D - 1)(D - 2)(A')^2 - \frac{4}{D - 2} (\phi')^2 + V = \frac{D - 1}{D - 3} \tilde{\Lambda} \exp(-2A) = 0,
\]  (5)

\[
(D - 2)A'' + \frac{4}{D - 2} (\phi')^2 + \frac{1}{D - 3} \tilde{\Lambda} \exp(-2A) + \frac{1}{2} Lf_\phi \delta(y - y_0) = 0.
\]  (6)
Here

\[ f \equiv \tilde{A} - \tilde{A} \exp[-2A(y_0)] \]  

is the effective brane tension. The scale \( L \), defined as

\[ L \equiv \frac{\tilde{M}_P^{D-3}}{\tilde{M}_P^{D-2}}, \]

plays the role of the crossover distance scale below which gravity is effectively \((D-1)\)-dimensional, while above this scale it becomes \(D\)-dimensional. Next, \( \tilde{A} \) is independent of \( x^\mu \) and \( y \). In fact, it is nothing but the cosmological constant of the \((D-1)\)-dimensional manifold, which is therefore an Einstein manifold, corresponding to the hypersurface \( \Sigma \). Our normalization of \( \tilde{A} \) is such that the \((D-1)\)-dimensional metric \( \tilde{g}_{\mu\nu} \) satisfies Einstein’s equations:

\[ \tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R} = -\frac{1}{2} \tilde{g}_{\mu\nu} \tilde{A}. \]  

Here we note that in the bulk (that is, for \( y \neq y_0 \)) one of the second order equations is automatically satisfied once the first-order equation (5) as well as the other second-order equation are satisfied. As usual, this is a consequence of Bianchi identities.

Note that by rescaling the coordinates \( x^\mu \) on the brane we can always set \( \exp[y_0/y_D] = 1 \). Then the \((D-1)\)-dimensional Planck scale is simply \( \tilde{M}_P \). Let \( \phi_0 \equiv \phi(y_0) \). Note that the above system of equations has smooth solutions for

\[ f(\phi) = f_\phi(\phi_0) = 0, \]

that is, if the brane cosmological constant and the brane tension are equal

\[ \tilde{A} = \Lambda, \]

and there is no \( \phi \) tadpole due to the brane. In particular, in these solutions \( \phi \) and \( A \) as well as their derivatives \( \phi' \) and \( A' \) are smooth.

Let us now discuss possible solutions of the above system of equations (4)–(6) for \( f(\phi) = f_\phi(\phi_0) = 0 \). To obtain an infinite volume solution, let us assume that the scalar potential has one AdS minimum located at \( \phi = \phi_- \) and one Minkowski minimum located at \( \phi = \phi_+ \) (without loss of generality we will assume that \( \phi_+ > \phi_- \)). Moreover, let us assume that there are no other extrema except for a dS maximum located at \( \phi = \phi_* \), where \( \phi_- < \phi_* < \phi_+ \), such that

\[ V(\phi_+) > V(\phi_-) \quad \text{and} \quad V(\phi_*) \approx |V(\phi_-)|. \]

This latter condition is necessary to sufficiently suppress the probability for nucleation of AdS bubbles in the Minkowski vacuum, which could otherwise destabilize the background [26]. Then we can have smooth domain walls interpolating between the two vacua. In fact, for \( \Lambda > 0 \) we have \( \phi(y) \to \phi_\pm \) as \( y \to \pm \infty \). On the other hand, for \( \Lambda > 0 \) we have \( \phi(y) \to \phi_\pm \) as \( y \to +\infty \), while \( \phi(y) \to \phi_- \) as \( y \to y_- \), where \( y_- < y_0 \) is finite (here for definiteness we have assumed that the domain wall approaches the Minkowski vacuum as \( y \to +\infty \)). As to the warp factor \( A \), it goes to \( -\infty \) as \( \phi \to \phi_- \) (if \( \Lambda = 0 \), then \( A \) goes to \( -\infty \) linearly with \( |y| \), while if \( \Lambda > 0 \), then \( A \sim \ln(y - y_-) \) as \( y \to y_- \)). On the other hand, if \( \Lambda = 0 \), then \( A \) goes to a constant as \( y \to +\infty \), while if \( \Lambda > 0 \), then \( A \) grows logarithmically with \( y \). In both cases the volume of the extra dimension is infinite as the integral

\[ \int dy \exp[-(D-1)A] \]

diverges. Moreover, there are no quadratically normalizable bulk graviton modes. Rather, for \( \tilde{A} = 0 \) we have a continuum of plane-wave normalizable bulk modes (with mass squared \( m^2 \geq 0 \)), while for \( \tilde{A} > 0 \) we have a mass gap in the bulk graviton spectrum, and the plane-wave normalizable modes are those with \( m^2 > m_1^2 \), where
Thus, without any additional assumptions consistent solutions with vanishing as well as positive brane cosmological constant exist for such potentials.

However, as was pointed out in [24], as long as the scalar potential $V(\phi)$ is non-trivial, bulk supersymmetry is incompatible with non-zero brane cosmological constant. Indeed, this immediately follows from the bulk Killing spinor equations (following from the requirement that variations of the superpartner $\lambda$ of $\phi$ and the gravitino $\psi_M$ vanish under the corresponding supersymmetry transformations), which in such backgrounds reduce to:

$$\phi' = \alpha W\phi, \quad A' = \beta W,$$

where $W$ is the superpotential,

$$\alpha \equiv \eta \frac{\sqrt{D-2}}{2}, \quad \beta \equiv -\eta \frac{2}{(D-2)^{3/2}},$$

and $\eta = \pm 1$.

Note that the system of equations (13) and (14) is compatible with the system of equations (4)–(6) if and only if $\tilde{\Lambda} = 0$, and the scalar potential is given by

$$V = W_\phi^2 - \gamma^2 W^2,$$

where

$$\gamma \equiv \frac{2\sqrt{D-1}}{D-2}.$$ 

Thus, bulk supersymmetry (note that the domain wall solution preserves 1/2 of the supersymmetries corresponding to the minima of $V$) is preserved if and only if the brane cosmological constant vanishes. We therefore conclude that even if brane supersymmetry is broken, bulk supersymmetry, which remains unbroken as the volume of the transverse dimension is infinite, ensures that the brane cosmological constant still vanishes in the model defined in (1).

Before we end this section, for illustrative purposes let us give an example of a domain wall of the aforementioned type. Let

$$W = \xi \left[ v^3 \phi - \frac{1}{3} \phi^3 - \frac{2}{3} v^3 \right],$$

where

$$\phi(y) = v \tanh \left( \alpha \xi (y - y_1) \right), \quad A(y) = \frac{2\beta}{3\alpha} v^2 \left[ \ln \left( \cosh (\alpha \xi (y - y_1)) \right) - \frac{1}{4} \cosh^2 (\alpha \xi (y - y_1)) \right] - \frac{2\beta}{3} \xi v^3 (y - y_1) + C,$$

where $y_1$ and $C$ are integration constants.

Finally, let us note that solutions with non-vanishing $f(\phi_0)$ and $f_\phi(\phi_0)$ do not interpolate between the AdS and Minkowski vacua. Thus, solutions with positive $f(\phi_0)$ asymptotically approach the AdS minimum on both sides of the brane, while solutions with negative $f(\phi_0)$ asymptotically approach the Minkowski minimum on both sides of the brane.

\footnote{Note that $f(\phi_0)$ is the effective brane tension. If $f(\phi_0) < 0$, then we have world-volume ghosts unless we assume that the brane is an “end-of-the-world” brane located at an orbifold fixed point. Thus, in solutions with $f(\phi_0) < 0$ the geometry of the $y$ dimension is that of $\mathbb{R}/\mathbb{Z}_2$.}
3. Brane matter sources

In this section we would like to study gravitational interactions between sources localized on the brane. To do this, let us start from the action (1), and study small fluctuations of the metric $G_{MN}$ and the scalar field $\phi$, which we will denote via $h_{MN}$ and $\varphi$, respectively, around the corresponding smooth domain wall solution (with vanishing brane cosmological constant) in the presence of brane matter sources.

In the following it will prove convenient to make the coordinate transformation $y \to z$ so that the background metric takes the form:

$$ds^2_D = \exp(2A)\left[\eta_{\mu\nu} \, dx^\mu \, dx^\nu + dz^2\right].$$

That is,

$$dy = \exp(A) \, dz,$$

where we have chosen the overall sign so that $z \to \pm \infty$ as $y \to \pm \infty$. Moreover, we can fix the integration constant upon solving (22) such that $y = y_0$ is mapped to $z = 0$. So from now on we will use the coordinates $x^M = (x^\mu, x^D) = (x^\mu, z)$, and prime will denote derivative with respect to $z$. Moreover, the capital Latin indices $M, N, \ldots$ are lowered and raised with the flat $D$-dimensional Minkowski metric $\eta_{MN}$ and its inverse, while the Greek indices $\mu, \nu, \ldots$ are lowered and raised with the flat $(D-1)$-dimensional Minkowski metric $\eta_{\mu\nu}$ and its inverse.

Also, instead of metric fluctuations $h_{MN}$, it will be convenient to work with $\tilde{h}_{MN}$ defined via

$$h_{MN} = \exp(2A)\tilde{h}_{MN}.$$  \hspace{1cm} (23)

It is not difficult to see that in terms of $\tilde{h}_{MN}$ the $D$-dimensional diffeomorphisms

$$\delta h_{MN} = \nabla_M \xi_N + \nabla_N \xi_M$$

are given by the following gauge transformations:

$$\delta \tilde{h}_{MN} = \partial_M \xi_N + \partial_N \xi_M + 2A' \eta_{MN} \xi^n n^n.$$  \hspace{1cm} (25)

Here for notational convenience we have introduced a unit vector $n^M$ with the following components: $n^\mu = 0$, $n^D = 1$.

3.1. Equations of motion

To proceed further, we need equations of motion for $\tilde{h}_{MN}$ and $\varphi$. Let us assume that we have matter localized on the brane, and let the corresponding conserved energy-momentum tensor be $T_{\mu\nu}$:

$$\partial^\nu T_{\mu\nu} = 0.$$  \hspace{1cm} (26)

The graviton field $\tilde{h}_{\mu\nu}$ couples to $T_{\mu\nu}$ via the following term in the action:

$$S_{\text{int}} = \int \Sigma d^{D-1}x \left[\frac{1}{2} T_{\mu\nu} \tilde{h}^{\mu\nu} + \frac{8}{D-2} \Box \varphi\right].$$  \hspace{1cm} (27)

(and not of $\tilde{R}$), with the orbifold fixed point identified with $y_0$ (then the corresponding solution on the covering space has the $Z_2$ symmetry required for the orbifold interpretation), and the brane is stuck at the orbifold fixed point.
where we have also included the corresponding coupling of \( \psi \) to the brane matter. Next, starting from the action \( S + S_\text{int} \) we obtain the following linearized equations of motion for \( \tilde{h}_{MN} \) and \( \psi \):

\[
\left\{ \partial_5 \partial^5 \tilde{h}_{MN} + \partial_M \partial_5 \tilde{h}^N - \partial_M \partial^5 \tilde{h}^N_{\text{SN}} - \partial_N \partial^5 \tilde{h}_{SM} - \eta_{MN} \left[ \partial_5 \partial^5 \tilde{h} - \partial^5 \partial^R \tilde{h}^R_{SN} \right] \right\} + (D - 2) A' \left[ \partial_5 \tilde{h}_{MN} - \partial_M \tilde{h}_{NS} - \partial_N \tilde{h}^S_{MS} \right] n^S + \eta_{MN} \left[ 2 \partial^R \tilde{h}^R_{RS} - \partial^5 \tilde{h} \right] n^S \\
- \eta_{MN} \tilde{h}_{SR} n^S n^R V \exp(2A) = \frac{8}{D - 2} \psi \left[ \eta_{MN} \partial_5 \partial^5 \psi - \partial_5 \partial^5 \psi + \partial_5 \partial^5 \psi \right] + \eta_{MN} \psi V_\phi \exp(2A) - M_p^{2-D} \tilde{T}_{MN} \delta(z).
\]

(28)

\[
\partial_5 \partial^5 \psi + (D - 2) A' \partial_5 \partial^5 \psi - \frac{D - 2}{8} \psi V_\phi \exp(2A) = \frac{1}{2} \psi \left[ 2 \partial^R \tilde{h}^R_{RS} - \partial^5 \tilde{h} \right] n^S \\
- \frac{D - 2}{8} \eta_{SR} n^S n^R V_\phi \exp(2A) = -M_p^{2-D} \tilde{\Theta} \delta(z).
\]

(29)

where \( \tilde{h} = \tilde{h}_{MN}^0, \tilde{T}_{MN} = T_{MN} + T_{MN}^{\text{brane}}, \tilde{\Theta} = \Theta + \Theta^{\text{brane}}. \) Here \( T_{MN}^{\text{brane}} \) and \( \Theta^{\text{brane}} \) are the corresponding brane contributions (which are linear in \( \tilde{h}_{MN} \) and \( \psi \)) coming from the first term in (1)\(^2\). Note that the only non-vanishing components of \( \tilde{T}_{MN} \) are \( \tilde{T}_{\mu\nu} \), and we have \( \partial^5 \tilde{T}_{\mu\nu} = 0. \)

The above equations of motion drastically simplify if we perform a gauge transformation (25) with

\[
\tilde{\xi}_M = n_M \left( \frac{\psi}{\phi^2} \right).
\]

(30)

The new equations of motion then read

\[
\left\{ \partial_5 \partial^5 \tilde{h}_{MN} + \partial_M \partial_5 \tilde{h}^N - \partial_M \partial^5 \tilde{h}^N_{\text{SN}} - \partial_N \partial^5 \tilde{h}_{SM} - \eta_{MN} \left[ \partial_5 \partial^5 \tilde{h} - \partial^5 \partial^R \tilde{h}^R_{SN} \right] \right\} + (D - 2) A' \left[ \partial_5 \tilde{h}_{MN} - \partial_M \tilde{h}_{NS} - \partial_N \tilde{h}^S_{MS} \right] n^S + \eta_{MN} \left[ 2 \partial^R \tilde{h}^R_{RS} - \partial^5 \tilde{h} \right] n^S \\
- \eta_{MN} \tilde{h}_{SR} n^S n^R V \exp(2A) = -M_p^{2-D} \tilde{T}_{MN} \delta(z).
\]

(31)

\[
\frac{1}{2} \psi \left[ 2 \partial^R \tilde{h}^R_{RS} - \partial^5 \tilde{h} \right] n^S - \frac{D - 2}{8} \eta_{SR} n^S n^R V_\phi \exp(2A) = -M_p^{2-D} \tilde{\Theta} \delta(z).
\]

(32)

Note that these equations of motion no longer contain \( \psi \). This has a simple physical interpretation [25]. The domain wall background spontaneously breaks translational invariance in the \( z \) direction. Since this invariance is a gauge symmetry, the corresponding Goldstone mode, which is given by configurations where \( \omega \equiv \psi/\phi^2 \) is independent of \( z \) [25]\(^1\), must be eaten in the corresponding Higgs mechanism. The field which eats the Goldstone mode is nothing but the graviphoton \( h_{\mu\nu} \) arising in the decomposition of the \( D \)-dimensional metric fluctuations in terms of \((D - 1)\)-dimensional fields [25]. Note, however, that with the above gauge fixing not only the Goldstone zero mode but all \( \psi \) modes have been eliminated. There is, however, a price we have to pay for this simplification. In particular, the residual gauge invariance which preserves the equations of motion (31) and (32) is given by

\[
\delta \tilde{h}_{MN} = \partial_M \tilde{\xi}_N + \partial_N \tilde{\xi}_M, \quad \tilde{\xi}_MN^S = 0.
\]

(33)

Note that here \( \tilde{\xi}_M \) need not be independent of \( z \). Under these residual gauge transformations the fields \( \tilde{h}_{\mu\nu}, A_\mu, \rho \), where \( A_\mu \equiv h_{\mu\nu}, \rho \equiv h_{DD}, \) transform as follows:

\[
\delta \tilde{h}_{\mu\nu} = \partial_\nu \tilde{\xi}_\mu + \partial_\mu \tilde{\xi}_\nu, \quad \delta A_\mu = \tilde{\xi}_\mu, \quad \delta \rho = 0.
\]

(34)

\(^2\) If the brane world-volume theory is not conformal, then we can \textit{a priori} expect that a kinetic term for the \( \psi \) field will also be generated on the brane (just as it happens for the graviton). Then \( \Theta^{\text{brane}} \) also contains a term proportional to \( \partial^5 \partial_\mu \psi \) along with the term proportional to \( f_\phi \partial_\mu \psi \). However, at the end of the day we will find that the brane world-volume theory is conformal, so these terms are not generated by quantum effects.

\(^3\) To see this, note that the translational Goldstone mode corresponds to fluctuations around the solution given by \( \phi(z + \omega(x^\mu)) = \phi(z) + \omega(x^\mu)) + \mathcal{O}(\omega^2). \)
This implies that we cannot gauge \( \rho \) away. We can, however, gauge \( A_\mu \) away. Thus, in the following we will use the gauge where \( A_\mu = 0 \). Note that after this gauge fixing the residual gauge transformations are given by

\[
\delta \tilde{h}_{\mu \nu} = \partial_\nu \xi_\mu + \partial_\mu \xi_\nu, \quad \delta \rho = 0, \quad \xi_\mu = 0.
\]

We now have the following equations of motion:

\[
\begin{align*}
\left[ \partial_\nu \partial^\nu H_{\mu \nu} + \partial_\mu \partial_\nu H - \partial_\mu \partial^\sigma H_{\sigma \nu} - \partial_\nu \partial^\sigma H_{\mu \sigma} - \eta_{\mu \nu} \left[ \partial_\eta \partial^\eta H - \partial^\eta \partial^\eta H_{\eta \rho} \right] \right] \\
+ \left[ H_{\mu \nu} - \eta_{\mu \nu} H_{\rho \sigma} + (D - 2) A' \right] \left[ H_{\nu \rho} - \eta_{\nu \rho} H_{\sigma \mu} \right] \\
+ \left[ \partial_\mu \partial_\nu \rho - \eta_{\mu \nu} \partial_\sigma \rho + \eta_{\mu \nu} \left[ (D - 2) A' \rho' - \rho V \exp(2A) \right] \right] = -M_p^{2-D} \tilde{T}_{\mu \nu} \delta(z), \tag{36}
\end{align*}
\]

\[
\begin{align*}
\left[ \partial_\mu \partial_\nu H_{\mu \nu} - \partial_\mu H \right] + (D - 2) A' \partial_\nu \rho = 0, \tag{37}
\end{align*}
\]

\[
\begin{align*}
- \left[ \partial_\mu \partial_\nu H_{\mu \nu} - \partial_\mu \partial_\nu H \right] + (D - 2) A' H' + \rho V \exp(2A) = 0, \tag{38}
\end{align*}
\]

\[
\begin{align*}
\phi' \left[ H' - \rho' \right] - \frac{D - 2}{4} \rho V \phi' \exp(2A) = -2M_p^{2-D} \tilde{\phi} \delta(z), \tag{39}
\end{align*}
\]

where \( H_{\mu \nu} = \tilde{h}_{\mu \nu} \), and \( H = H^\mu_{\mu} \).

Not all of the above equations are independent. First, differentiating (36) with \( \tilde{\partial}^\mu \) (and taking into account that \( \partial_\mu \tilde{T}_{\mu \nu} = 0 \)), we obtain an equation which is identically satisfied once we take into account (37) together with the equations of motion for \( A \) and \( \phi \). Next, taking the trace in (36), we obtain an equation which together with (37), (38) and the equations of motion for \( A \) and \( \phi \) gives the following equation:

\[
\frac{4}{D - 2} \left( \phi' \right)^2 \left[ H' - \rho' \right] - \rho V \phi' \exp(2A) = M_p^{2-D} A' \tilde{T} \delta(z), \tag{40}
\]

where \( \tilde{T} = \tilde{T}^\mu_{\mu} \). This equation is compatible with (39) if and only if

\[
\tilde{\phi} = - \frac{D - 2}{8} A'(0) \tilde{T}. \tag{41}
\]

Thus, we already see that the coupling of the brane matter to the bulk scalar cannot be arbitrary but is determined in terms of the trace of the energy-momentum tensor. (Note that neither \( A' \) nor \( \phi' \) vanish anywhere in the backgrounds we consider here, including the location of the brane \( z = 0 \).)

Finally, let us discuss (39). It came from the equation of motion for \( \varphi \), which was a second order equation. However, after we eliminated \( \varphi \) itself, this equation became a first order equation in terms of \( H \) and \( \rho \). This then implies that the source term on the right-hand side of (39) must vanish or else \( H - \rho \) will be discontinuous. Thus, we have arrived at the conclusion that consistency of the above equations implies that we must have

\[
\tilde{T} = \tilde{\phi} = 0. \tag{42}
\]

This, in particular, implies that the brane world-volume theory is generically expected to be conformal in this setup. Indeed, it is not difficult to see that (42) can be satisfied if \( T = T^\mu_{\mu} \) as well as \( \Theta \) vanish. To ensure conformality of the matter localized on the brane then generically requires that the brane world-volume theory itself be conformal. On the other hand, if this is not the case then to satisfy (42) \( T \) and \( \Theta \) (in the best case where \( f_{\varphi \varphi}(\varphi_0) = 0 \)) must be the same up to a non-vanishing constant, which generically need not be the case.

\[4\] To see this, note that, once we perform the gauge transformation (25) with the gauge parameter given in (30), \( \tilde{T}_{\mu \nu} \) contains a term proportional to \((D - 3) \left[ \partial_\mu \partial_\nu - \eta_{\mu \nu} \partial_\rho \partial^\rho \right] \). Assuming \( D \neq 2, 3 \), we then have that on the brane \( \partial^\mu \partial_\mu \omega \) is proportional to \( T \) with a non-vanishing coefficient. On the other hand, the condition \( \Theta = 0 \) gives a second order differential equation on the brane for \( \omega \) with a source term proportional to \( \Theta \). Hence the aforementioned conclusion about the relation between \( T \) and \( \Theta \).
Let us verify that, if (42) is satisfied, the above system of equations does have a consistent solution for $H_{\mu\nu}$ and $\rho$. It is not difficult to show that such a solution indeed exists, and is given by (note that $p^\mu \tilde{T}_{\mu\nu}(p) = 0$)
\begin{align}
\rho &= 0, \\
H_{\mu\nu}(p, z) &= M_p^{2-D} \tilde{T}_{\mu\nu}(p) \Omega(p, z),
\end{align}
where we have performed a Fourier transform with respect to the coordinates $x^\mu$ (and the corresponding momenta are $p^\mu$). Also, let us Wick rotate to the Euclidean space (where the propagator is unique). The function $\Omega(p, z)$ is a solution to the following equation ($p^2 \equiv p^\mu p_\mu$):
\begin{equation}
\Omega''(p, z) + (D - 2) A' \Omega'(p, z) - p^2 \Omega(p, z) = -\delta(z)
\end{equation}
subject to the boundary conditions (for $p^2 > 0$)
\begin{equation}
\Omega(p, z \rightarrow \pm \infty) = 0.
\end{equation}
The above solution describes a gravitational field of conformal matter localized on the brane.

### 3.2. Additional evidence

In this subsection we would like to give additional evidence that the condition $e_T(p) \leq D_0$ (from which it follows that $e_T(p) \leq 0$) is indeed necessary. To begin with, let us perform the aforementioned Fourier transform in (36)–(39), and Wick rotate to the Euclidean space. The equations of motion for the Fourier transformed quantities read:
\begin{align}
&\left\{ \frac{p^2}{2} H_{\mu\nu} + p_\mu p_\nu H - p_\mu p^\sigma H_{\sigma\nu} - p_\nu p^\sigma H_{\mu\sigma} - \eta_{\mu\nu} \left[ \frac{p^2}{2} H - p^\sigma p^\rho H_{\sigma\rho} \right] \right\}' + (D - 2) A' \left[ H_{\mu\nu} - \eta_{\mu\nu} H' \right] \\
&\quad + \left\{ -p_\mu p_\nu p^2 + \eta_{\mu\nu} \left[ (D - 2) A' \rho' + \rho \left( p^2 - V \exp(2A) \right) \right] \right\} = -M_p^{2-D} \tilde{T}_{\mu\nu}(p) \delta(z),
\end{align}
\begin{align}
&\left[ p^\mu H_{\mu\nu} - p_\nu H \right]' + (D - 2) A' p_\nu \rho = 0, \\
&\left[ p^\mu p^\nu H_{\mu\nu} - p^2 H \right]' + \left[ (D - 2) A' H' + \rho V \exp(2A) = 0, \\
&\frac{4}{D - 2}(\phi')^2 \left[ H' - \rho' \right] - \frac{D - 2}{4} \rho V \phi' \exp(2A) = M_p^{2-D} A' \tilde{T}(p) \delta(z),
\end{align}
where we have taken into account (41).

Let us assume that $\tilde{T}(p) \neq 0$. Then the most general tensor structure for the fields $H_{\mu\nu}$ and $\rho$ can be parameterized in terms of four functions $a$, $b$, $c$, $d$ as follows:
\begin{align}
\rho &= M_p^{2-D} d \tilde{T}(p), \\
H_{\mu\nu} &= M_p^{2-D} \left\{ a \tilde{T}_{\mu\nu}(p) + [b \eta_{\mu\nu} + c \eta_{\mu\nu}] \tilde{T}(p) \right\}.
\end{align}
Plugging this back into the equations of motion, we obtain six equations for four unknowns $a$, $b$, $c$, $d$. However, as should be clear from the above discussion, two of them are identically satisfied once we take into account the other four (as well as the equations of motion for $A$ and $\phi$). After some straightforward computations we obtain the following system of four independent equations:
\begin{align}
a'' + (D - 2) A' a' - p^2 a &= -\delta(z), \\
(D - 2) A' d &= a' + (D - 2) b', \\
A' \left[ (D - 2) p^2 c'' - a' \right] &= p^2 \left[ a + (D - 2) b \right] - \frac{4}{D - 2} (\phi')^2 d, \\
a + (D - 3) b - c'' - (D - 2) A' c' + d &= 0.
\end{align}
Note that from the first equation it follows that \( a(p,z) = \Omega(p,z) \).

Let \( w \equiv a + (D - 2)b \). Using the above equations for \( a, b, c, d \) after some straightforward computations we obtain the following second-order equation for \( w \):

\[
w'' + \left( (D - 2)A' - \left[ \ln(F) \right]' \right) w' - p^2 w = -F \delta(z),
\]

where \( F(z) \) is the following function:

\[
F \equiv \frac{(A')^2}{(A')^2 - A''}.
\]

Solutions to the above equation for \( w \) have some peculiar properties. To expose them, we need to study the asymptotic behavior of the function \( \ln(F) \).

To begin with, it is not difficult to see that

\[
\ln(F) \equiv -\beta \frac{W^2}{\alpha (W_{\phi})^2}.
\]

where the superpotential \( W \) as well as constants \( \alpha, \beta \) were defined in Section 2. It then follows that

\[
[\ln(F)]' = 2\alpha \exp(A) \frac{(W_{\phi})^2 - WW_{\phi\phi}}{W}.
\]

Let us compute this function in the example discussed at the end of Section 2. In that example the superpotential is given by (18). We then have

\[
[\ln(F)]' = -2\alpha \xi v \exp(A) \frac{3 + 2\phi + \phi^2}{2 + \phi}.
\]

where \( \phi \equiv \phi/v \). For definiteness let us assume that \( \alpha \xi v > 0 \). Then at \( z \to +\infty \) the domain wall solution approaches the Minkowski vacuum where \( \phi = +1 \), while at \( z \to -\infty \) it approaches the AdS vacuum where \( \phi = -1 \). It then follows that \( \ln(F) \)' is always negative on the solution. Moreover, at \( z \to +\infty \) (where \( A \) goes to a constant and \( A' \) goes to zero) \( \ln(F) \)' goes to a constant, which we will denote by \(-2\xi \). Then for large positive \( z \) \( w \) is well approximated by the solution to the following equation:

\[
w'' + 2\xi w' - p^2 w = 0
\]

subject to the boundary condition \( w(z \to +\infty) = 0 \). This solution is given by

\[
w(z) = \text{const.} \times \exp(-\lambda z),
\]

where (the other root of the corresponding quadratic equation is negative)

\[
\lambda \equiv \xi + \sqrt{\xi^2 + p^2}.
\]

Note that for \( p \to 0 \) \( \lambda \) does not vanish but approaches \( 2\xi \). This implies that even at zero momentum there is a non-trivial solution to (57)\(^5\). In particular, it is given by \( w(z) = \tilde{w}(0) \) for \( z < 0 \), \( w(z) = \tilde{w}(z) \) for \( z \geq 0 \), where \( \tilde{w}(z) \) is the solution of the equation

\[
\tilde{w}'' + \left( (D - 2)A' - \left[ \ln(F) \right]' \right) \tilde{w}' - p^2 \tilde{w} = 0
\]

\(^5\) Here we should point out that this does not occur if we consider domain walls arising for runaway type of potentials discussed in [24]. Nonetheless, the fact that a non-trivial solution of (57) does exist in the above example if we assume \( \tilde{F} \neq 0 \) might be considered as (at least indirect) evidence that \( \tilde{F} = 0 \) condition is indeed necessary. At any rate, if this condition is not satisfied, as we have already pointed out, there is a discontinuity in \( H - \rho \), which appears to be inconsistent.
subject to the boundary conditions $\tilde{w}(z \to +\infty) = 0$, and $\tilde{w}'(0) = -F(0)$ (note that $F(z)$ is always positive). The fact that such a solution always exists for $v \ll 1$ can be seen from the fact that in this case $(D-2)A^I \ll -[\ln(F)]^I$.

The existence of a non-trivial solution at $p^2 = 0$ indicates an inconsistency in the system. Note that if we have $\tilde{T} = 0$ to begin with, then we do not have the same system of equations for $a, b, c, d$ as above. In fact in this case there is no inconsistency, and we have a consistent solution discussed in the previous subsection.

4. Remarks

Thus, as we see, in the setup of [24], where the brane cosmological constant is protected by bulk supersymmetry, to have consistent couplings of the bulk scalar and gravity to matter localized on the brane, it appears to be necessary that the latter is conformal. This then implies that (generically) the brane world-volume theory should itself be conformal. The fact that the brane cosmological constant vanishes is then a trivial consequence of conformal invariance of the brane world-volume theory. However, what appears to be non-trivial is that unbroken bulk supersymmetry in this setup (where the volume of the extra dimension is infinite) actually protects conformality of the brane world-volume theory (which a priori need not even be supersymmetric).

In this context one might hope to use this setup as a possible realization of the conformal approach to phenomenology [27] (also see [28]). However, it is still unclear how one could possibly have the brane conformal invariance broken around TeV while having much larger Planck scale on the brane.

Acknowledgements

I would like to thank Gregory Gabadadze, Juan Maldacena, Tom Taylor and Cumrun Vafa for valuable discussions. Parts of this work were completed while I was visiting at Harvard University and New York University. This work was supported in part by the National Science Foundation. I would also like to thank Albert and Ribena Yu for financial support.

References

   hep-ph/9807522;
     P.H. Frampton, Phys. Rev. D 60 (1999) 085004;
     P.H. Frampton, Phys. Rev. D 60 (1999) 121901;
On some new warped brane world solutions in higher dimensions

Seif Randjbar-Daemi a,*, Mikhail Shaposhnikov b

a International Center for Theoretical Physics, Trieste, Italy
b Institute of Theoretical Physics, University of Lausanne, CH-1015 Lausanne, Switzerland

Received 11 August 2000; accepted 7 September 2000

Editor: L. Alvarez-Gaumé

Abstract

We present new solutions of higher dimensional Einstein’s equations with a cosmological constant that localize gravity on branes which are transverse to Ricci-flat manifolds or to homogeneous spaces with topologically non-trivial solutions of gauge field equations. These solutions are relevant for the localization of chiral fermions on a brane.

Although the non-compact internal spaces [1–4] have been considered in the past, the recent interest, inspired by ideas from superstring theory, emphasizes the notion that our large-scale universe may be regarded as a brane embedded in a higher-dimensional manifold. The gravity may be localized on a brane by the Randall–Sundrum mechanism [5], while the observed particles are attached to a brane because of dynamics of string theory [6] or by field-theoretic effects [1,7–10]. It thus becomes relevant to look for general solutions of the D-dimensional field equations. It is expected that a richer geometrical and topological structure of the subspaces spanned by extra dimensions will lead to more interesting physical consequences on the brane world. A number of warped solutions have already been found in [2,3,11–19].

In this note, we consider a D = D1 + D2 + 1-dimensional system of gravity — Yang–Mills system and look for warped solutions of the form

$$ds^2 = e^{A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{B(r)} g_{mn}(y) dy^m dy^n + dr^2,$$

where x^0, x^1, …, x^{D_1-1} cover the brane world M_{D_1}, and y^1, …, y^{D_2} cover an internal space K with a metric g_{mn}(y). The metric with D_2 = 0 and A = c|r|, c < 0 is the Randall–Sundrum case [5], the one with D_2 = 1 and A = B = c|r|, c < 0, is the case of local string [13], D_2 = 1 and A = c|r|, c < 0, B = const corresponds to a global string [12]. When D_2 ≥ 2 the approximate solutions with scalar fields [14] or exact bulk solutions with p-form fields [16] have all assumed that K = S^{D_2}. In this note we shall consider cases where K is not restricted to being a sphere. These general cases become important if we want to localize chiral fermions to the brane world M_{D_1} [10].

In the presence of Yang–Mills fields we shall assume that K is a symmetric homogeneous space G/H with a G-invariant metric g_{mn}(y) defined on it. We shall also assume that the gauge group has a non-trivial intersection with H. For example, for G/H = S^4 it is sufficient to assume that the gauge group contains an SU(2) subgroup. On such spaces
one can construct \( G \)-invariant solutions to the Yang–Mills equations [20], giving

\[
\tilde{F}_{mp} \cdot \tilde{F}_p^p = \frac{e^{-B}}{D_2} F^2 g_{mn},
\]

where, by virtue of \( G \)-invariance,

\[
F^2 \equiv g^{mp} g^{aq} \tilde{F}_{mn} \cdot \tilde{F}_{pq}
\]

must be a constant. The non-trivial equations to be solved can then be put into the form

\[
A'' + \frac{D_1}{2} A'^2 + \frac{D_2}{2} A'B' = \frac{4k^2}{D-2} \left( -\Lambda + \frac{F^2}{4g^2} e^{-2B} \right),
\]

\[
B'' + \frac{D_2}{2} B'^2 + \frac{D_1}{2} A'B' = \frac{4k^2}{D-2} \left( -\Lambda - \frac{2D - D_1}{D_2} \frac{F^2}{4g^2} e^{-2B} \right) + ke^{-B},
\]

\[
D_1 A'' + D_2 B'' + \frac{D_1}{2} A'^2 + \frac{D_2}{2} B'^2 = \frac{4k^2}{D-2} \left( -\Lambda + \frac{F^2}{4g^2} e^{-2B} \right),
\]

where \( \Lambda \) and \( k^2 \) are \( D \)-dimensional cosmological and Newtonian constants respectively, \( g \) is the gauge coupling. Also the constant \( k \) in Eq. (5) is the curvature scalar of \( K \) defined by \( R_{mn} = k g_{mn} \). It can be shown that only two out of the three equations above are independent. It should also be noted that these equations are valid in the bulk, the brane core region are independent. It should also be noted that these equations are valid in the bulk, the brane core region are independent.

At this point it is convenient to consider the cases \( k = 0 \) and \( k \neq 0 \) separately.

(i) \( k = 0 \). In this case \( K \) is a Ricci flat manifold. Well known examples are tori and Calabi–Yau manifolds. Setting \( F^2 = 0 \), one can find all solutions of Eqs. (4)–(6), generalizing the solutions found in [2,3,16] for \( D_1 = 4, D = 6 \), where the \( k \)-term is trivially absent. The general result is:

\[
A(r) = a \log[z'(r)] + b \log[z(r)],
\]

\[
B(r) = c \log[z'(r)] + d \log[z(r)],
\]

where

\[
a = \frac{2}{D-1} \left( 1 - \sqrt{\frac{D_2(D-2)}{D_1}} \right),
\]

\[
b = \frac{2}{D-1} \left( 1 + \sqrt{\frac{D_2(D-2)}{D_1}} \right),
\]

\[
c = \frac{2}{D-1} \left( 1 - \sqrt{\frac{D_1(D-2)}{D_2}} \right),
\]

\[
d = \frac{2}{D-1} \left( 1 + \sqrt{\frac{D_1(D-2)}{D_2}} \right),
\]

and

\[
z(r) = \text{Re}(\alpha e^{\gamma r} + \beta e^{-\gamma r}).
\]

where \( \alpha \) and \( \beta \) are arbitrary constants.

It is easy to verify that for a negative cosmological constant there is a simple solution with \( A = B = -\gamma r \), where \( \gamma \) is not given by (10) but \( \gamma = \sqrt{-\frac{k^2 A}{2(D-2)}}, \)

As in the standard RS solution, we can extend the range of \( r \) to \((-\infty, +\infty)\) and write the solution as \( A = B = -\gamma |r| \). In this case \( A'' = B'' = -2\gamma \delta(r) \).

The field equations in the core region will then relate the tension of the brane to \( \gamma \), along the lines discussed in [5,13,16]. At infinity, \( r \to \infty \), the square of the curvature tensor is singular, as follows from the general expression, given in [3]. The only exception is when the internal space is flat, for example, it can be a torus. In this case the solution is non-singular even as \( r \to \infty \). For the general solution to make physical sense the singularity should be smoothened by, say, string theory effects.

(ii) \( k \neq 0 \). In this case the \( G \)-invariant solutions of Yang–Mills equations can be constructed as in [21]. One can have a regular solution in the bulk with the structure \( AdS_{D_1+1} \times G/H \) with \( A = -cr \) and \( B \) a constant, similar to the one considered in [16] for a compactification on a monopole for \( D_2 = 2 \). One can consider two possibilities. In the first case we can treat \( r \) as a radial co-ordinate, varying from zero to infinity. Then this solution is singular at \( r = 0 \) and requires the addition of a \( G \) invariant brane, residing at this point. The tensions on the brane must then
be fine tuned accordingly. In the second case $r$ may vary from $-\infty$ to $+\infty$. As the Einstein equations are symmetric with respect to the transformation $r \to -r$, a solution, leading to the localization of gravity, looks like $A = -c|r|$ and requires the presence of a brane as well.

An interesting question arises whether one can construct a braneless solution that is regular at $r = 0$ and which approaches the bulk solution just described for large values of $r$. Arguing along the lines of Ref. [3], this seems to be impossible, at least if $-\infty < r < \infty$, when the $Z_2$ symmetry is imposed. To this end it is convenient to rewrite Eqs. (4)–(6) as a single equation for $B$ only,

$$B'' = \frac{1}{2(D-2)(D-1)D_2e^{2B}} \left\{ (D-2)D_2^2 \frac{e^{2B}}{e^B} \right.$$
$$+ 2(D_1-1) \left( 4 - 2D + D_1 \right) \frac{k^2 F^2}{g^2}$$

$$- D_2 \left( 2k - Dk + 4 e^B k^2 A \right) e^B \left. \right\} \pm S \right\}, \quad (11)$$

where

$$S = D_2 \sqrt{D_1(D-2)B'} e^B \left\{ D_2(D-2)B^2 e^{2B} \right.$$}
$$- 2(D_1-1) \left( 2D - 2D_1 - 3 \right) \frac{k^2 F^2}{g^2}$$

$$+ 4(D_1-1) e^{2B} k^2 A$$

$$+ D_2 \left( 2 - D \right) \left( k + 4 e^B k^2 A \right) e^B \right\}^{1/2}. \quad (12)$$

Now, this equation describes a motion of a particle in a potential $V(B)$ which is obtained by setting $B' = 0$ in Eq. (12) and integrating with respect to $B$.

$$V(B) = k e^{-B} \frac{1}{(D-2)}$$

$$\times \left\{ \frac{1}{2D_2} \left( 2D - D_1 - 4 \right) \frac{k^2 F^2}{g^2} e^{-2B} \right.$$}

$$- 4k^2 A B \right\}. \quad (13)$$

with quite complicated friction force. The regularity of solution at $r = 0$ requires $B'(0) = 0$, while $B \to \text{const}$ at infinity. In a mechanical analogy the particle should start moving with zero velocity and reach the maximum of the potential at “time” $r \to \infty$. This is only possible if the potential (13) has two extrema.

This requirement leads to a certain constraints on the parameters $\frac{1}{c^2} \frac{F^2}{g^2}$ and on the dimensionalities $D_1$ and $D_2$. The latter happen to be not consistent with reality of the quantity $S$ defined in Eq. (12) at the maximum of the potential.

Both types of solutions (i) and (ii) can be used to obtain localized chiral fermions in $M_{D_1}$. As argued in [10] for the case $K = K_3$ and $D = 9$, the index of the Dirac operator $\gamma^i$ acting in the transverse space is two, which implies the existence of two families of chiral fermions in $M_4$. Alternatively, if we choose $K = S^4$ and $F$ an $SU(2)$ instanton configuration on $S^4$ then we obtain $\frac{7}{2} t(t + 1)(2t + 1)$ generations of chiral fermions on $M_4$, where $t$ is the $SU(2)$ spin of the fermions [21].

In conclusion, we found new solutions that localize gravity on a brane with non-trivial transverse spaces and gauge instanton backgrounds. It remains to be seen if a realistic theory incorporating fields of the Standard Model can be constructed on these solutions.

**Acknowledgements**

We wish to thank T. Gherghetta for helpful discussions. S.R.-D. is thankful to IPT-UNIL for hospitality. This work was supported by the FNRS, contract no. 21-55560.98.

**References**


Which action for brane worlds?

Rainer Dick a,b

a Department of Physics and Engineering Physics, University of Saskatchewan, 116 Science Place, Saskatoon, SK S7N 5E2, Canada
b Sektion Physik der Ludwig-Maximilians-Universität, Theresienstr. 37, 80333 München, Germany

Received 7 July 2000; accepted 16 August 2000
Editor: M. Cvetic

Abstract

In his pioneering work on singular shells in general relativity, Lanczos had derived jump conditions across energy-momentum carrying hypersurfaces from the Einstein equation with codimension 1 sources. However, on the level of the action, the discontinuity of the connection arising from a codimension 1 energy-momentum source requires to take into account two adjacent space–time regions separated by the hypersurface.

The purpose of the present note is to draw attention to the fact that Lanczos’ jump conditions can be derived from an Einstein action but not from an Einstein–Hilbert action.

1. Introduction

Recently, a particular class of cosmological models commonly denoted as brane-world models attracted a lot of attention. These models are essentially based on two assumptions:

- Our universe may be described as a time-like four-dimensional submanifold \( \Sigma \) of a \((1 + 3 + n)\)-dimensional bulk space–time \( \mathcal{S} \), with \( n \geq 1 \) additional space-like dimensions.
- The additional space-like dimensions can only be probed by gravity and eventually some non-standard matter degrees of freedom, but standard model particles cannot leave our observable universe \( \Sigma \).

Predecessors of this kind of cosmological scenarios rely on dynamical binding mechanisms for low energy matter to an effectively four-dimensional submanifold, which has some finite but small extension in the transverse dimensions. Dynamical mechanisms for explaining such scenarios have been proposed already by Akama [1], by Rubakov and Shaposhnikov [2], by Visser [3], and by Gibbons and Wiltshire [4], and the corresponding transversally “thick” universes have also attracted much attention recently [5], see also [6,7] and references therein.

The other extreme, which was partly motivated from string theory, consists of 3-branes \( \Sigma \) which have no transverse extension at all and are strictly codimension-\( n \) submanifolds [8–14]. Here the confinement of matter to \( \Sigma \) is not necessarily a dynamical phenomenon, but imposed axiomatically through the assumption that matter degrees contribute only a hypersurface integral over \( \Sigma \) to an action \( \mathcal{S} \) which also contains bulk terms for gravity and eventually a few other bulk degrees of freedom. Such an axiomatic distinction between hypersurface and bulk

E-mail address: dick@dansas.usask.ca (R. Dick).

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.
PII: S0370-2693(00)01058-3
degrees of freedom may seem strange at first sight, but a priori there is nothing mathematically inconsistent with it, and so there is no a priori reason to rule out such scenarios. ¹

It has been mentioned already that such (1 + 3)-dimensional submanifolds go by the name 3-branes, but referring to the old literature on singular time-like 3-manifolds in 1 + 3 dimensions (e.g., [32,33]) another appropriate term would be hypersurface layers. Layers denote hypersurfaces with discontinuous extrinsic curvature across the hypersurface due to the presence of energy-momentum on the hypersurface.

The purpose of this note is to draw attention to the fact that an Einstein action instead of an Einstein–Hilbert action does yield the same jump conditions across a hypersurface layer as the Einstein equation.

The following notation is used:

\[ f(x)|_{x=a} = f(a), \quad \int_{a}^{b} f(x) \, dx = f(b) - f(a). \]

It is helpful to start with a toy model from electrodynamics before we address the Einstein–Hilbert action in Section 3.

2. A toy model: Electrodynamics with planar codimension 1 sources

Electrodynamics in 1 + 3 dimensions with codimension 1 sources located on the plane \( x^3 = 0 \) is described by an action

\[ S = -\frac{1}{4} \int dt \int d^2x \int dx^3 F^{\mu\nu} F_{\mu\nu} + \int dt \int d^2x (j \cdot A + j^0 A_0)|_{x^3=0}, \]

where \( d^2x = dx^1 \, dx^2 \) and all vectors are two-dimensional vectors: \( j \cdot A = j^1 A_1 + j^2 A_2 \).

Without much ado we write down the equations of motion which follow from \( \delta S = 0 \):

\[ \partial_\mu F^{\mu\nu} = -j^\nu \delta(x^3), \quad (1) \]

implying, in particular,

\[ \lim_{\epsilon \to 0} \left[ F^{3\nu}|_{x^3=0} \right]^{\epsilon}_{-\epsilon} = -j^\nu |_{x^3=0}. \quad (2) \]

Of course, this can be confirmed from a more careful evaluation of the variation of \( S \):

\[ S = -\frac{1}{4} \lim_{\epsilon \to 0} \left( \int dt \int d^2x \int dx^3 F^{\mu\nu} F_{\mu\nu} + \int dt \int d^2x \int dx^3 F^{\mu\nu} F_{\mu\nu} \right) \]

\[ + \int dt \int d^2x (j \cdot A + j^0 A_0)|_{x^3=0}, \]

whence

\[ \delta S = \lim_{\epsilon \to 0} \left( \int dt \int d^2x \int dx^3 \delta A_\nu \partial_\mu F^{\mu\nu} + \int dt \int d^2x \int dx^3 \delta A_\nu \partial_\mu F^{\mu\nu} \right) \]

\[ - \lim_{\epsilon \to 0} \left( \int dt \int d^2x (\delta A \cdot j + \delta A_0 j^0)|_{x^3=0} \right) \]

¹ Whether or not we find a posteriori reasons is another story, of course. But this can only be clarified by investigations of (post-)Newtonian limits [15–19] and of cosmological implications of these models [11,20–31].
Therefore, (1) and the jump condition (2) indeed imply $\delta S = 0$. However, this does not work with the Einstein–Hilbert action.

3. A first Ansatz for the action in brane models

For simplicity we pretend that we can cover a $(1 + 4)$-dimensional space–time by a single coordinate patch $x^M$ which is Gaussian close to the brane: $x^0 = t$, $x^j$ ($1 \leq j \leq 3$) are tangential to the world-volume of the brane, and $x^5$ is normal: $g_{55} = 0$ on the brane, $0 \leq \mu \leq 3$. It is known that the geodesic distance from the brane provides such a coordinate system locally, whence the brane is localized at $x^5 = 0$. If this coordinate system cannot be extended to all of the five-dimensional space–time (which is what one expects), we have to glue together several patches with appropriate transition functions to formulate action principles. However, the difficulty that we encounter with the Einstein–Hilbert action is related only to the boundary conditions across the brane, and therefore we write the Einstein–Hilbert Ansatz for the brane action as

$$S_{\text{EH}} = \int dt \int d^4x \int dx^5 \sqrt{-g} \left( \frac{m^3}{2} R - \Lambda \right) + \int dt \int d^4x \mathcal{L} \bigg|_{x^5 = 0},$$

where we assume that the brane Lagrangian $\mathcal{L}$ contains no genuine gravitational terms: Derivatives of the metric appear in $\mathcal{L}$ only through covariant derivatives on fermions and eventually massive vector fields.

One might expect an Einstein equation to emerge from (3):

$$R_{MN} - \left( \frac{1}{2} R - \frac{\Lambda}{m^3} \right) g_{MN} = - \frac{2}{m^3 \sqrt{-g}} \frac{\delta (x^5)}{\delta g_{MN}} \frac{\delta \mathcal{L}}{\delta g_{MN}}.$$  \hspace{1cm} (4)

However, a naive derivation of (4) from (3) would have to assume continuity of normal derivatives across the brane, in \textit{a posteriori} contradiction to (4).

To clarify this and to reveal which equations would really follow from stationarity of $S_{\text{EH}}$, we write it more carefully as

$$S_{\text{EH}} = \lim_{\epsilon \to +0} \left( \int dt \int d^4x \int d^5x \sqrt{-g} \left( \frac{m^3}{2} R - \Lambda \right) \right. \hspace{1cm}$$

$$\left. + \int dt \int d^4x \int d^5x \sqrt{-g} \left( \frac{m^3}{2} R - \Lambda \right) \right) + \int dt \int d^4x \mathcal{L} \bigg|_{x^5 = 0}. \hspace{1cm} (5)$$

Variation of the metric then yields

$$\delta S_{\text{EH}} = \frac{m^3}{2} \lim_{\epsilon \to +0} \left( \int dt \int d^4x \int d^5x \sqrt{-g} \delta g^{MN} \left( R_{MN} - \frac{1}{2} g_{MN} R + \frac{\Lambda}{m^3} g_{MN} \right) \right.$$  \hspace{1cm} $$+ \int dt \int d^4x \int d^5x \sqrt{-g} \delta g^{MN} \left( R_{MN} - \frac{1}{2} g_{MN} R + \frac{\Lambda}{m^3} g_{MN} \right) \right)$$

$$\left. + \frac{m^3}{2} \lim_{\epsilon \to +0} \int dt \int d^4x \int d^5x \left[ \sqrt{-g} \left( g^{55} \delta \Gamma^5_{MN} - g^{55} \delta \Gamma^M_{5N} \right) \right] \bigg|_{x^5 = -\epsilon} \right.$$  \hspace{1cm} $$+ \int dt \int d^4x \delta g^{MN} \frac{\delta \mathcal{L}}{\delta g^{MN}} \bigg|_{x^5 = 0}.$$
\[ \frac{m^3}{2} \lim_{\epsilon \to -0} \left( \int dt \int d^3 x \int d^5 \sqrt{-g} \delta g^{MN} \left( R_{MN} - \frac{1}{2} g_{MN} R + \frac{A}{m^3} g_{MN} \right) \right) \]

\[ + \int dt \int d^3 x \int d^5 \sqrt{-g} \delta g^{MN} \left( R_{MN} - \frac{1}{2} g_{MN} R + \frac{A}{m^3} g_{MN} \right) \]

\[ + \frac{m^3}{4} \lim_{\epsilon \to -0} \int dt \int d^3 x \left[ \sqrt{-g} (3 \delta g^{\mu \nu} g^{55} \delta g_{\mu \nu} - 2 \delta g^{\mu S} g^{55} \delta g_{S55} \right. \]

\[ - \delta g^{55} \delta g_{\mu \nu} + 2 \delta g^{55} \delta g_{\mu \nu} \delta g^{\mu \nu} \left] \right|_{x^5 = -\epsilon} + \int dt \int d^3 x \delta g^{MN} \frac{\delta L}{\delta g^{MN}} \bigg|_{x^5 = 0}. \]  

(6)

The jump conditions following from \( \delta S_{EH} = 0 \) are incompatible with the jump conditions following from (4) (see Eq. (9) below). In fact, \( \delta S_{EH} = 0 \) would even require a traceless energy-momentum tensor on the brane if \( \delta L/\delta g^{55} = 0 \).

In an attempt to infer the jump conditions following from (4) from an action principle, we will consider the Einstein action next.

4. An Einstein action for brane models

The Einstein action proves more suitable in boundary models [16], and may also be better adapted to brane models:

\[ S_E = \frac{m^3}{2} \lim_{\epsilon \to -0} \left( \int dt \int d^3 x \int d^5 \sqrt{-g} \left( \frac{m^3}{2} g^{MN} (\Gamma^K_{LM} \Gamma^L_{KN} - \Gamma^K_{KL} \Gamma^L_{MN}) - \Lambda \right) \right) \]

\[ + \int dt \int d^3 x \int d^5 \sqrt{-g} \left( \frac{m^3}{2} g^{MN} (\Gamma^K_{LM} \Gamma^L_{KN} - \Gamma^K_{KL} \Gamma^L_{MN}) - \Lambda \right) \]

\[ + \int dt \int d^3 x \frac{\delta L}{\delta g^{MN}} \bigg|_{x^5 = 0} \]

\[ = S_{EH} - \frac{m^3}{2} \lim_{\epsilon \to -0} \int dt \int d^3 x [\sqrt{-g} (g^{MN} \Gamma^K_{MN} - g^{55} \Gamma^K_{MN})] \bigg|_{x^5 = -\epsilon}, \]  

(7)

with variation under changes of the metric:

\[ \delta S_E = \frac{m^3}{2} \lim_{\epsilon \to -0} \left( \int dt \int d^3 x \int d^5 \sqrt{-g} \delta g^{MN} \left( R_{MN} - \frac{1}{2} g_{MN} R + \frac{A}{m^3} g_{MN} \right) \right) \]

\[ + \int dt \int d^3 x \int d^5 \sqrt{-g} \delta g^{MN} \left( R_{MN} - \frac{1}{2} g_{MN} R + \frac{A}{m^3} g_{MN} \right) \]

\[ - \frac{m^3}{4} \lim_{\epsilon \to -0} \int dt \int d^3 x \left[ 2 \sqrt{-g} \left( \delta g^{MN} \Gamma^K_{MN} - \delta g^{55} \Gamma^K_{MN} \right) \right. \]

\[ - \sqrt{-g} \delta g^{MN} g_{MN} (g^{KL} \Gamma^K_{KL} - g^{55} \Gamma^K_{KL}) \bigg|_{x^5 = -\epsilon} + \int dt \int d^3 x \delta g^{MN} \frac{\delta L}{\delta g^{MN}} \bigg|_{x^5 = 0}. \]
\[
\delta S_E = 0 \quad \text{thus yields a five-dimensional Einstein space in the bulk}
\]

\[
R_{MN} = \frac{2A}{3m^3} \delta g_{MN},
\]

and the five-dimensional analog of the Lanczos equations [32,33]:

\[
\lim_{\epsilon \to 0} \left( \partial_5 g_{\mu\nu} \right)_{x^5 = -\epsilon} = \frac{2}{m^3} \sqrt{\delta S_5} \left( T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} g^{\alpha\beta} T_{\alpha\beta} \right) \bigg|_{x^5 = 0},
\]

i.e. exactly the equations that one infers from the Einstein equation \(^2\) (4). Here the brane energy-momentum tensor is defined via

\[
T_{\mu\nu} = -\frac{2}{\sqrt{-\det(g_{\alpha\beta})}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} \bigg|_{x^5 = 0}.
\]

Another advantage of the Einstein action is the disappearance of \(\delta g^{55}\) on the brane, implying that the Einstein action complies with the usual assumption that the brane Lagrangian \(\mathcal{L}\) depends only on the induced metric on the brane.

5. Conclusion

The jump conditions following from the Einstein equation with brane sources imply stationarity of the Einstein action with brane sources, but not of the Einstein–Hilbert action with brane sources. Furthermore, stationarity of the Einstein action complies with brane Lagrangians \(\mathcal{L}\) which do not depend on the normal component of the metric.

One might be concerned about diffeomorphism invariance since the Einstein action is only invariant under \(IGL(5)\) transformations. However, we have seen that stationarity of the Einstein action is equivalent to the fully covariant Einstein equation (4) (remembering that \(x^5\) is a geodesic distance, i.e. a well-defined geometric object). Therefore, besides the numerical value of the action itself, no classical results inferred from the use of an Einstein action depend on the coordinate system.

\(^2\) A further equation on the brane appears if the brane is a boundary of space–time [16]: In this case the term \(\sim \delta g^{55}\) cannot be cancelled by continuity across the brane and requires

\[
\left. g^{55} \partial_5 g^{55} \right|_{x^5 = 0} = \left. g^{\alpha\beta} \partial_5 g_{\alpha\beta} \right|_{x^5 = 0} = \left. \delta \mathcal{L} \right|_{x^5 = 0}.
\]

i.e. \(g^{55} \sim -\det(g_{\alpha\beta})\) on a boundary.
Note added

An important reference on early investigations in brane cosmology is Chamblin and Reall [34]. These authors had also already recognized the difficulty with the Einstein action for thin branes and added a Gibbons–Hawking term to cure the problem.

References

Abstract

We investigate the role of quantum fluctuations in the system composed of two branes bounding a region of AdS. It is shown that the modulus effective potential generated by quantum fluctuations of both brane and bulk fields is incapable of stabilizing the space naturally at the separation needed to generate the hierarchy. Consequently, a classical stabilization mechanism is required. We describe the proper method of regulating the loop integrals and show that, for large brane separation, the quantum effects are power suppressed and therefore have negligible affects on the bulk dynamics once a classical stabilization mechanism is in place.

Recently, Randall and Sundrum [1] proposed a novel mechanism for addressing the hierarchy problem. In their model, the Standard Model fields are confined to one of two 3-branes which are endpoints of an $S^1/Z_2$ orbifold spatial dimension. A negative bulk cosmological constant generates an AdS metric in the five-dimensional spacetime

$$ds^2 = e^{-2kr_c(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2,$$  \hspace{1cm} (1)

where $k$ is a parameter of order the Planck scale which is related to the AdS radius of curvature and $r_c$ determines the length of the orbifold. The coordinate $\phi \in [-\pi, \pi]$ parameterizes the fifth dimension, with the point $(x, \phi)$ and $(x, -\phi)$ identified, and the two 3-branes reside at the orbifold fixed points $\phi = 0, \pi$. Because of the exponential factor in Eq. (1), a field with Lagrangian mass parameter $m_0$ that is confined to the brane at $\phi = \pi$ (the “TeV brane”) will have a physical mass $m = m_0 e^{-kr_c\pi}$. If all mass scales in the theory are of order the Planck scale and $kr_c \sim 12$, then the observed mass $m$ is in the TeV range. In this way, the hierarchy between the TeV and the Planck scale is generated purely through gravitational effects.

The Randall–Sundrum (RS) scenario contains a modulus field that determines the size of the $S^1/Z_2$ orbifold extra dimension. This scalar arises as one of the massless fluctuations about the background AdS geometry, and it is encoded in the five-dimensional metric

$$ds^2 = e^{-2k|\phi|T(x)} g_{\mu\nu}(x) dx^\mu dx^\nu - T^2(x) d\phi^2,$$  \hspace{1cm} (2)

where the field $g_{\mu\nu}$ is the four-dimensional graviton and $T(x)$ is the modulus, or “radion” field whose VEV $r_c = \langle T \rangle$ determines the length of the orbifold according to Eq. (1). Dimensional reduction of the
five-dimensional Einstein–Hilbert action for Eq. (2) leads to an effective action for the massless fields [2,3]

\[
S = \frac{2M^3}{k} \int d^4x \sqrt{-g} (1 - (\varphi/f)^2) R + \frac{1}{2} \int d^4x \sqrt{-g} \partial_\mu \varphi \partial^\mu \varphi, \tag{3}
\]

where \( R \) is the Ricci scalar constructed from \( g_{\mu\nu} \) and we have defined \( \varphi = f \exp(-k_\pi T) \) with \( f = \sqrt{24M^3/k} \).

To account for the observed discrepancy between the gravitational and electroweak scales, we need \( k_{r_c} \sim 12 \). However, there is nothing in Eq. (3) that stabilizes the VEV of the radion \( \varphi \). Some additional dynamics must be introduced to make \( k_{r_c} \sim 12 \) without fine tuning parameters. As suggested in [4], introducing a bulk scalar with appropriate interaction terms on the branes can induce a radion potential that has an acceptable minimum without severe tuning of the model parameters. Although in [4] the scalar profile was treated as a perturbation on the background metric, its back reaction on the spacetime geometry can be included [5] without changing the qualitative features of the stabilization mechanism.

The analysis of Refs. [4,5] was purely at the classical level. However, quantum fluctuations of fields which propagate in the bulk or on the TeV brane will also generate contributions to the effective radion potential.\(^1\) In this paper we explore the possibility that it is these quantum corrections which stabilize the radion. We calculate the effective potential arising from bulk fields as well as fields confined to the TeV brane. For the confined fields we calculate using three different regulators, and show clearly that the effective cut-off on the brane is indeed of order TeV. After proper regularization, the sole effect of the brane field fluctuations is the renormalization of the brane tension.

\(^1\) In fact, the original RS solution necessitates one fine tuning of parameters to keep the radion potential flat and the resulting metric static. This fine tuning problem goes away in the presence of some additional radion stabilization dynamics. There is a second fine tuning, related to the cosmological constant problem, about which the RS solution has nothing to say.

\(^2\) Fields on the Planck brane, at \( \varphi = 0 \), do not couple directly to the radion. We expect that their contribution to the one-loop effective potential is suppressed relative to the sources mentioned above.

1. Quantum corrections to the radion potential

First, we consider the contribution to the radion potential coming from a field on the TeV brane. We will show that the fluctuations of fields on the brane serve only to renormalize the brane tension. Given the subtlety in the regularization procedure we will calculate using three different regulators.

For concreteness, we will take a scalar field theory confined to the TeV brane. First we shall compute the effective potential using dimensional regularization. In \( n = 4 - \epsilon \) dimensions, the action is given by

\[
S = \frac{1}{2} \int d^n x a^n \left( \frac{1}{a^2} (\partial h)^2 - m_0^2 h^2 \right), \tag{4}
\]

where \( m_0 \) is of order the Planck scale, and the powers of \( a \) multiplying the kinetic and mass terms come from the induced metric on the brane. To compute the radion potential, take \( a \) constant and rescale \( h \rightarrow ah \). The rescaled field has a canonically normalized kinetic term and an effective mass \( m = am_0 \). The effective potential obtained from integrating out \( h \) can be trivially expressed as the zero point energy in the presence of a constant \( \varphi \) field configuration:

\[
V = \frac{1}{2} \mu^{4-n} \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \sqrt{k^2 + a^2m_0^2}, \tag{5}
\]

with \( \mu \) an arbitrary mass scale which has been introduced to keep \( V \) a four-dimensional energy density. The resulting expression,

\[
V = -\frac{1}{2} \mu^{4-n} \frac{\Gamma \left(-\frac{n}{2}\right)}{(4\pi)^{n/2}} (m_0 a)^n, \tag{6}
\]

contains a divergent piece that must be absorbed into a local counterterm. Such a counterterm is provided by the brane tension on the TeV brane

\[
S_{ct} = -\int d^n x a^n \delta V \mu^{n-4}, \tag{7}
\]
which is generally covariant in \( n \) dimensions. Comparing this with our result, we see that \( V \) is in fact pure counterterm: the effect of the scalar \( h \) is simply to renormalize the brane tension. Given a bare mass of order the Planck scale (this is the appropriate choice for our set of coordinates), there are no large logs for \( \mu \approx M_{\text{Pl}} \).

An alternative way of understanding this result is to regulate the divergent integral using a physical (coordinate invariant) cutoff \( \Lambda \). The vacuum energy for \( h \) is then, for \( \Lambda \gg m_0 \)

\[
V = \frac{1}{2} \int_0^{\Lambda a} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + a^2m_0^2} \]

\[
= \frac{a^4}{32\pi^2} \left[ 2A^4 + 2\Lambda^2 m_0^2 + m_0^4 \ln \left( \frac{m_0}{2A} \right) \right],
\]

(8)

which simply induces a shift in the brane tension. Note that the coordinate cutoff on the momentum integral is rescaled by a factor of \( a \) with respect to the physical cutoff. Had we used an \( a \)-independent cutoff on the momentum integral, we would have generated terms in the effective action proportional to \( (\Lambda m_0 a)^2 \). On the other hand, the rescaled cutoff yields results that are consistent with four-dimensional general covariance on the brane, and which are in agreement with dimensional regularization.

The same conclusion can be reached by using a Pauli-Villars regulator. To get a consistent result, the regulator fields must couple to the induced metric on the TeV brane in the same way as our scalar field. Performing the calculations in two dimensions for simplicity, we make the subtraction

\[
V \to V - \frac{1}{2(M_1^2 - M_2^2)} \int \frac{d^2k}{(2\pi)^2} \]

\[
\times \left[ \left( m^2 - M_2^2 \right) \sqrt{k^2 + M_1^2} \right. \]

\[
+ \left. \left( M_1^2 - m^2 \right) \sqrt{k^2 + M_2^2} \right].
\]

(9)

Where all the masses, including the regulator masses get rescaled by the warp factor. Performing the momentum integral, it is easily seen that all logarithmic dependence on \( a \) cancels from the regulated expression. The remaining dependence on \( a \) is a pure counterterm.

The quantum fluctuations of bulk fields also contribute to the radion effective potential. Decomposing the bulk field into four-dimensional Kaluza–Klein modes, the potential can again be expressed as a sum over zero point energies

\[
V = (-1)^F \frac{g}{2} \sum_n \int \frac{d^3-x}{(2\pi)^3} \sqrt{k^2 + m_n^2},
\]

(10)

where \( F = 0, 1 \) for bosons and fermions respectively, and \( g \) is the number of physical polarizations of the Kaluza–Klein modes. In this equation, the dependence on \( a \) enters through the Kaluza–Klein masses \( m_n \). Defining \( m_n = ak x_n \), the above becomes

\[
V = (-1)^{F+1} \frac{g^4 a^4}{32\pi^2} \left( \frac{k^2 a^2}{4\pi \mu^2} \right)^{-\epsilon/2}
\]

\[
\times \Gamma(-2 + \epsilon/2) \sum_n x_n^{4-\epsilon}.
\]

(11)

We now evaluate Eq. (11) for a bulk scalar field with action

\[
S_b = \frac{1}{2} \int d^4x \int d\phi \sqrt{G}
\]

\[
\times \left( G^{AB} \partial_A \Phi \partial_B \Phi - \left( m^2 + \alpha \sigma'' \right) \phi^2 \right),
\]

(12)

where \( G_{AB} \) with \( A, B = \mu, \phi \) is given by Eq. (1). Because \( \sigma'' = 2kr \beta (\phi - \beta (\phi - \pi)) \), the parameter \( \alpha \) controls a possible mass term on the boundaries of the space. Such mass terms arise if the field \( \Phi \) is a component of a supermultiplet on AdS5 with one dimension compactified on an \( S^1/Z_2 \) orbifold (see [8]). It is found in [8] that the roots \( x_n \) satisfy

\[
j_v(x_n) y_v(\alpha x_n) - j_v(\alpha x_n) y_v(x_n) = 0,
\]

(13)

where \( v = \sqrt{4 + m^2/k^2} \), \( j_v(z) = (2 - \alpha) J_v(z) + z J'_v(z) \), and \( y_v \) is given by the same expression with \( Y_v \), replacing \( J_v \). (See [8,9] for other work on the Kaluza–Klein reduction of bulk fields.) The \( a \) dependence from the sum over \( x_n \) in Eq. (11), can be calculated by zeta function regularization techniques [10], which we now review.

First, convert the sum into a contour integral

\[
\sum_n \frac{x_n^{-s}}{2\pi i} \int_{C} dz z^{-s-1} \ln \left[ j_v(z) y_v(az) - j_v(az) y_v(z) \right],
\]

(14)

which is valid for \( \text{Re} \ s > 1 \). In this equation, \( C \) is a contour between arcs of radius \( \delta \) (chosen to avoid
a possible pole at \( z = 0 \) and \( R \to \infty \) which circles the roots \( x_n \) in a counterclockwise manner. Our goal is to perform the analytic continuation of the RHS of Eq. (14) to a neighborhood of \( s = -4 \). To do this, split the contour into \( C_+ \) and \( C_- \), its portions above and below the real axis respectively. On each contour, the asymptotic expansion of the argument of the logarithm is

\[
Z_v(z, a) \equiv j_v(z) \gamma_v(z a) - j_v(a z) \gamma_v(z)
\]

\[
\sim \mp \frac{i}{\pi} \frac{e^{\pm i z(1-a)}}{z} \ln \left[ 1 + \mathcal{O}(1/z) \right]. \tag{15}
\]

We now add and subtract the logarithm of the RHS of this expression to the contour integral above, which yields

\[
\sum_n \chi_n^{-s} = \frac{s}{2 \pi i} \sum_{C_+} \int \frac{dz}{z} z^{-s-1}
\]

\[
\times \ln \left[ \pm \frac{i}{\pi} \frac{e^{\pm i z(1-a)}}{z} Z_v(z, a) \right]
\]

\[
- \frac{s}{2 \pi i} \sum_{C^-} \int \frac{dz}{z} z^{-s-1}
\]

\[
\times \ln \left[ \pm \frac{i}{\pi} \frac{e^{\pm i z(1-a)}}{z} \right]. \tag{16}
\]

The first line is now defined for \( \text{Re} \, s > -1 \), while the second is still only defined for \( \text{Re} \, s > 1 \). However, for the second term in Eq. (16), we are free to deform the contour \( C \) into a straight line running parallel to the imaginary axis from \( z = i \infty + \delta \) to \( z = -i \infty + \delta \). The result is

\[
\sum_n \chi_n^{-s} = \frac{s}{2 \pi i} \sum_{C_+} \int \frac{dz}{z} z^{-s-1}
\]

\[
\times \ln \left[ \pm \frac{i}{\pi} \frac{e^{\pm i z(1-a)}}{z} \right] Z_v(z, a)
\]

\[- \frac{s}{\pi} \sum_{C^-} \int \frac{dz}{z} z^{-s-1}
\]

\[
\times \ln \left[ \frac{2(1-a)}{1-s} \delta^{-1} + \frac{\pi}{2s} \delta^{-s} \right]. \tag{17}
\]

Since the second line of this equation provides its own analytic continuation, we can now extend the definition of the sum on the LHS to \(-1 < \text{Re} \, s < 0\). In this region, it is safe to take the limit \( \delta \to 0 \). Then second term above vanishes. To evaluate the piece left over, we can take the straight line contour along the imaginary axis. The result, valid for \(-1 < \text{Re} \, s < 0\), is

\[
\sum_n \chi_n^{-s} = \frac{s}{\pi} \sin \left( \frac{\pi s}{2} \right) \int_0^\infty dt t^{3-s-1}
\]

\[
\times \ln \left[ \frac{2 \sqrt{a}}{t} e^{-t(1-a)} \right] \left( k_v(t) i_v(at) - k_v(at) i_v(t) \right), \tag{18}\]

where \( i_v(t) = (2 - a) I_v(t) + t I'_v(t) \) and \( k_v(t) \) is defined in the same way with \( K_v(t) \) instead of \( I_v(t) \).

Eq. (18) still needs to be extended to a neighborhood of \( s = -4 \). For \( s = -4 + \epsilon \), it can be written as

\[
\sum_n \chi_n^{-s} = -2 \epsilon \int_0^\infty dt t^{3+\epsilon} \ln \left[ \left( k_v(t) i_v(at) - k_v(at) i_v(t) \right) \right]
\]

\[
\times \ln \left( \frac{2 \sqrt{a}}{t} e^{-t(1-a)} \right)
\]

\[
+ \int_0^\infty dt t^{3+\epsilon} \ln \left( \frac{2 \sqrt{a}}{t} e^{-t(1-a)} \right)
\]

\[
+ \frac{1}{a^{4-\epsilon}} \int_0^\infty dt t^{3+\epsilon} \ln \left( \frac{2 \sqrt{a}}{t} e^{-t(1-a)} \right). \tag{19}\]

Because of the overall factor of \( a^{4-\epsilon} \) in Eq. (11), the second two terms in this expression yield contributions that go as \( a^{4-\epsilon} \) or independent of \( a \) respectively. The term that is independent of \( a \) can be absorbed into the renormalization of the Planck brane tension. As we discussed in the case of a TeV brane fields, \( a^{4-\epsilon} \) can also be cancelled by a local counterterm. The first term in the brackets is well defined at \( s = -4 \). Taking the limit \( \epsilon \to 0 \), we end up with

\[
V = V_h + V_e a^4 + \frac{k^4 a^4}{16 \pi^2} \int_0^\infty dt t^{3} \ln \left[ 1 - \frac{k_v(t) i_v(at)}{k_v(at) i_v(t)} \right]. \tag{20}\]

where \( V_{h,e} \) are shifts in the brane tensions. For \( a \ll 1 \), the \( a \) dependence in the above equation is

\[
\int_0^\infty dt t^{3} \ln \left[ 1 - \frac{k_v(t) i_v(at)}{k_v(at) i_v(t)} \right]
\]

\[
= \frac{2}{\Gamma^2(v)} \left( \frac{v - \alpha + 2}{\alpha + v - 2} \right) \left( \frac{a}{2} \right)^{2v}
\]
\begin{align}
\times \int_0^\infty dt t^{2v+3} \frac{k_v(t)}{i_v(t)} + \mathcal{O}(a^{2v+2})
\end{align}
\tag{21}

if \alpha + \nu \neq 2. For \alpha + \nu = 2
\begin{align}
\int_0^\infty dt t^3 \ln \left[ 1 - \frac{k_v(t)}{k_v(at)} \right] 
= \frac{2(v-1)}{\Gamma(v)^2} \left( \frac{a}{2} \right)^{2v-2} \int_0^\infty dt t^{2v+1} \frac{k_v(t)}{i_v(t)} + \cdots,
\end{align}
\tag{22}

with terms of order \(a^{2v}\) for \(v \neq 2\) and \(a^4 \ln a\) for \(v = 2\) not shown. Incidentally, the eigenvalues \(\lambda_n\) for bulk fields of higher integer spin satisfy equations that are identical to that of the bulk scalar except for the values of \(v\) and \(\alpha\) [8]. It follows immediately that in those cases the \(\alpha\) dependence is similar to that in Eq. (20). Furthermore, we can use the scalar result to calculate the effective potential in these cases as well. For instance, the contribution from a bulk \(U(1)\) gauge field can be obtained by taking \(v = 1\) and \(\alpha = 1\). To calculate the effective potential due to metric fluctuations, decompose the metric into a sum of four-dimensional scalar, vector, and transverse traceless modes. The metric scalar and vector contributions are as described above, while the transverse traceless piece generates a term like Eq. (20) with \(v = 2, \alpha = 0\).

In addition to contributions from fields in the bulk and on the TeV brane, the vacuum energy receives corrections from loops of the radion itself. These can be computed from the effective four-dimensional Lagrangian
\begin{align}
\mathcal{L} = \frac{f^2}{2} (\partial a)^2 - \delta V \cdot a^4,
\end{align}
\tag{23}

where \(\delta V\) is a small classical shift in the TeV brane tension relative to the value which generates the background metric. The one-loop effective potential generated by the radion is
\begin{align}
V = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2 a^2} 
= \frac{a^4}{32\pi^2} \left[ 2\Lambda^4 + \Lambda^2 \bar{m}^2 + \bar{m}^4 \ln \left( \frac{\bar{m}}{2\Lambda} \right) \right],
\end{align}
\tag{24}

where \(\bar{m}^2 = 12\delta V \cdot / f^2\). Note that as in the case of TeV brane fields, we have used an \(a\)-dependent cut-off on the momentum integral. It is not immediately clear that this is the correct cut-off to use in the dimensionally reduced theory, which provides an effective description of the physics at energy scales for which the fifth dimension cannot be resolved. However, had we not used the rescaled cut-off, we would have obtained cut-off dependent terms that are proportional to \(a^2\) in the effective potential. No counterterm exists to absorb such terms. On the other hand, the rescaled cut-off yields \(V \propto a^4\), which is a pure counterterm that can be absorbed into the TeV brane tension.

We can also see this result using dimensional regularization. The dimensionally reduced theory becomes
\begin{align}
\mathcal{L} = \frac{f_n^\nu}{2} (\partial \bar{a})^2 - \delta V_n \mu^{n-4} \bar{a}^4,
\end{align}
\tag{25}

where \(f_n\) has dimensions of mass. Introducing a canonically normalized radion field \(\varphi = (f_n a)^{(n-2)/2}\), the vacuum energy scales as
\begin{align}
V \sim \left( \frac{\delta V_n \mu^{n-4}}{f_n^{n-2}} \right)^{n/2} \varphi^{n-2} = \left( \frac{\delta V_n \mu^{n-4}}{f_n^{n-2}} \right)^{n/2} a^n,
\end{align}
\tag{26}

which simply renormalizes the TeV brane tension. Finally, one could also use a Pauli–Villars regulator. To avoid cutoff-dependent terms that cannot be absorbed into counterterms, the regulator masses should scale with \(a\) in the same way as masses on the TeV brane. In this case, the resulting effective potential is proportional to \(a^4\) in agreement with the two other methods described here.

2. Conclusion

In this paper we have calculated the quantum effective potential for the radion in compactified AdS5. By explicitly performing the computation using three different regulators, we have shown that fields confined to the TeV brane give no non-trivial contributions to the potential. In particular, we find that in dimensional regularization, the disappearance of any contribution that scales as \(a^4 \ln a\) is not due to a rescaling of the regulator mass \(\mu\) by a factor of \(a\). Instead, it can be traced to the fact the proper generally covariant counterterm in \(n = 4 - \epsilon\) dimensions includes this term. Likewise, general covariance requires that within a cutoff regularization procedure, one should use the rescaled cutoff \(f_n a\), leading to \(V \propto a^4\).
The contribution due to bulk fields yields a non-trivial dependence on the warp factor $a$. However, as in the case of confined fields, no terms of the form $a^4 \ln a$ are generated. Beyond the pure counterterm $a^6$, bulk fields generate terms that are suppressed in the large $r_c$ regime. For instance, a massless bulk field yields terms of the form $a^6$ as well as the finite log term $a^8 \ln a$. This $a$ dependence is too weak to generate an exponentially small value of $a$ without having to choose unnatural values of the brane tensions. Because of this, a classical stabilization mechanism is needed. Our results for the bulk fields disagree with the results of [7], who finds enhanced power dependence on $a$, but agree with those of [6]. However, our method and interpretations seem to differ from this reference. We have shown how to properly calculate the vacuum energy in the effective theory by first dimensionally reducing and then summing over modes.

Finally, we have also included the quantum effects of the radion field itself. We found that as in the case of TeV brane fields, the correct momentum space cutoff should be rescaled by a factor of $a$. While this is quite natural for brane fields it is not obvious that this had to be so for the radion field, since in the dimensionally reduced theory we have integrated over the fifth dimension, and there is no single preferred “scale” $\exp(-k r_c \phi)$.

**Acknowledgements**

The authors benefited from conversations with P. Horava, K. Intrilligator and H. Ooguri. We thank M. Wise for helping us find an error on an earlier version of this paper. This work was supported in part by the Department of Energy under grant numbers DE-FG03-92-ER 40701 and DOE-ER-40682-143.

**References**

Noncommutative quantum gravity

J.W. Moffat

Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

Received 31 July 2000; received in revised form 22 August 2000; accepted 31 August 2000

Editor: M. Cveti

Abstract

The possible role of gravity in a noncommutative geometry is investigated. Due to the Moyal $*$-product of fields in noncommutative geometry, it is necessary to complexify the metric tensor of gravity. We first consider the possibility of a complex Hermitian, nonsymmetric $g_{\mu \nu}$ and discuss the problems associated with such a theory. We then introduce a complex symmetric (non-Hermitian) metric, with the associated complex connection and curvature, as the basis of a noncommutative spacetime geometry. The spacetime coordinates are in general complex and the group of local gauge transformations is associated with the complex group of Lorentz transformations $CSO(3,1)$. A real action is chosen to obtain a consistent set of field equations. A Weyl quantization of the metric associated with the algebra of noncommuting coordinates is employed.

1. Introduction

The concept of a quantized spacetime was proposed by Snyder [1], and has received much attention over the past few years [2–6]. There has been renewed interest recently in noncommutative field theory, since it makes its appearance in string theory, e.g., noncommutative gauge theories describe the low energy excitations of open strings on D-branes in a background two-form $B$ field [5,7–12]. Noncommutative Minkowski space is defined in terms of spacetime coordinates $x^\mu$, $\mu = 0, \ldots, 3$, which satisfy the following commutation relations

$$[x^\mu, x^\nu] = i \theta^{\mu \nu},$$

where $\theta^{\mu \nu}$ is an antisymmetric tensor. In what follows, we can generally extend the results to higher dimensions $\mu = 0, \ldots, d$.

An associative noncommuting algebra, $M^4_d$, is constructed with elements given by ordinary continuous functions on $M^4$ and with a deformed product of functions given by the Moyal bracket or $*$-product of functions [13]

$$f(x) * g(x) = \exp \left[ \frac{i}{2} \theta^{\mu \nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} \right] f(x + \alpha) g(x + \beta) |_{\alpha = 0, \beta = 0}$$

$$= fg + \frac{i}{2} \theta^{\mu \nu} \partial_\mu f \partial_\nu g + O(\theta^2).$$

Conventional field theories are generalized to noncommutative spacetime by replacing the usual product of fields by the Moyal bracket or $*$-product.

Since the $*$-product of fields involves an infinite number of derivatives, the resulting field theories are nonlocal. We know that self-consistent nonlocal quantum field theories exist, which are finite, gauge invariant, and unitary to all orders of perturbation theory [19]. The lack of commutativity of the spacetime
coordinates gives rise to a spacetime uncertainty relation
\[ \Delta x^\mu \Delta x^\nu \geq \frac{1}{2} |\theta^{\mu\nu}|. \]  
\[ (3) \]

It appears that in a perturbative context, the noncommutative theories have a unitary S-matrix for space–space noncommutativity, \( \theta^{ij} = 0 \), while the S-matrix is not unitary for spacetime noncommutativity, \( \theta^{ij} = 0 \) [14]. Moreover, there are indications that conventional renormalizable field theories remain renormalizable, when generalized to noncommutative spacetimes [15,16].

The product of two operators \( \hat{f} \) and \( \hat{g} \) can be defined and can be shown to lead to the Moyal \( \ast \)-product [17]
\[ f(x) \ast g(x) = \frac{1}{(2\pi)^4} \int d^4k d^4p \]
\[ \times \exp \left[ i(k_\mu + p_\mu)x^\mu - \frac{i}{2} k_{\mu\nu} p_{\nu} \right] \]
\[ \times \hat{f}(k) \hat{g}(p) \]
\[ = \exp \left[ i \frac{\partial}{\partial x^\mu} \theta^{\mu\nu} \frac{\partial}{\partial y^\nu} \right] \]
\[ \times f(x)g(y)|_{y \to x}, \]  
\[ (4) \]
where \( \hat{f}(k) \) is the Fourier transform
\[ \hat{f}(k) = \frac{1}{(2\pi)^4} \int d^4x \exp(-ik_\sigma x^\sigma) f(x). \]  
\[ (5) \]

The noncommutative Yang–Mills action is defined by
\[ S_{\text{YM}} = -\frac{1}{4} \int d^4x F^{a\mu\nu} \ast F^{a\mu\nu}, \]  
\[ (6) \]
where
\[ F^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ie^{abc} A^b_\mu \ast A^c_\nu. \]  
\[ (7) \]
The action is invariant under the gauge transformations
\[ A^a_\mu \rightarrow [U \ast A^a_\mu \ast U^{-1} - \partial_\mu U \ast U^{-1}]^a, \]  
\[ (8) \]
where
\[ U \ast U^{-1} = U^{-1} \ast U = 1. \]  
\[ (9) \]

The definition (7) of \( F^{a\mu\nu} \) requires the gauge fields to be complex and these fields should be invariant under the transformations of a general group of noncommutative gauge transformations NCU(3, 1). When we contemplate a noncommutative extension of spacetime, we do not appear to be able to proceed with the quantization of fields as in commutative quantum field theory, in which the classical fields are real for neutral charged fields and complex for charged fields. Moreover, the classical field variables are treated in the quantization procedure as operators in a Hilbert space, and they satisfy commutation or anticommutation relations. Within this standard scenario, spacetime plays a passive role as far as quantization is concerned.

Let us now consider the role of gravity in a noncommutative spacetime. We can employ the Weyl quantization procedure [17,18] to associate the metric tensor operator \( \hat{g}_{\mu\nu} \) with the classical metric \( g_{\mu\nu} \). This prescription can be used to associate an element of the noncommutative algebraic structure \( M_x \), which defines a noncommutative space in terms of the coordinate operators \( \hat{x} \). Using the Fourier transform of the metric
\[ \hat{g}_{\mu\nu}(k) = \frac{1}{(2\pi)^2} \int d^4x \exp(-ik_\sigma x^\sigma) g_{\mu\nu}(x), \]  
\[ (10) \]
we can define the metric operator
\[ \hat{g}_{\mu\nu}(\hat{x}) = \frac{1}{(2\pi)^2} \int d^4k \exp(ik_\sigma \hat{x}^\sigma) \hat{g}_{\mu\nu}(k). \]  
\[ (11) \]
Thus, the operators \( \hat{g}_{\mu\nu} \) and \( \hat{x} \) replace the variables \( g_{\mu\nu} \) and \( x \). If the \( \hat{x} \) have complex symmetry properties, then the \( \hat{g}_{\mu\nu} \) will inherit these properties for a classical metric \( g_{\mu\nu} \).

When we turn to the geometry of spacetime in the presence of matter, the situation with regards to noncommutativity of spacetime coordinates is much more problematic. In this case, it seems unlikely that we can retain our conventional notions of a real pseudo-Riemannian space within a real manifold. If we define the metric of spacetime in terms of vierbeins \( v^a_\mu \) using the Moyal \( \ast \)-product
\[ g_{\mu\nu} = v^a_\mu \ast v^b_\nu \eta_{ab}, \]  
\[ (12) \]
where \( a, b = 0, \ldots, 3 \) denote the flat, fiber bundle tangent space (anholonomic) coordinates, and \( \eta_{ab} \) denotes the Minkowski metric; \( \eta_{ab} = \text{diag}(1, -1, -1, -1) \), then the metric of spacetime is forced to be complex.

We shall investigate two possibilities for complex gravity. First we consider the possibility that the fundamental tensor \( g_{\mu\nu} \) is complex Hermitian satisfying
$g^\dagger_{\mu\nu} = g_{\mu\nu}$, where $\dagger$ denotes the Hermitian conjugate. This was recently proposed by Chamseddine [20,21], as a possible way to complexify the gravitational metric. The second possibility we shall consider is a complex pseudo-Riemannian geometry, based on a complex symmetric (non-Hermitean) tensor $g_{\mu\nu}$ [22–24].

2. Nonsymmetric gravity

The nonsymmetric field extension of Einstein gravity has a long history. It was originally proposed by Einstein, as a unified field theory of gravity and electromagnetism [25,26]. But it was soon realized that the antisymmetric part $g_{\mu\nu}^T$ in the decomposition

$$g_{\mu\nu} = g_{\mu\nu} + g_{[\mu\nu]}$$

(13)

could not describe physically the electromagnetic field. It was then suggested that the nonsymmetric field structure describes a generalization of Einstein gravity, known in the literature as the nonsymmetric gravitational theory (NGT) [27–32].

NGT faces some difficulties which have their origin in the lack of a clear-cut gauge symmetry in the antisymmetric sector of the theory. Even in the linear approximation, the antisymmetric field equations are not invariant under the transformation

$$g^\prime_{\mu\nu} = g_{\mu\nu} + \partial_\lambda \lambda_\mu - \partial_\mu \lambda_\nu,$$

(14)

where $\lambda_\mu$ is an arbitrary vector field. This lack of overall gauge symmetry in the antisymmetric sector of the theory gives rise to two basic problems [33–35]. In the weak antisymmetric field approximation, an expansion about a classical general relativity (GR) background, reveals that there are ghost poles, tachyons and higher-order poles associated with the asymptotic boundary conditions [33]. However, this problem can be resolved by a careful choice of the NGT action [31,32]:

$$S_{NGT} = \int d^4 x \sqrt{-g} \left[ \right.$$

$$\left. g^{\mu\nu} R_{\mu\nu}(W) - 2\lambda - \frac{1}{4} \mu^2 g^{\mu\nu} g_{[\nu\mu]} \
- \frac{1}{6} g^{\mu\nu} W_\mu W_\nu \right].$$

(15)

Here, we choose units so that $G = c = 1$, $g = \text{Det}(g_{\mu\nu})$, $\lambda$ is the cosmological constant, $\mu$ is a mass associated with the skew field $g_{[\mu\nu]}$ and $R_{\mu\nu}(W)$ is the contracted curvature tensor:

$$R_{\mu\nu}(W) = \partial_\beta W^\beta_{\mu\nu} - \frac{1}{2} \left( \partial_\alpha W^\alpha_{\mu\beta} + \partial_\beta W^\beta_{\nu\alpha} \right) - W^\beta_{\alpha\beta} W_{\mu\nu} + W^\beta_{\nu\beta} W_{\mu\alpha},$$

(16)

defined in terms of the unconstrained nonsymmetric connection

$$W^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \frac{2}{3} \delta^\lambda_{\mu} W_\nu,$$

(17)

where

$$W_\mu = \frac{1}{2} (W^\lambda_{\mu\lambda} - W^\lambda_{\lambda\mu}).$$

(18)

Eq. (17) leads to the result

$$\Gamma^\lambda_{[\mu\lambda]} = 0.$$

(19)

A nonsymmetric matter source is added to the action

$$S_M = -8\pi \int d^4 x \sqrt{-g} T_{\mu\nu},$$

(20)

where $T_{\mu\nu}$ is a nonsymmetric source tensor.

This action will lead to a physically consistent Lagrangian and field equations for the antisymmetric field in the linear, weak field approximation, with field equations of the form of a massive Proca-type theory which is free of tachyons, ghost poles and higher-order poles [31,32].

However, the problems do not end here with this form of NGT. A Hamiltonian constraint analysis performed on NGT by Clayton, showed that when the NGT field equations are expanded about a classical GR background, the resulting theory is unstable [34,35]. This problem appears to be a generic feature of any fully geometrical NGT-type of theory, including the vierbein derivation of NGT based on the Hermitian fundamental tensor [20,29,30]

$$g_{\mu\nu} = e^a_{\mu} e^b_{\nu} \eta_{ab},$$

(21)

where $e^a_{\mu}$ is a complex vierbein. Basically, this result means that we cannot consider GR as a sensible limit of NGT for weak antisymmetric fields.

Another serious problem with the complex version of NGT, in which $g_{[\mu\nu]} = i f_{[\mu\nu]}$ and $f_{[\mu\nu]}$ is a real antisymmetric tensor, is that the linear, weak field approximation to the NGT field equations produces generic, negative energy ghost poles. This prompted a proposal
that the nonsymmetric vierbeins be described by hyperbolic complex variables [29,30]. For a sesquilinear, hyperbolic complex \( g_{\mu
u} \), there exists a local \( GL(4,R) \) gauge symmetry, which corresponds to \( g_{\mu\nu} \) preserving rotations of generalized linear frames in the tangent bundle. This symmetry should not be confused with the linear (global) subgroup \( GL(4) \) of the diffeomorphism group of the manifold \( M^4 \) under which NGT is also invariant. The hyperbolic complex vierbeins are defined by

\[
e^a_\mu = \Re(e^a_\mu) + \omega \Im(e^a_\mu),
\]

(22)

while the \( g_{\mu\nu} \) is given by

\[
g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab} = e^a_\mu e^a_\nu, \]

(23)

where \( \omega^2 = +1 \) is the pure imaginary element of the hyperbolic complex Clifford algebra \( \Omega \) [36]. The \( g_{\mu\nu} \) and the connexion \( \Gamma^a_{\mu\nu} \) are hyperbolic complex Hermitian in \( \mu \) and \( \nu \), while the spin connection \( (\Omega_\sigma)_{ab} \) is hyperbolic complex skew-Hermitian in \( a \) and \( b \). The hyperbolic complex unitary group \( U(3,1,\Omega) \) is isomorphic to \( GL(4,R) \). The spin connection \( (\Omega_\sigma)_{ab} \) is invariant under the \( GL(4) \) transformations provided

\[
(\Omega_\sigma)_{ab} \rightarrow \left[U_G \Omega_\sigma U_G^{-1} - (\partial_x U_G)U_G^{-1}\right]_{ab},
\]

(24)

where \( U_G \) is an element of the unitary group \( U(3,1,\Omega) \). The curvature tensor \( (R_{\mu\nu})^a_b \) is invariant under the \( GL(4) \) transformations when

\[
(R_{\mu\nu})^a_b \rightarrow U^a_G (R_{\mu\nu})^a_b (U^{-1})^d_G. \]

(25)

The field equations can be found from the action [29,30]

\[
S_{grav} = \int d^4x \ e R(e),
\]

(26)

where \( e = \sqrt{-g} \) and \( R = e^{\mu\nu} \bar{e}^{\rho\sigma} (R_{\mu\nu})_{\rho\sigma} \). Although the particle spectrum is now free of negative energy ghost states in the weak field approximation, the theory still suffers from the existence of dipole ghost states due to the asymptotic boundary conditions for \( g_{\mu\nu} \), unless we use the action (15), rewritten in the language of vierbeins. However, a Hamiltonian constraint analysis for this theory will still reveal serious instability problems [34,35].

3. Complex symmetric Riemannian geometry

We shall now consider choosing a complex manifold of coordinates \( M^4_c \) and a complex symmetric metric defined by [22–24]

\[
g_{\mu\nu} = s_{\mu\nu} + a_{\mu\nu},
\]

(27)

where \( a_{\mu\nu} = ib_{\mu\nu} \) and \( b_{\mu\nu} \) is a real symmetric tensor. This corresponds to having two copies of a real metric in an eight-dimensional space. The real diffeomorphism symmetry of standard Riemannian geometry is extended to a complex diffeomorphism symmetry under the group of complex coordinates transformations with \( e^\mu = x^\mu + iy^\mu \). The metric can be expressed in terms of a complex vierbein \( E^a_\mu = \Re(E^a_\mu) + i \Im(E^a_\mu) \) as

\[
g_{\mu\nu} = E^a_\mu E^b_\nu \eta_{ab}. \]

(28)

The real contravariant tensor \( s^{\mu\nu} \) is associated with \( s_{\mu\nu} \) by the relation

\[
S_{\mu\nu} s_{\mu\sigma} = \delta_{\sigma}^\nu, \]

(29)

and also

\[
S^{\mu\nu} g_{\mu\sigma} = \delta_{\sigma}^\nu. \]

(30)

With the complex spacetime is also associated a complex symmetric connection

\[
\Gamma^\lambda_{\mu\nu} = \Delta^\lambda_{\mu\nu} + \Omega^\lambda_{\mu\nu}, \]

(31)

where \( \Omega^\lambda_{\mu\nu} \) is purely imaginary.

We shall determine the \( \Gamma^\lambda_{\mu\nu} \) according to the \( g_{\mu\nu} \) by the equations

\[
g_{\mu\nu;\lambda} = \partial_\lambda g_{\mu\nu} - g_{\rho\mu} \Gamma^\rho_{\nu\lambda} - g_{\rho\nu} \Gamma^\rho_{\mu\lambda} = 0. \]

(32)

By commuting the two covariant differentiations of an arbitrary complex vector \( A_{\mu} \), we obtain the generalized curvature tensor

\[
R^a_{\mu\nu;\lambda} = -\partial_\lambda \Gamma^a_{\mu\nu} + \partial_\nu \Gamma^a_{\mu\lambda} + \partial_\mu \Gamma^a_{\nu\lambda} - \Gamma^b_{\lambda\nu} \Gamma^a_{\mu\rho} - \Gamma^b_{\lambda\mu} \Gamma^a_{\nu\rho} \]

(33)

and a contracted curvature tensor \( R_{\mu\nu} = R^a_{\mu\nu;\rho} \)

\[
R_{\mu\nu} = A_{\mu} + B_{\mu\nu}, \]

(34)

where \( B_{\mu\nu} \) is a purely imaginary tensor. From the curvature tensor, we can derive the four complex (eight real) Bianchi identities

\[
(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R)_{;\nu} = 0. \]

(35)
We must choose a real action to guarantee a consistent set of field equations. There is, of course, a degree of arbitrariness in choosing this action due to the complex manifold of coordinate transformations and the complex Riemannian geometry. We shall choose \[ S_{\text{grav}} = \frac{1}{2} \int d^4 x \left[ g^{\mu \nu} R_{\mu \nu} + \text{compl. conj.} \right] \]
\[ = \int d^4 x \left[ s^{\mu \nu} A_{\mu \nu} + a^{\mu \nu} B_{\mu \nu} \right], \tag{36} \]
where \( g^{\mu \nu} = \sqrt{-g} g^{\mu \nu} = s^{\mu \nu} + a^{\mu \nu} \). The variation with respect to \( s^{\mu \nu} \) and \( a^{\mu \nu} \) yields the twenty field equations of empty space
\[ A_{\mu \nu} = 0, \quad B_{\mu \nu} = 0, \tag{37} \]
or, equivalently, the ten complex field equations
\[ R_{\mu \nu} = 0. \tag{38} \]
We can add a real matter action to (36):
\[ S_{\text{matter}} = -4 \pi \int d^4 x \left[ g^{\mu \nu} T_{\mu \nu} + \text{compl. conj.} \right], \tag{39} \]
where \( T_{\mu \nu} = S_{\mu \nu} + C_{\mu \nu} \) is a complex symmetric source tensor, and \( S_{\mu \nu} \) and \( C_{\mu \nu} \) are real and pure imaginary tensors, respectively.

We shall assume that the line element determining the physical gravitational field is of the real form
\[ ds^2 = s_{\mu \nu} dx^\mu dx^\nu. \tag{40} \]
Let us consider the static spherically symmetric line element
\[ ds^2 = a dt^2 - \eta dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{41} \]
where the real \( a \) and \( \eta \) are functions of real \( r \) only. Our complex fundamental tensor \( g_{\mu \nu} \) is determined by
\[ g_{11}(r) = -\mu(r) = -[\eta(r) + i \zeta(r)], \]
\[ g_{22}(r) = s_{22}(r) = -r^2, \]
\[ g_{33}(r) = s_{33}(r) = -r^2 \sin^2 \theta, \]
\[ g_{00}(r) = y(r) = \alpha(r) + i \beta(r). \tag{42} \]
Solving the \( I_{\mu \nu} \) from Eq. (32) and substituting into the field equation (38), we get the solutions
\[ \alpha = 1 - \frac{2m}{r}, \quad \beta = \frac{2\epsilon}{r}, \tag{43} \]
where \( 2m \) and \( 2\epsilon \) are constants of integration, and we have imposed the boundary conditions
\[ \alpha \to 1, \quad \eta \to 1, \quad \beta \to 0, \quad \xi \to 0, \tag{44} \]
as \( r \to \infty \). Solving for \( \eta(r) \) and \( \zeta(r) \) from the solution \( \mu(r) = 1/\gamma(r) \) we get \[ d s^2 = \left( 1 - \frac{2m}{r} \right) d t^2 - \frac{1 - \frac{2m}{r}}{(1 - \frac{2m}{r})^2 + \frac{4\epsilon^2}{r^2}} d r^2 \]
\[ - r^2 (d \theta^2 + \sin^2 \theta d \phi^2), \tag{45} \]
\[ \zeta = - \frac{2\epsilon}{(1 - \frac{2m}{r})^2 + \frac{4\epsilon^2}{r^2}}. \tag{46} \]
When \( \epsilon \to 0 \), we regain the Schwarzschild solution of Einstein gravity.

The weak field approximation obtained from the expansion about Minkowski spacetime
\[ g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu}, \tag{47} \]
where \( h_{\mu \nu} = p_{\mu \nu} + i k_{\mu \nu} \), leads to an action which is invariant under the local, linear gauge transformation
\[ h'_{\mu \nu} = h_{\mu \nu} + \partial_\mu \theta_\nu + \partial_\nu \theta_\mu, \tag{48} \]
where \( \theta_\mu \) is an arbitrary complex vector. Thus, two spin 2 massless gravitons describe the complex bimetric gravity field and there are two light cones associated with the spacetime. The physical spin 2 graviton is described by a real mixture of the basic spin two particles associated with \( p_{\mu \nu} \) and \( k_{\mu \nu} \). Moreover, the physical null cone with \( ds^2 = 0 \) will be a real mixture of the two null cones associated with \( s_{\mu \nu} \) and \( a_{\mu \nu} \).

The linearized action \( S_{\text{grav}} \), obtained from (36) in the weak field approximation, contains negative energy ghost states coming from the purely imaginary part of the metric \( k_{\mu \nu} \). To avoid this unphysical aspect of the theory, we can base the geometry on a hyperbolic complex metric
\[ g_{\mu \nu} = s_{\mu \nu} + \omega a_{\mu \nu}, \tag{49} \]
where \( \omega \) is the pure imaginary element of a hyperbolic complex Clifford algebra \( \Omega \) [36]. We have \( \omega^2 = +1 \) and for \( z = x + \omega y \), we obtain \( z \bar{z} = x^2 - y^2 \) where \( \bar{z} = x - \omega y \), so that there exist lines of zeros, \( z \bar{z} = 0 \), in the hyperbolic complex space and it follows that \( \Omega \) forms a ring of numbers and not a field as for the usual system of complex numbers with the pure imaginary element \( i = \sqrt{-1} \). The linearized action \( S_{\text{grav}} \) should now have positive energy and with the invariance under the hyperbolic complex group of transformations \( CSO(3, 1, \Omega) \) the gravitons should be free of ghost states.
4. Noncommutative complex symmetric gravity

We must now generalize the complex symmetric gravity (CSG) to noncommutative coordinates by replacing the usual products by Moyal $\star$-products. We shall use the complex vierbein formalism based on the metric (28) and a complex spin connection $(\omega_{\mu})_{ab}$, since this formalism is closer to the standard gauge field formalism of field theory.

The complex symmetric metric is defined by

$$g_{\mu\nu} = E^a_\mu \star E^b_\nu \eta_{ab}. \quad (50)$$

The spin connection is subject to the gauge transformation

$$(\omega_\mu)_b \rightarrow [U_C \star \omega_\mu \star U_C^{-1} - (\partial_\mu U_C) \star U_C^{-1}]_b,$$  

where $U_C$ is an element of a complex noncommutative group of orthogonal transformations $NCSO(3,1)$.

The curvature tensor is now given by

$$(R_{\mu\nu})^d_b = \partial_{[\mu} (\omega_\nu)_b^a - \partial_{\nu} (\omega_\mu)_b^a + (\omega_\mu)_b^a \star (\omega_\nu)_b^c - (\omega_\nu)_b^c \star (\omega_\mu)_b^a,$$  

which transforms as

$$(R_{\mu\nu})^d_b \rightarrow U^{d}_{C\mu} \star (R_{\mu\nu})^c_d \star (U^{-1}_{C\nu})^b_c. \quad (53)$$

The real action is

$$S_{\text{grav}} = \frac{1}{2} \int d^4x \left[ \sqrt{-g} \star E \star (E^a_\mu \star (R_{\mu\nu})^b_\nu \star E^{\mu\nu} + \text{compl.conj.}) \right]. \quad (54)$$

where $E = \sqrt{-g}$. The action $S_{\text{grav}}$ is locally invariant under the transformations of the complex noncommutative, fiber bundle tangent space group $NCSO(3,1)$, i.e., the group of complex noncommutative homogeneous Lorentz transformations. It has been shown by Bonora et al. [37] that attempting to define a noncommutative gauge theory corresponding to a subgroup of $U(n)$ is not trivially accomplished. This is true in the case of a string–brane theory configuration. We must find a $NCSO(3,1)$, which reduces to $CSO(3,1)$ and, ultimately, to $SO(3,1)$ when $\theta \rightarrow 0$. Bonora et al., were able to show that it is possible to impose constraints on gauge potentials and the corresponding gauge transformations, so that ordinary commutative orthogonal and symplectic gauge groups were recovered when the deformation parameter $\theta$ vanishes. These constraints were defined in terms of a generalized gauge theory charge conjugation operator, and a generalization of connection-based Lie algebras in terms of an antiautomorphism in the corresponding $C^\star$-algebra. The problem of deriving an explicit description of $NCSO(d,1)$ will be investigated in a future article.

To avoid possible problems with negative energy ghost states, we can develop the same formalism based on the hyperbolic complex vierbein $E^a_\mu = \text{Re}(E^a_\mu) + \omega \text{Im}(E^a_\mu)$ and a hyperbolic complex spin connection and curvature. The noncommutative spacetime is defined by

$$[x^\mu, x^\nu] = \omega \theta^{\mu\nu}, \quad (55)$$

and a hyperbolic complex Moyal $\diamond$-product in terms of the pure imaginary element $\omega$:

$$f(x) \diamond g(x) = \exp \left[ \frac{\omega}{2} \theta^{\mu\nu} \frac{\partial}{\partial \alpha^\mu} \frac{\partial}{\partial \beta^\nu} \right] \times f(x + \alpha)g(x + \beta)|_{\alpha=\beta=0} = fg + \frac{\omega}{2} \theta^{\mu\nu} \partial_\mu f \partial_\nu g + \mathcal{O}(\theta^2). \quad (56)$$

A hyperbolic complex vierbein formalism can be developed along the same lines as the complex vierbein formalism and the invariance group of transformations $NCSO(3,1, C)$ is extended to $NCSO(3,1, \Omega)$. The noncommutative field equations and the action contain an infinite number of derivatives, and so constitute a nonlocal theory at the quantum level. When the coordinates assume their classical commutative structure, then the theory regains standard, classical causal properties. The nonlocal nature of the noncommutative quantum gauge field theory and quantum gravity theory presents potentially serious problems. Such theories can lead to instabilities, which render them unphysical [19,38], and in the case of spacetime noncommutativity, $\theta^{ij} \neq 0$, the perturbative S-matrix may not be unitary [14]. Moreover, it is unlikely that one can apply the conventional, canonical Hamiltonian formalism to quantize such gauge field theories. Of course, we know that the latter formalism already meets serious obstacles in applications to quantum gravity for commutative spacetime.
5. Conclusions

The deformed product of functions given by the Moyal product of functions leads to complex gauge fields and a complex Riemannian or non-Riemannian geometry. We first considered the possibility that the fundamental tensor $g_{\mu\nu}$ is Hermitian nonsymmetric, leading to a nonsymmetric gravitational theory (NGT). This theory does not at present form a viable gravitational theory, because of instability difficulties that arise when an expansion is performed about a GR background. The $g_{\mu\nu}$ had to be chosen to be hyperbolic complex to avoid basic negative energy ghost problems. Even though this hyperbolic complex nonsymmetric theory could be shown to be self-consistent as far as asymptotic boundary conditions are concerned and to be free of higher-order ghost poles, the instability problems discovered by Clayton [34,35] remain. It is possible that these instability problems can be removed by some new formulation of NGT, but so far no one has succeeded in discovering such a formulation.

We then investigated a complex symmetric (non-Hermitian) $g_{\mu\nu}$ on a complex manifold with the local gauge group of complex Lorentz transformations $CSO(3, 1)$ (or $CSO(d, 1)$ in $(d + 1)$-dimensions). From a real action, we obtained a consistent set of field equations and Bianchi identities in a torsion-free spacetime. By assuming that the physical line element was given by the real form (40), we derived a static spherically symmetric solution with two gravitational “charges” $2m$ and $2\epsilon$. When $\epsilon = 0$, we regained the standard Schwarzschild solution of GR, so there is a well-defined classical gravity limit of the theory, which agrees with all the known experimental data. We also considered a complex geometry based on a hyperbolic complex metric and connection. This geometry would avoid potential problems of negative ghost states in the linearized equations of gravity.

By formulating the vierbein and spin connection formalism on a flat tangent space by means of complex vierbeins and a complex spin connection and curvature tensor, we generalized the complex symmetric geometry to noncommuting coordinates by replacing the usual products by Moyal $\star$-products and complex gauge transformations, and by Moyal $\diamondsuit$-products for a hyperbolic complex noncommutative geometry. We extended the complex group of gauge transformations $CSO(3, 1)$ of the commutative spacetime to a noncommutative group of complex orthogonal gauge transformations $NCSO(3, 1, C)$. For the geometry based on a noncommutative Clifford algebra $\Omega$, the invariance group was extended to the hyperbolic complex group $NCSO(3, 1, \Omega)$.

The CSG theory we have developed can be the basis of a quantum gravity theory, which contains classical GR as the limit of CSG when the new gravitational charge $\epsilon \to 0$. It would be interesting to consider possible experimental consequences of CSG for strong gravitational fields, for gravitational wave experiments and for cosmology.

A noncommutative quantum gravity theory as well as a noncommutative quantum gauge field theory pose potential difficulties, because of the nonlocal nature of the interactions at both the nonperturbative and perturbative levels. A self-consistent nonlocal quantum gravity theory has been formulated which is perturbatively finite, gauge invariant and unitary to all orders [19], but the theory is based on a special choice of entire functions used to generalize the standard point particle propagators. It is not clear whether such a physically consistent quantum gauge field theory or quantum gravity theory, based on noncommutative coordinates, can be found at either the perturbative or nonperturbative levels. This is an open question that requires more intensive investigation. It remains to be seen whether our standard physical notions about the nature of spacetime can be profoundly changed at the quantum level, without leading to unacceptable physical consequences.

Acknowledgements

I thank Michael Clayton for helpful discussions. This work was supported by the Natural Sciences and Engineering Research Council of Canada.

References

Living inside a hedgehog: higher-dimensional solutions that localize gravity

Tony Gherghetta, Ewald Roessl, Mikhail Shaposhnikov*

Institute of Theoretical Physics, University of Lausanne, CH-1015 Lausanne, Switzerland

Received 10 July 2000; accepted 11 August 2000

Abstract

We consider spherically symmetric higher-dimensional solutions of Einstein’s equations with a bulk cosmological constant and $n$ transverse dimensions. In contrast to the case of one or two extra dimensions we find no solutions that localize gravity when $n > 3$, for strictly local topological defects. We discuss global topological defects that lead to the localization of gravity and estimate the corrections to Newton’s law. We show that the introduction of a bulk “hedgehog” magnetic field leads to a regular geometry and localizes gravity on the 3-brane with either a positive, zero or negative bulk cosmological constant. The corrections to Newton’s law on the 3-brane are parametrically the same as for the case of one transverse dimension. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

A lot of attention has been devoted recently to alternatives [1–6] of Kaluza–Klein compactification [7]. In particular, our spacetime can be associated with some topological defect — 3-brane, embedded in a higher-dimensional spacetime with non-compact extra dimensions. It is usually assumed that the matter fields are localized on the brane because of the specific dynamics of solitons in string theory — $D$-branes [8]. Moreover, in Ref. [6] it was shown that the gravity of a domain wall in 5-dimensional anti-de Sitter (AdS) spacetime has a 4-dimensional character for the particles living on the brane, provided that the domain wall tension is fine tuned to a bulk cosmological constant. The corrections to Newton’s gravity law are generically small for macroscopic scales (see, however, [9] for a more complicated construction involving several branes). A similar statement is true for a local string living in 6-dimensional AdS space [10] (or more general constructions [11]).

The aim of the present paper is to generalize the results of Ref. [10] to the case when the number $n$ of transverse dimensions is larger than two. This happens to be not as trivial as one expects. The reason is that the transverse spaces with $n \leq 2$, and $n \geq 3$ extra dimensions are qualitatively different, at least in the spherically symmetric setup we are interested in. In contrast to the case with $n \geq 3$, for $n = 1$ the extra space is flat while for $n = 2$ the extra space can be curved, but is still conformally flat. In the absence of a brane the possibilities of compactification were studied in [2] for $n = 2$ and in [12] for general $n$.

We will consider three different possibilities. The first possibility is called a strictly local defect. By
strictly local we mean the situation when the stress-energy tensor of the defect is zero outside the core (or, for the more realistic situation of a “fat” brane, exponentially falling outside the core). Here, we were not able to find any geometry leading to the compactification of gravity — contrary to the $n = 1$ and $n = 2$ cases.

The second possibility is related to the so-called global defects. In this case one assumes that there exists a scalar field with, say $O(N)$ ($N \geq n$) global symmetry which is spontaneously broken. Outside the string core this field may have a hedgehog type configuration, which gives a specific contribution to the energy–momentum tensor outside the defect. This case was studied in [13, 14] for $n = 2$ and in [15] for higher dimensions, where the solutions with an exponential warp factor were found. We compute the corrections to Newton’s law in this case and study the boundary conditions at the core of the global string. Furthermore, a generalization of these metric solutions with nonzero components $T^\rho_\rho = \delta^\rho_\rho f_0(\rho)$, $T^\rho_\theta = f_0(\rho)$, and $T^\theta_\theta = f_0(\rho)$.

Here we have introduced three source functions $f_0$, $f_\rho$, and $f_\theta$ which depend only on the radial coordinate $\rho$ and by spherical symmetry all the angular source functions are identical, where we have defined $\theta = \theta_{n-1}$. Using the metric ansatz (2) and the stress-energy tensor (4), the Einstein equations become

$$\frac{3}{2} \frac{\sigma''}{\sigma} + \frac{3}{4} (n - 1) \frac{\sigma'}{\sigma} \frac{\gamma'}{\gamma} + \frac{1}{8} (n - 1)(n - 4) \frac{\gamma''}{\gamma}$$

$$+ \frac{1}{2} (n - 1) \frac{\gamma''}{\gamma} - \frac{1}{2 \gamma'}(n - 1)(n - 2) = - \frac{1}{M^n_{D}} (A + f_0(\rho)) + \frac{\Lambda_{\text{phys}}}{M^2_{D}} \frac{1}{\sigma},$$

$$\frac{3}{2} \frac{\sigma''}{\sigma} + (n - 1) \frac{\sigma'}{\sigma} \frac{\gamma'}{\gamma} + \frac{1}{8} (n - 1)(n - 2) \frac{\gamma''}{\gamma^2}$$

$$- \frac{1}{2 \gamma'}(n - 1)(n - 2)$$

$$= - \frac{1}{M^n_{D}} (A + f_\rho(\rho)) + 2 \frac{\Lambda_{\text{phys}}}{M^2_{D}} \frac{1}{\sigma},$$

$$\frac{3}{2} \frac{\sigma''}{\sigma} + \frac{1}{2} \frac{\sigma'}{\sigma} + (n - 2) \frac{\sigma'}{\sigma} \frac{\gamma'}{\gamma} + \frac{1}{8} (n - 2)(n - 5) \frac{\gamma''}{\gamma^2}$$

$$+ \frac{1}{2} (n - 2) \frac{\gamma''}{\gamma} - \frac{1}{2 \gamma'}(n - 2)(n - 3)$$

$$= - \frac{1}{M^n_{D}} (A + f_\theta(\rho)) + \frac{\Lambda_{\text{phys}}}{M^2_{D}} \frac{1}{\sigma},$$

where the $'$ denotes differentiation $d/d\rho$ and the Einstein equations arising from all the angular components simply reduce to the one angular equation (7). The constant $\Lambda_{\text{phys}}$ represents the physical 4-dimensional cosmological constant, where

$$R^{(4)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{(4)} = \frac{\Lambda_{\text{phys}}}{M^2_{D}} g_{\mu\nu}. $$

The system of equations (5)–(7) for $f_i = 0$ was first derived in [12] and describes the generalization of

2. Einstein equations with a 3-brane source

In $D$-dimensions the Einstein equations with a bulk cosmological constant $A_D$ and stress-energy tensor $T_{AB}$ are

$$R_{AB} - \frac{1}{2} g_{AB} R = \frac{1}{M^n_{D}} (A_D g_{AB} + T_{AB}),$$

where $M_D$ is the reduced $D$-dimensional Planck scale. We will assume that there exists a solution that respects 4D Poincaré invariance. A $D$-dimensional metric satisfying this ansatz for $n$ transverse spherical coordinates with $0 \leq \rho < \infty$, $0 \leq \theta_{n-1}, \ldots, \theta_2 < \pi$ and $0 \leq \theta_1 < 2\pi$, is

$$ds^2 = \sigma(\rho) g_{\mu\nu} dx^\mu dx^\nu - d\rho^2 - \gamma(\rho) d\Omega^2_{n-1},$$

where the metric signature of $g_{\mu\nu}$ is $(+, -, -, -)$ and $d\Omega^2_{n-1}$ is defined recursively as

$$d\Omega^2_{n-1} = d\theta_{n-1}^2 + \sin^2 \theta_{n-1} d\Omega^2_{n-2},$$

with $d\Omega^2_{0} = 0$. At the origin $\rho = 0$ we will assume that there is a 3-brane, whose source is described by a magnetic field $T^\mu_\rho$ with nonzero components $T^\rho_\rho = \delta^\rho_\rho f_0(\rho)$, $T^\rho_\theta = f_0(\rho)$, and $T^\theta_\theta = f_0(\rho)$.
the setup considered in [2,6,10,15], to the case where there are \( n \) transverse dimensions, together with a nonzero cosmological constant in 4-dimensions and stress-energy tensor in the bulk. If we eliminate two of the equations in (5)–(7) then the source functions satisfy

\[
f'_\rho = 2 \frac{\sigma'}{\sigma} (f_0 - f_\rho) + \frac{n-1}{2} \frac{\gamma'}{\gamma} (f_0 - f_\rho),
\]

which is simply a consequence of the conservation of the stress-energy tensor \( D_M T^M_N = 0 \). In general the Ricci scalar corresponding to the metric ansatz (2) is

\[
R = 4 \frac{\sigma''}{\sigma} + \frac{\sigma'^2}{\sigma^2} + 2(n-1) \frac{\gamma'}{\gamma} \frac{\gamma'}{\gamma} + (n-1) \frac{\gamma''}{\gamma} \\
+ \frac{1}{n-1} (n-4) \frac{\gamma'^2}{\gamma^2} + (n-1)(n-2) \frac{1}{\gamma} - \frac{4 A_{\text{phys}}}{n M^2_p}.
\]

The boundary conditions at the origin of the transverse space are assumed to be

\[
\sigma'|_{\rho=0} = 0, \\
(\sqrt{\gamma})'|_{\rho=0} = 1 \quad \text{and} \quad \gamma'|_{\rho=0} = 0,
\]

which is consistent with the usual regular solution in flat space. We have set \( \sigma(0) = A \), where \( A \) is a constant. Following [17], we can integrate over the disk of small radius \( \epsilon \) containing the 3-brane, and define various components of the brane tension per unit length as

\[
\mu_i = \frac{\epsilon}{\int_0^\infty d\rho \sigma^2 \gamma^{(n-1)/2} f_i(\rho)},
\]

where it is understood that the limit \( \epsilon \to 0 \) is taken. The equations (13) and (14) are the general conditions relating the brane tension components to the metric solution of the Einstein equations (5)–(7), and lead to nontrivial relationships between the components of the brane tension per unit length. In particular, these conditions on the brane tension components reduce to the relations obtained for \( n = 2 \) [10]. Furthermore, by analogy with the solution for local strings we can identify (13) as the gravitational mass per unit length and (14) as the angular deficit per unit length. Thus the source for the 3-brane, in general curves the transverse space.

From the Einstein term in the \( D \)-dimensional Lagrangian we can obtain the effective four-dimensional Planck mass. Using the spherically symmetric metric ansatz (2), the four-dimensional reduced Planck mass is given by

\[
M^2_p = 4 \pi M^2_D \int_0^\infty d\rho \sigma \gamma^{(n-1)/2},
\]

where \( A_0 \) is the surface area of an \( n \)-dimensional unit sphere. We are interested in obtaining solutions to the Einstein equations (5)–(7) such that a finite four-dimensional Planck mass is obtained. This leads to various possible asymptotic behaviours for the metric warp factors \( \sigma \) and \( \gamma \) in the limit \( \rho \to \infty \). Below, we will concentrate only on the case when the 4-dimensional cosmological constant is zero, \( A_{\text{phys}} = 0 \).

### 3. Strictly local defect solutions

First, let us assume that the functions \( f_i(\rho) \) are zero outside the core of the topological defect. In order to obtain a finite 4-dimensional Planck scale, one requires a solution of the system of equations (5)–(7) for which the function \( \sigma \gamma^{(n-1)/2} \) goes to zero when \( \rho \to \infty \). For \( n = 1 \) and \( n = 2 \) the solutions are known to exist, see [6] and [10], correspondingly. However, when \( n \geq 3 \) the structure of the equations is qualitatively different, because now there is a \( 1/\gamma \) term. Thus, there is no simple generalization of the solutions found for \( n = 1, 2 \).

To neutralize the effect of the \( 1/\gamma \) term, one can look for asymptotic solutions for which \( \gamma \) is a positive...
constant. However, one can easily check that the system of equations (5)–(7) does not allow a solution for which \( y \) tends to a constant when \( \rho \to \infty \), and \( \sigma \) is a negative exponential.

Alternatively, we can assume that there is an asymptotic solution for which \( y \to \infty \) but \( \sigma \) tends to zero faster than \( y^{(n-1)/2} \) and omit the troublesome \( 1/y \) term from the equations of motion. In this case the set of equations (5)–(7) can be simply reduced to a single equation, as in 6d case [2,10]:

\[
z'' = -\frac{dU(z)}{dz}, \tag{16}
\]

where the potential \( U(z) \) is given by

\[
U(z) = \frac{(n+3)}{4(n+2)} \frac{\Lambda_D}{M^n_D} z^2. \tag{17}
\]

With this parametrisation the metric functions \( \sigma(\rho) \) and \( y(\rho) \) can be written in terms of \( z(\rho) \) as

\[
\sigma = |z'|^{2/\sqrt{(n+2)(n-1)}}/(n+3)
\times |z|^{2n/(n+2)(n-1)},
\]

\[
y = |z'|^{6/(1-n+2\sqrt{(n+2)(n-1)})}
\times |z|^{6/[(n+2)(n-1)]}.
\tag{18}
\]

Solving Eq. (16) with the potential (17) gives the general solution

\[
z(\rho) = d_1 e^{-\frac{1}{2}(n+3)\epsilon \rho} + d_2 e^{\frac{1}{2}(n+3)\epsilon \rho}, \tag{20}
\]

where \( \epsilon^2 = -8\Lambda_D/((n+2)(n+3)M^n_D) \), \( d_1, d_2 \) are constants and we take \( \Lambda_D < 0 \). This solution is a generalization of the pure exponential solution considered earlier and in Ref. [15], see below. In this picture we can think of particle motion under the influence of the potential (17) with position \( z(\rho) \) and “time” \( \rho \).

Since \( 1 - n + 2\sqrt{(n+2)(n-1)} \) and \( 1 - n + 2\sqrt{(n+2)(n-1)} < 0 \), the metric factor \( \gamma \) can be large in two cases. In the first case \( z(\rho) \) is zero for some \( \rho_0 \). However, this point is only a coordinate singularity (the Ricci scalar is regular at this point) and the metric can be extended beyond \( \rho_0 \), leading then to an exponentially rising solution for both \( \sigma \) and \( y \). This, unfortunately, is not interesting for compactification. In the second case both \( z' \) and \( z \) are non-zero and increase exponentially for large \( \rho \). Thus, there is no possibility of a finite Planck scale in this case either.

Similarly, we were unable to find solutions in the reverse case when \( y \) vanishes at infinity. Moreover, even if such solutions were to exist, they would likely lead to a singular geometry (naked singularity), because the Ricci scalar contains a \( 1/y \) term, see Eq. (10). This was indeed shown to be the case for solutions with regular geometries at \( \rho = 0 \) in [12].

4. Bulk scalar field

4.1. Global topological defects

The other possibility is to consider defects with different types of “hair”, i.e., with non-zero stress-energy tensor outside the core of the defect. We start with global topological defects. In fact, we have little to add to this question as it has been extensively studied in [14,15], so we just list a number of explicit solutions (see also [18–20]). Again, for simplicity we will restrict to the case where the four-dimensional cosmological constant \( \Lambda_\text{phys} = 0 \), and assume that outside the core \( \rho > \epsilon \)

\[
\sigma(\rho) = e^{-\gamma \rho}. \tag{21}
\]

We have chosen the arbitrary integration constant, which corresponds to an overall rescaling of the coordinates \( x^\mu \), such that \( \lim_{\epsilon \to 0} \sigma(\epsilon) = 1 \).

Consider \( n \) scalar fields \( \phi^a \) with a potential

\[
V(\phi) = \lambda (\phi^a \phi^a - v^2)^2, \tag{22}
\]

where \( v \) has mass dimension \((n+2)/2\). Then the potential minimum is at \( \phi^a \phi^a = v^2 \). The defect solution has a “hedgehog” configuration outside the core

\[
\phi^a(\rho) = vd^a, \tag{23}
\]

where \( d^a \) is a unit vector in the extra dimensions, \( d^a = \cos \theta_{n-1}, \ d^{a-1} = \sin \theta_{n-1} \cos \theta_{n-2}, \ldots \).

The scalar field gives an additional contribution to the stress-energy tensor in the bulk with components

\[
T^\nu_\mu = (n-1) \frac{v^2}{2\gamma} \delta^\nu_\mu, \tag{24}
\]

\[
T^\rho_\rho = (n-1) \frac{v^2}{2\gamma}, \tag{25}
\]

\[
T^0_0 = (n-3) \frac{v^2}{2\gamma}. \tag{26}
\]
where $R_A$ and confirmed by looking at the other curvature invariants, and in fact has a singularity at $n$ which diverges when $R_{cylinder}$.
The transverse geometry of this solution is that of a regular geometry in the bulk [14,15] solution leads to the localization of gravity and a $D/D_0$ exponential solution to the coupled set of equations (5)–(7) can then be found with eliminated from the system of equations (5)–(7) and the

\[ R_{AB} R_{AB} - R_{ABCD} R^{ABCD} \]

Only for the 5d and 6d cases do we obtain a constant curvature anti-de Sitter space. The appearance of a singularity is similar to the case of the global-string defect [13].
The metric solution (31) can also be written in the form

\[ ds^2 = \gamma^2 g_{\mu\nu} dx^\mu dx^\nu - \gamma^2 R_0^2 \Omega_{n-1}^2 - \frac{4}{c^2} \gamma dz^2, \]

where $\gamma = \exp(\gamma_0)$. In these coordinates the origin $\rho = 0$ is now mapped to $\rho = 1$ and the 3-brane source is spread around the surface area of a $n$-dimensional sphere of radius $R_0$. This confirms our previous suggestion [10] that for $n \geq 2$ transverse dimensions the 3-brane can be identified with $\mathcal{M}_{n+2}/S^{n-1}$, where $\mathcal{M}_{n+2}$ is a $(n+2)$-brane.

Requiring that our exponential solution satisfy the boundary conditions (13) and (14), leads to the relation

\[ \mu_0 + \mu_\rho = \frac{1}{2} (n+2) R_0^{n-1} M_D^{n+2}, \]

where $\mu_0$ satisfies

\[ \mu_0 = \mu_0 + A^2 M_D^4 \delta_{n2}, \]

and $\mu_\rho$ remains undetermined. Thus for $n > 2$ we simply have $\mu_0 = \mu_\rho$.

4.2. Corrections to Newton’s law

For the solution (27), (28) the corrections to Newton’s law are parametrically the same as for 5d case, since $\gamma$ is a constant. On the other hand, the singular solution (31) will ultimately require that the singularity is smoothed by string theory corrections (perhaps similar to the nonsingular deformations considered in [21]). Assuming that this is the case, then the corrections to Newton’s law on the 3-brane can be calculated by generalizing the calculation presented in [6, 10] (see also [22]).

In order to see that gravity is only localized on the 3-brane, let us now consider the equations of motion for the linearized metric fluctuations. We will only concentrate on the spin-2 modes and neglect the scalar modes, which needs to be taken into account together with the bending of the brane [23]. The vector modes are massive as follows from a simple modification of the results in Ref. [24]. For a fluctuation of the

Now, for $A_D < 0$ and $v^2 > (n-2) M_D^{n+2}$ the following solution leads to the localization of gravity and a regular geometry in the bulk [14,15]

\[ c = \frac{2(-A_D)}{(n+2) M_D^{n+2}}, \]

\[ \gamma = \frac{1}{c^2} \left( \frac{v^2}{M_D^{n+2}} - n + 2 \right), \]

provided that the brane tension components satisfy the conditions

\[ -c\sqrt{\gamma^{n-1}} = \frac{2}{(n+2) M_D^{n+2}} \times \left( (n-2) \mu_0 - \mu_\rho - (n-1) \mu_\sigma \right), \]

and

\[ A^2 \delta_{n2} = \frac{1}{(n+2) M_D^{n+2}} (4 \mu_0 + \mu_\rho - 3 \mu_\sigma). \]

The transverse geometry of this solution is that of a cylinder, $R_+ \times S_{n-1}$ with $R_+$ being the half-line and $S_{n-1}$ being an $n-1$-sphere.

When $v^2 > (n-2) M_D^{n+2}$ the $1/\gamma$ terms are eliminated from the system of equations (5)–(7) and the exponential solution to the coupled set of equations (5)–(7) can then be found with

\[ \gamma(\rho) = R_0^2 \sigma(\rho), \quad c = \frac{8(-A_D)}{(n+2)(n+3) M_D^{n+2}}, \]

where $R_0$ is an arbitrary length scale. As expected, the negative exponential solution (21) requires that $A_D < 0$. Notice that the exponential solution (31) only requires the “hedgehog” scalar field configuration in the bulk for transverse spaces with dimension $n \geq 3$. No such configuration is needed for the 5d [6] and 6d cases [10], which only require gravity in the bulk.

The Ricci scalar corresponding to the negative exponential solution with vanishing four-dimensional cosmological constant is

\[ R = (n+3)(n+4) - (n-1)(n-2) \frac{e^{2\psi}}{R_0^2}, \]

which diverges when $\rho = \infty$. Thus we see that for $n \geq 3$ the space is no longer a constant curvature space and in fact has a singularity at $\rho = \infty$. This is also confirmed by looking at the other curvature invariants, $R_{AB} R_{AB}$ and $R_{ABCD} R^{ABCD}$.
form \( h_{\mu\nu}(x, z) = \Phi(z)h_{\mu\nu}(x) \) where \( z = (\rho, \theta) \) and 
\[ \partial^2 h_{\mu\nu}(x) = m^2 h_{\mu\nu}(x) \] 
we can separate the variables by defining \( \Phi(z) = \sum_{\ell,m} \phi_m(\rho) \psi_L(\phi, \theta) \). The radial modes satisfy the equation [24]

\[ -\frac{1}{\sigma} \gamma^{(n-1)/2} \partial_\rho \left[ \sigma^2 \gamma^{(n-1)/2} \partial_\rho \phi_m \right] = m^2 \phi_m, \]  
(36)

where \( m^2 = m^2 + \Delta^2 / R_0^2 \) contains the contributions from the angular momentum modes \( \ell \). The differential operator (36) is self-adjoint provided that we impose the boundary conditions

\[ \phi_m(0) = \phi_m'(\infty) = 0, \]  
(37)

where the modes \( \phi_m \) satisfy the orthonormal condition

\[ A_n \int_0^\infty d\rho \sigma \gamma^{(n-1)/2} \phi_m \phi_n = \delta_{mn}. \]  
(38)

Using the specific solution (21), (31), the differential operator (36) becomes

\[ \phi_m'' - \frac{(n + 3)}{2} c \phi_m' + m^2 c^2 \phi_m = 0. \]  
(39)

This equation is the same as that obtained for the 5d domain wall solution [6], when \( n = 1 \) and the local stringlike solution [10] when \( n = 2 \). We see that each extra transverse coordinate augments this coefficient by \( 1/2 \). When \( m = 0 \) we clearly see that \( \phi_0(\rho) = \text{constant} \) is a solution. Thus we have a zero-mode tensor fluctuation which is localized near the origin \( \rho = 0 \) and is normalizable.

The contribution from the nonzero modes will modify Newton’s law on the 3-brane. In order to calculate this contribution we need to obtain the wavefunction for the nonzero modes at the origin. The nonzero mass eigenvalues can be obtained by imposing the boundary conditions (37) on the solutions of the differential equation (39). The solutions of (39) are

\[ \phi_m(\rho) = e^{\frac{(n+3)}{2} \rho} \left[ C_1 J_\ell \left( \frac{2m}{c} e^{\frac{\rho}{2}} \right) + C_2 Y_\ell \left( \frac{2m}{c} e^{\frac{\rho}{2}} \right) \right], \]  
(40)

where \( C_1, C_2 \) are constants and \( J_{\ell} \) and \( Y_{\ell} \) are the usual Bessel functions. Imposing the boundary conditions (37) at a finite radial distance cutoff \( \rho = \rho_{\text{max}} \) (instead of \( \rho = \infty \)) will lead to a discrete mass spectrum, where for \( k = 1, 2, 3, \ldots \), we obtain

\[ m_k \simeq c \left( k + \frac{n}{4} \right) \frac{\pi}{2} e^{-\frac{n}{2} \rho_{\text{max}}}. \]  
(41)

With this discrete mass spectrum we find that in the limit of vanishing mass \( m_k \),

\[ \phi_{m_k}^2(0) = \frac{1}{A_n R_0} \frac{\pi c}{8} \left( \frac{n + 1}{\Gamma^2[(n + 3)/2]} \right) \left( \frac{m_k}{c} \right)^n e^{-\frac{n}{2} \rho_{\text{max}}}, \]  
(42)

where \( \Gamma[x] \) is the gamma-function. On the 3-brane the gravitational potential between two point masses \( m_1 \) and \( m_2 \), will receive a contribution from the discrete nonzero modes given by

\[ \Delta V(r) \simeq G_N \frac{m_1 m_2}{r} \frac{\pi (n + 1)}{4 \Gamma^2[(n + 3)/2]} \times \sum_k e^{-m_k r} \left( \frac{m_k}{c} \right)^n e^{-\frac{n}{2} \rho_{\text{max}}}, \]  
(43)

where \( G_N \) is Newton’s constant. In the limit that \( \rho_{\text{max}} \to \infty \), the spectrum becomes continuous and the discrete sum is converted into an integral. The contribution to the gravitational potential then becomes

\[ V(r) \simeq G_N \frac{m_1 m_2}{r} \left[ 1 + \frac{1}{2} e^{n+1} \frac{n + 1}{\Gamma^2[(n + 3)/2]} \times \int_0^\infty dm \ m^n e^{-mr} \right] \]  
(44)

\[ = G_N \frac{m_1 m_2}{r} \left[ 1 + \frac{\Gamma[n+2]}{2 \Gamma^2[(n + 3)/2]} \times \frac{1}{(cr)^{n+1}} \right]. \]  
(45)

Thus we see that for \( n \) transverse dimensions the correction to Newton’s law from the bulk continuum states grows like \( 1/(cr)^{n+1} \). This correction becomes more suppressed as the number of transverse dimensions grows, because now the gravitational field of the bulk continuum modes spreads out in more dimensions and so their effect on the 3-brane is weaker.
5. Bulk \( p \)-form field

The global topological defects considered in the previous section inevitably contain massless scalar fields — Nambu–Goldstone bosons associated with the spontaneous breakdown of the global symmetry. Thus, the stability of these configurations is far from being obvious.

We will now consider the possibility of introducing other types of bulk fields (\( p \)-form field \( \Lambda_{\mu_1 \cdots \mu_p} \)), which directly lead to a regular geometry. The stability of the corresponding configurations may be insured simply by the magnetic flux conservation. The \( D \)-dimensional action is

\[
S = \int d^Dx \sqrt{|g|} \left( \frac{1}{2} M^{\mu_1 \cdots \mu_{p+1}} R - \frac{\Lambda_D}{M^{\mu_1 \cdots \mu_{p+1}}} \right) + (-1)^p \frac{1}{4} F^{\mu_1 \cdots \mu_{p+1}} F_{\mu_1 \cdots \mu_{p+1}}.
\]

(46)

The energy–momentum tensor associated with the \( p \)-form field configuration is given by

\[
T^A_B = (-1)^{p+1} \left( \frac{1}{4} g^A_B F_{\mu_1 \cdots \mu_{p+1}} F^{\mu_1 \cdots \mu_{p+1}} - \frac{p}{2} F^A_{\mu_1 \cdots \mu_{p-1}} F_B^{\mu_1 \cdots \mu_{p+1}} \right).
\]

(47)

A solution to the equation of motion for the \( p \)-form field when \( p = n - 2 \) is

\[
F_{\theta_1 \cdots \theta_{n-1}} = Q (\sin \theta_{n-1})^{(n-2)} \cdots \sin \theta_2,
\]

(48)

where \( Q \) is the charge of the field configuration and all other components of \( F \) are equal to zero. In fact, this “hedgehog” field configuration is the generalization of the magnetic field of a monopole. The stress–energy tensor associated with this \( p \)-form field in the bulk is

\[
T^\mu_\nu = \left( \frac{n-1}{4} - Q^2 \right) g^\mu_\nu,
\]

(49)

\[
T^\rho_\theta = - T^\theta_\rho = \frac{(n-1)!}{4} Q^2 g^{\rho \theta}.
\]

(50)

Let us assume a solution outside the core (\( \rho > \epsilon \)) of the form

\[
\sigma(\rho) = e^{-\rho^2} \quad \text{and} \quad \gamma = \text{constant},
\]

(51)

where we have again chosen the arbitrary integration constant such that \( \lim_{\epsilon \to 0} \sigma(\epsilon) = 1 \). With this ansatz we see from the Ricci scalar that the transverse space will have a constant curvature and the effective four-dimensional Planck constant will be finite. If we substitute this ansatz and also include the contribution of the \( p \)-form bulk field to the stress–energy tensor, the Einstein equations (5)–(7), with \( A_{\text{phys}} = 0 \) are reduced to the following two equations for the metric factors outside the 3-brane source

\[
\frac{(n-1)!}{\gamma^{n-1}} - \frac{Q^2}{2} (n-2)(n+2) + \frac{A_D}{M_D^{n+2}} = 0,
\]

\[
c^2 = \frac{1}{2} A_D M_D^{n+2} + \frac{1}{4} C(n-2)^2.
\]

(52)

(53)

We are interested in the solutions of these two equations which are exponential, \( c^2 > 0 \) and do not change the metric signature, \( \gamma > 0 \). Remarkably, solutions to these equations exist for which these conditions can be simultaneously satisfied. Let us first consider the case \( n = 2 \). Then the solutions reduce simply to

\[
\frac{Q^2}{\gamma} = - \frac{A_6}{M_6^4},
\]

\[
c^2 = - \frac{1}{2} A_6 M_6^4.
\]

(54)

(55)

Thus, for \( A_6 < 0 \) we see that there is solution satisfying (51). This solution is different from the local string defect [10]. In this case the brane tension components satisfy

\[
\mu_0 = \mu_0 + \left( 1 - \frac{Q}{2 \sqrt{2}} \right) A_2 M_6^4,
\]

(56)

where \( \mu_\rho \) remains undetermined. This reduces to the condition satisfied by the local string solution when \( Q = 0 \).

Next we consider the case \( n = 3 \). Only the ‘+’ solution to the quadratic equation (52) gives rise to a solution with both \( c^2 > 0 \) and \( \gamma > 0 \). This solution can be written in the form

\[
\frac{Q^2}{\gamma} = \frac{1}{4} \left[ \frac{5}{2} + \sqrt{\left( \frac{5}{2} \right)^2 - 8 Q^2 \frac{A_7}{M_7^2}} \right],
\]

(57)

\[
Q^2 c^2 = \frac{1}{16} \left[ \frac{5}{2} - 8 Q^2 \frac{A_7}{M_7^2} + \sqrt{\left( \frac{5}{2} \right)^2 - 8 Q^2 \frac{A_7}{M_7^2}} \right].
\]

(58)
When $Q^2 A_7/M_7^3 < 25/32$ we obtain real solutions which are plotted in Fig. 1. In fact requiring $c^2 > 0$ we see that there are solutions not only for $A_7 < 0$, but also for $A_7 > 0$, provided that $Q^2 A_7/M_7^3 < 1/2$. Thus, the bulk cosmological constant does not need to be negative in order to localize gravity.

Similarly, solutions for which $c^2 > 0$ and $\gamma > 0$ exist for values of $n \geq 3$. Again, we find that solutions with both positive, zero and negative bulk cosmological constants exist. In general for these type of solutions, the 3-brane tension components satisfy

$$\mu_0 = \mu_0 - c \gamma^{(n-1)/2} M_D^{n+2},$$

(59)

where $\mu_0$ remains undetermined.

The nice property of these solutions (51) is that since $\gamma$ is a constant the Ricci scalar does not blow up at any point in the transverse space. In particular for the $n = 2$ solution the Ricci scalar is $R = -5/2 A_6/M_6^4$, while for $n = 3$ it is

$$R = -\frac{1}{2Q^2} \left[ \frac{35}{16} + c^2 A_7^2 M_7^2 + \frac{7}{8} \left( \frac{5}{2} - 8Q^2 A_7^2 M_7^2 \right) \right].$$

(60)

The space is again a constant curvature space, but it is not necessarily anti-de Sitter. This is also confirmed by checking the higher-order curvature invariants $R_{AB} R^{AB}$ and $R_{ABCD} R^{ABCD}$.

Finally, we see from (36) that the equation of motion for the spin-2 radial modes using the solution (51) is qualitatively similar to the 5d case. The constant $\gamma$ factor effectively plays no role in the localization of gravity. Thus, the corrections to Newton’s law will be suppressed by $1/r^2$ for all solutions $n \geq 3$. This is easy to understand since the geometry of the extra dimensions is simply $\mathbb{R}_+ \times S_{n-1}$, as in the global defect case, and there is just one non-compact dimension for all $n \geq 3$. In the special case when $A_D = 0$, the metric solution we have found corresponds to the near-horizon metric of a class of extremal nondilatonic black branes [25].

Another possible solution for the $p$-form in the bulk includes

$$F_{\mu_1 \cdots \mu_n} = \epsilon_{\mu_1 \cdots \mu_n} \kappa(\rho),$$

(61)

where

$$\kappa(\rho) = Q \gamma^{(n-1)/2} \frac{1}{\sigma^2} (\sin \theta_{n-1})^{(n-2)} \cdots \sin \theta_2. $$

(62)

In this case the contribution to the stress-energy tensor is

$$T_{\nu} = \frac{n!}{4} \frac{Q^2}{\sigma^4} \delta_{\nu},$$

(63)

$$T_{\rho} = T_0^\rho = -\frac{n!}{4} \frac{Q^2}{\sigma^4}. $$

(64)

This contribution does not appear to make the solution of the Einstein equations any easier.

The above two $p$-form solutions have only included components in the transverse space. If we also require the $p$-form field to transform nontrivially under the 3-brane coordinates, then we can have

$$F_{01230 \cdots 0} = Q (\sin \theta_{n-1})^{(n-2)} \cdots \sin \theta_2,$$

(65)

where all other components of $F$ are zero. The components of the stress-energy tensor for this field configuration are

$$T_{\nu} = \frac{(n+3)!}{4} \frac{Q^2}{\sigma^4 \gamma^{n-1}} \delta_{\nu},$$

(66)

$$T_{\rho} = -T_0^\rho = -\frac{(n+3)!}{4} \frac{Q^2}{\sigma^4 \gamma^{n-1}}. $$

(67)

Again, there is no simple solution of the Einstein equations with the inclusion of this contribution.

Finally, one can also generalize the solution (61)

$$F_{\mu \nu \alpha_1 \cdots \alpha_n} = \epsilon_{\mu \nu \alpha_1 \cdots \alpha_n} \kappa(\rho),$$

(68)

where

$$\kappa(\rho) = Q \sigma^2 \gamma^{(n-1)/2} (\sin \theta_{n-1})^{(n-2)} \cdots \sin \theta_2,$$

(69)
and the stress-energy tensor
\[ T^M_N = \frac{(n+4)!}{4} Q^2 \delta^M_N. \] (70)
is a constant. Thus, we see that the addition of a \((n + 4)\)-form field is equivalent to adding a bulk cosmological constant.

6. Conclusion

We have seen that higher-dimensional solutions exist which can localize gravity to the 3-brane. The generalization of the exponential solution found in Refs. [6,10], only exists when a scalar field with “hedgehog” configuration is added to the bulk. In this case the transverse space no longer has constant curvature, and furthermore it develops a singularity at \( \rho = \infty \). The corrections to Newton’s law on the 3-brane are suppressed by \( 1/\rho^N \). If, however \( r \) is a constant then regular solutions with an exponential warp factor do exist.

However, if instead a \((n - 1)\)-form field configuration is added in the bulk, which generalizes the magnetic field of a monopole, then solutions which localize gravity can be found for positive, zero and negative bulk cosmological constant. In this case, the transverse space has constant curvature but is not an anti-de Sitter space. The corrections to Newton’s law have the same form as in the original model [6]. Furthermore, the addition of an \((n + 4)\)-form field in the bulk is equivalent to adding a bulk cosmological constant.

Given that \( p \)-form fields are generic in string theories, it would be interesting to study whether the exponential solutions that we have found here can be realized in an effective supergravity theory (see also [26]). It is encouraging that the embedding of dilatonic “global cosmic strings” in string theory has recently been considered in [21,27].

Acknowledgements

We wish to thank E. Poppitz, S. Randjbar-Daemi, P. Tinyakov and V. Rubakov for helpful discussions.

This work was supported by the FNRS, contract no. 21-55560.98.

References

Large $N$ limit of higher derivative extended $CP(N)$ model

Taichi Itoh $^{a,b}$, Phillial Oh $^{a,*}$

$^{a}$ Department of Physics and Institute of Basic Science, Sungkyunkwan University, Suwon 440-746, South Korea
$^{b}$ Department of Physics, Kyungpook National University, Taegu 702-701, South Korea

Received 21 June 2000; accepted 2 September 2000

Editor: M. Cvetič

Abstract

We construct a fourth-order derivative $CP(N)$ model in $1+1$ dimensions by incorporating the topological charge density squared term into the Lagrangian. We quantize the theory by reformulating with auxiliary fields and then performing the path integral explicitly. We discuss the renormalizability in the large $N$ limit and relevance of the effective action with axion physics.

© 2000 Elsevier Science B.V. All rights reserved.

PACS: 11.10.Gh; 11.15.-q; 11.15.Pg; 11.30.-j; 14.80.Mz

1. Introduction

The $(1+1)$-dimensional $CP(N)$ model has been studied intensively in the large $N$ limit [1]. It is known to be exactly soluble, and exhibits interesting phenomena such as dynamical mass generation and asymptotic freedom which is also one of the essential properties of gauge theories [1]. Therefore it is important to extend the $CP(N)$ model and investigate various properties. In this Letter, we consider a higher derivative extension of the model. More specifically, we invent a fourth-order derivative interaction by utilizing the topological charge density on the $CP(N)$. One of the merit of our higher derivative theory is that it permits an auxiliary field formalism and the path integral can be performed explicitly. The theory has only logarithmic divergence in the large $N$ limit, still it remains nonrenormalizable due to the lack of counter terms. But we find that the effective theory is well-defined and describes a two-dimensional axion interacting with Maxwell theory.

We start with the coadjoint orbit approach to nonlinear sigma model [2] for some generality and introduce a coadjoint orbit variable $Q = gKg^{-1}$ on $G/H$, where $g \in G$ and $K$ which belongs to the Lie algebra $\mathcal{H}$ of $H$ is the centralizer for $\mathcal{H}$. In order to construct higher derivative theory, we consider $I \equiv \epsilon^\mu\nu\partial_\mu Q \partial_\nu Q$ which is the topological charge density on the coadjoint orbit $G/H$. The topological charge $T = c_1 \int d^2x \ I$ with some normalization constant $c_1$ is completely characterized by the homotopy group $\pi_1(H)$. It is well known that this group is in general given by (sum of) additive group of integers depending on the nature of the centralizer $K$, and the topological charge $T$ characterizes the instanton solution of the nonlinear sigma model described by the field $Q$ [3]. Our higher derivative model is constructed...
by adding the fourth derivative term $t^2$ to the nonlinear sigma model:

$$S = \frac{1}{g^2} \int d^2x \left( -\frac{1}{2} (\partial_\mu Q \partial^\mu Q) + \frac{t^2}{2M^2} \right),$$

(1)

where $(\cdots)$ denotes trace and $M$ is some mass scale in the theory. In $SU(2)$ case, the above model reduces to the two-dimensional low energy effective description of QCD which was proposed by Faddeev and Niemi [4]. Also, it is related with Skyrmion model, because $t^2$ can be written as $t^2 \sim (J_{\mu\nu} J^{\mu\nu})$, $J_{\mu\nu} = [\partial_\mu Q, \partial_\nu Q]$.

Let us focus on $CP(N)$ case and study the large $N$ limit of (1) by using the path integral quantization method. We will show that the above higher derivative theory admits an exact path integration in the large $N$ limit. We will show that the above model reduces to the two-dimensional low energy effective description of QCD which was proposed by Faddeev and Niemi [4]. Also, it is related with Skyrmion model, because $t^2$ can be written as $t^2 \sim (J_{\mu\nu} J^{\mu\nu})$, $J_{\mu\nu} = [\partial_\mu Q, \partial_\nu Q]$.

Let us focus on $CP(N)$ case and study the large $N$ limit of (1) by using the path integral quantization method. We will show that the above higher derivative theory admits an exact path integration in the large $N$ limit. We will show that the above model reduces to the two-dimensional low energy effective description of QCD which was proposed by Faddeev and Niemi [4]. Also, it is related with Skyrmion model, because $t^2$ can be written as $t^2 \sim (J_{\mu\nu} J^{\mu\nu})$, $J_{\mu\nu} = [\partial_\mu Q, \partial_\nu Q]$.

Let us focus on $CP(N)$ case and study the large $N$ limit of (1) by using the path integral quantization method. We will show that the above higher derivative theory admits an exact path integration in the large $N$ limit. We will show that the above model reduces to the two-dimensional low energy effective description of QCD which was proposed by Faddeev and Niemi [4]. Also, it is related with Skyrmion model, because $t^2$ can be written as $t^2 \sim (J_{\mu\nu} J^{\mu\nu})$, $J_{\mu\nu} = [\partial_\mu Q, \partial_\nu Q]$.
where we separate the Goldstone boson mass $m^2$ from the original Lagrangian.

The vacuum polarization functions are given by

$$
S_{\text{eff}} = \int d^2 x \mathcal{L} + i \frac{1}{2} \sum_{n=1}^{\infty} \left( \frac{1}{n} \right)^2 \text{Tr} \left[ -\partial^2 - m^2 \right] n.
$$

After some straightforward calculations with momentum cutoff $\Lambda$ in a gauge invariant way, the effective action up to quadratic terms ($n = 1, 2$) is obtained as

$$
S_{\text{eff}} = N \int d^2 x \left[ \frac{1}{N g^2} \mathcal{L}_{\text{bare}}^{\text{bare}} \left[ -\partial^2 - m^2 - \Lambda \right] \right] z
- \frac{1}{2} \frac{M^2}{N g^2} b^2 + \left( m^2 + \frac{\lambda}{\Lambda} \right) \left[ \frac{1}{N g^2} - \frac{4 \pi^2}{m^2} \right]
- \frac{1}{4 \pi} m^2 + \frac{1}{2} \lambda \Pi_\lambda \left( i \partial \right) \tilde{\lambda}
- \frac{1}{4} F_{\mu \nu} \Pi A (i \partial) F^{\mu \nu} + \frac{1}{2} b \Pi b A (i \partial) \epsilon^{\mu \nu} F_{\mu \nu}
- \frac{1}{4} b \bar{\partial}^2 \Pi b A (i \partial) b.
$$

The scale invariance condition $\Lambda dm/d\Lambda = 0$ leads us to the Callan–Symanzik $\beta$-function

$$
\beta(g) = \frac{dg}{d\Lambda} = -\frac{N g^3}{4 \pi},
$$

which shows the asymptotically free behavior of the coupling. In the original $CP(N)$ the only divergence is the one which arises in the large $N$ effective action through a tadpole diagram coupled with $\tilde{\lambda}$, so that the scale invariance condition $\Lambda dm/d\Lambda = 0$ is enough to achieve the cutoff independent theory. Since $m$ is scale invariant, the gap equation (14) suggests the renormalization of coupling is given by

$$
\frac{1}{N} V_{\text{eff}} = -\frac{m^2}{N g^2} + \frac{m^2}{4 \pi} \left( \frac{A^2}{m^2} + 1 \right).
$$

We notice that $m$ can be independent of the ultraviolet (UV) cutoff $\Lambda$ by imposing $\Lambda$ dependence on the coupling $g$. In fact the scale invariance condition $\Lambda dm/d\Lambda = 0$ leads us to the Callan–Symanzik $\beta$-function

$$
\beta(g) = \frac{dg}{d\Lambda} = -\frac{N g^3}{4 \pi},
$$

which shows the asymptotically free behavior of the coupling. In the original $CP(N)$ the only divergence is the one which arises in the large $N$ effective action through a tadpole diagram coupled with $\tilde{\lambda}$, so that the scale invariance condition $\Lambda dm/d\Lambda = 0$ is enough to achieve the cutoff independent theory. Since $m$ is scale invariant, the gap equation (14) suggests the renormalization of coupling is given by

$$
\frac{1}{N} V_{\text{eff}} = -\frac{m^2}{N g^2} + \frac{m^2}{4 \pi} \left( \frac{A^2}{m^2} + 1 \right).
$$

where $g_R$ is the renormalized coupling at the reference energy scale $\mu$.

In the extended model, however, logarithmic divergences arise in the coefficients of the induced extra terms, $b \bar{\partial}^2 b$ and $\bar{b} \epsilon^{\mu \nu} F_{\mu \nu}$ which do not have their counter terms in the classical action. Therefore our theory is not renormalizable though the coupling $g$ can be renormalized in the same way as in the original $CP(N)$ model. Let us look at how this argument works in the large $N$ effective action (9). The induced kinetic terms of $A_\mu$ and $\lambda$ are UV finite in themselves so that we do not need wave function renormalization.
for them. Then the third term in the right-hand side of Eq. (9) becomes UV finite through Eq. (17) from which the $Z$ factor for the coupling can be read

$$Z^{-1} = \frac{s_R}{g^2} = 1 + \frac{Ng^2}{4\pi} \ln \frac{A^2}{\mu^2}. \quad (18)$$

The kinetic term of $z$ has to be UV finite in itself so that we see

$$\frac{1}{Ng^2} z^\dagger \left[ -\partial^2 - m^2 \right] z = \frac{1}{Ng^2} z_R^\dagger \left[ -\partial^2 - m^2 \right] z_R. \quad (19)$$

where $z_R$ has been introduced through $z = Z_c^{1/2} z_R$ and $Z_c$ is thereby determined as $Z_c \equiv Z$ in order to cancel the $Z$ factor from the coupling renormalization. Thus we realize that $\Gamma$ in the effective action has to remain invariant through renormalization procedure. This forces all of $A_\mu$, $\lambda$ and $b$ to be unchanged through renormalization. Thus we obtain another renormalization condition

$$\frac{M^2}{Ng^2} = \frac{M_R^2}{Ng_R^2}. \quad (20)$$

which determines the $Z$ factor for $M^2$ as $Z_M \equiv Z$. On the other hand, we do not have any degrees of freedom to subtract the logarithmic divergences which arise in the kinetic term of $b$ and in the mixing term between $b$ and $A_\mu$. This makes our extended model nonrenormalizable even after the $1/N$ resummation.

Even though the theory is not renormalizable, the effective theory is a well-defined renormalizable theory. In fact, by using the gap equation (14), the logarithmic $\Lambda$ dependence of $\Pi_{b\Lambda}(i\partial)$ in (9) can be completely eliminated in favor of the $1/N$ counting rule $Ng^2 \equiv$ fixed, and we see that the effective action for $A_\mu$ and $b$ field is given by

$$S_{\text{eff}}[A_\mu, b] = \frac{1}{g^2} \int d^2x \left[ \frac{c}{4} \left( \partial_\mu b \right) \left( \partial^\mu b \right) + \frac{c}{2} b \epsilon^{uv} F_{\mu v} - \frac{M^2}{2} b^2 \right] - \frac{N}{48\pi m^2} \int d^2x F_{\mu \nu} F^{\mu \nu}, \quad (21)$$

where we have expanded the vacuum polarization functions with respect to $p^2 / 4m^2$ after the analytic continuation from $p^2 < 0$ to $0 \leq p^2 < 4m^2$. Here, the constant $c \equiv Ng^2 / 4\pi - 1$ confines the physical region of coupling to $g^2 > 4\pi/N$. This is a two-dimensional model in which the Maxwell field $A_\mu$ interacts with a pseudoscalar field $b$ with $b^* F$ axion type interaction [6]. We note that the potential prefers $\langle b \rangle = 0$ for the minimum, and the effect of $CP$ violating term $\langle b \rangle^* F$ will be suppressed.

The above argument has meaning only when we suppose that the large $N$ effective gauge theory has instanton solutions [7]. However, these solutions are usually destroyed by the next to leading order corrections in $1/N$. Recall also even though the original $CP(N)$ model has instanton solutions [3], the inclusion of our higher derivative term does not admit any instanton solutions. Nevertheless, the aforementioned suppression mechanism may provide a useful toy model for the strong $CP$ problem [8].

4. Conclusion

In summary, we have constructed a higher derivative $CP(N)$ model and quantized it by using the path integral method. We have illustrated that in the large $N$ limit, the ultraviolet divergence can be completely isolated into a logarithmic divergence but the theory remains nonrenormalizable due to the lack of counter terms. However, we have found that the effective action is a well-defined renormalizable theory, and it describes a massless gauge field interacting with massive axion. It would be interesting to check whether such a composite higher derivative axion model could have some relevance in higher dimensional axion physics.

Acknowledgements

T.I. was supported by KOSEF Postdoctoral Fellowship and Korea Research Center for Theoretical Physics and Chemistry, P.O. was supported by the Samsung Research Fund, Sungkyunkwan University, 1999. This work was also partially supported by BK21 Physics Research Program.

References


M.S. Turner, Phys. Rep. 197 (1990) 1;
The (2 + 1)-dimensional NJL model at finite temperature

Thomas Appelquist a,*, Myck Schwetz b

a Department of Physics, Yale University, New Haven, CT 06511, USA
b Department of Physics, Boston University, Boston, MA 02215, USA

Received 2 August 2000; received in revised form 1 September 2000; accepted 15 September 2000

Editor: H. Georgi

Abstract

We describe properties of (2 + 1)-dimensional Nambu–Jona-Lasinio (NJL) models at finite temperature, beginning with the model with a discrete chiral symmetry. We then consider the model with a continuous $U(1) \times U(1)$ chiral symmetry, describing the restoration of the symmetry at finite temperature. In each case, we compute the free energy and comment on a recently proposed constraint based upon it. We conclude with a brief discussion of NJL models with larger chiral symmetries.

1. Introduction

The behavior of QCD at finite temperature and density has received renewed attention recently with the coming of the RHIC experimental program. The study of this phase structure is difficult for any strongly coupled quantum field theory, and it can be helpful to examine the problem in nontrivial but tractable models. In particular, theories in less than four spacetime dimensions can offer interesting and complex behavior as well as tractability, often in the form of large-$N$ expansions. In the case of three spacetime dimensions, they can even be directly physical, describing various planar condensed matter systems. Models of interest for the study of finite temperature and density involve fermionic degrees of freedom and fall into two broad classes. One utilizes four-fermion interactions of the Nambu–Jona-Lasinio (NJL), or Gross–Neveu, form, and the other includes three dimensional gauge theories and closely related Thirring models.

We restrict our attention in this paper to NJL models at finite temperature $T$. The features we discuss are interesting in their own right and will play an important role in understanding temperature–density phase diagrams for these models. All the analysis will make use of a $1/N_f$ expansion where $N_f$ is the number of fermion species. The theories under consideration are renormalizable in the $1/N_f$ expansion unlike in the loop expansion [2]. The expansion is reliable for all temperatures $T$ except those in the vicinity of a certain critical temperature $T_c$.

The question of interest is the spontaneous breaking of chiral symmetry. Although this is in some ways easier to handle in three spacetime dimensions (3D) than in four, there is an important subtlety in the case of continuous symmetry. The Coleman–Mermin–Wagner (CMW) theorem stipulates that the sponta-
neous breaking of a continuous symmetry cannot happen in 3D at finite $T$ [1]. This statement is related to the fact that it is impossible to write down a consistent theory of massless scalars in 2D. If a spontaneous breaking of continuous symmetry were to happen at finite $T$, then one would be faced with this problem at momentum scales below $T$, i.e., it would be impossible to construct an effective 2D theory of the Goldstone bosons’ zero-modes. We then turn to the 3D NJL model with a continuous restoration at all with a continuous symmetry; in Section 3, we treat the NJL model and apriori may or may not satisfy this condition. This problem does not arise. The broken symmetry at finite temperature $T_c$ remains broken at finite $T$ up to a critical value $T_c$. We then turn to the 3D NJL model with a continuous $U(1) \times U(1)$ chiral symmetry, exploring symmetry restoration at all $T > 0$. A critical temperature $T_c$ continuous to exist, but it now marks a transition from the ordinary symmetric phase at high temperature to a low temperature Kosterlitz–Thouless phase that is also chirally symmetric. The broken phase exists only at $T = 0$.

For each model, we also compute the thermodynamic free energy and enumerate the thermodynamic degrees of freedom. In a recent paper [3], the free energy $F(T)$ was used as the basis for a proposed constraint on the behavior of asymptotically free theories. It was observed that for any theory governed by a fixed point in the ultraviolet (UV) or infrared (IR), the dimensionless quantity $f(T) = -2\pi I_3 [F(T) - F(0)] / T^3$ approaches a finite value in the corresponding limit ($f_{UV} = f(T \rightarrow \infty)$ and $f_{IR} = f(T \rightarrow 0)$), and counts the (effectively massless) degrees of freedom if the fixed point is trivial. It was noted that $f_{UV} \geq f_{IR}$ for all known asymptotically free theories in which the IR behavior is also free, or weakly interacting, allowing $f_{IR}$ to be computed. It was conjectured that this is true for all asymptotically free theories. The theories considered in this paper are governed by nontrivial UV fixed points and apriori may or may not satisfy this condition.

The outline of the paper is as follows: in Section 2, we review the NJL model with a discrete chiral symmetry; in Section 3, we treat the NJL model with a continuous $U(1) \times U(1)$ chiral symmetry; we summarize and briefly describe NJL models with larger chiral symmetries in Section 4.

2. Discrete chiral symmetry

The NJL model with a discrete chiral symmetry for $N_f$ copies of Dirac fermions is described by the following Lagrangian:

$$\mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{g_0^2}{2N_f} (\bar{\psi} \gamma^\mu \gamma^5 \psi)^2, \quad (1)$$

where $\Psi$ may be taken to be a $4N_f$-component fermion field. The discrete symmetry is $\Psi \rightarrow \gamma_5 \Psi$, and $g_0$ is the coupling. We analyze the model using the $1/N_f$ expansion.

We work first to leading order and then note that for this discrete-symmetry model the qualitative behavior is not modified in higher orders. It is convenient to introduce an auxiliary field $\sigma$ coupled to $\bar{\psi}\psi$. The finite temperature effective potential as a function of $\sigma$ may be computed to this order by integrating out the fermions:

$$V_{\text{eff}}(\sigma)/N_f = \frac{\sigma_0^2}{2g_0^2} + i \text{Tr} \ln(i\sigma - \sigma_0), \quad (2)$$

with the second term evaluated at finite temperature.

The only cutoff dependence is in the renormalization of $g_0$, and it may be carried out at zero temperature. Extremizing the zero-temperature effective potential $\partial V_{\text{eff}}/\partial \sigma = 0$ gives

$$\frac{1}{g_0^2} = 4 \int^\Lambda (2\pi)^3 \frac{1}{p^2 + \sigma_0^2} = \frac{2}{\Lambda^2} \left( 1 - \sigma_0 \tan^{-1} \frac{\Lambda}{\sigma_0} \right), \quad (3)$$

where $\Lambda$ is an ultraviolet cutoff and where the extremal value $\sigma_0$ describes the fermion mass gap at zero temperature. As $g_0^2 \Lambda$ approaches the critical value $\pi^2/2$ from above, $\sigma_0/\Lambda \rightarrow 0$. Equivalently, the cutoff may be removed holding $\sigma_0$ fixed providing that $g_0^2 \Lambda$ is tuned to $\pi^2/2$. With this prescription, it can be seen that the high energy behavior of the theory is described by a nontrivial but weak ultraviolet fixed point. That is, a finite effective four-fermion interaction $\tilde{g}^2(p)/2N_f$ is induced such that for $p > \sigma_0$, the dimensionless quantity $\tilde{g}^2(p) \cdot p \rightarrow O(1)$.

---

1 We adopt the notation of [2] for $\gamma$-matrices: $\gamma^\mu = \sigma^\mu \otimes (1 \ 0) \quad \text{and} \quad i\gamma_5 = (0 \ -1) \quad \text{in} \ (2 + 1)$-dimensional Minkowski space with $(1, -1, -1)$ signature.
To leading order in $1/N_f$, the zero-temperature effective potential is now
\[ V_{\text{eff}}(\sigma)/N_f = \frac{1}{3\pi} \sigma^3 - \frac{\sigma_0}{2\pi} \sigma^2 - T^3 \int_{0}^\infty \frac{dx}{\pi} \times \ln \left(1 + \exp \left[-\sqrt{x + \frac{\sigma^2}{T^2}}\right]\right). \]  
(5)

This leads to the critical temperature $T_c = \sigma_0/(2\ln 2)$, above which $\sigma_T$ vanishes. The existence of a finite $T_c$ for a discrete symmetry in $(2+1)$ dimensions is perfectly acceptable and this qualitative feature is not changed by the higher order terms in the $1/N_f$ expansion.

Before discussing higher order terms, we comment on the thermodynamic free energy of this model which is given by $F(T) = V_{\text{eff}}(\sigma_T)$. Then, to leading order in $1/N_f$, the $f(T)$ defined in the introduction will take the form
\[ f(T) = -\frac{2N_f}{\pi (3T)^3} \left[ \frac{\sigma_T^3}{3} - \frac{\sigma_0}{2} \sigma_T^2 + \frac{\sigma_0^3}{6} \right] + 2T^3 \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^3} \left(1 + k \frac{\sigma_T}{T}\right) \exp \left[-k \frac{\sigma_T}{T}\right]. \]  
(7)

where $\sigma_T$ is determined by the gap equation (6). It can be shown that this function increases monotonically from $f_{\text{IR}} = 0$ at $T = 0$ to $f_{\text{UV}} = 3N_f$ at $T = \infty$. The infrared value, $f_{\text{IR}} = 0$, is expected because a mass gap develops for $T < T_c$. The ultraviolet value, $f_{\text{UV}} = 3N_f$, counts the number of fermionic degrees of freedom ($4N_f$) times the Fermi–Dirac factor $(3/4)$.

This is expected because the model is governed in the ultraviolet by a weak UV fixed point — there will be higher order corrections in the $1/N_f$ expansion. Monotonicity may be established by noting that
\[ \frac{df(\sigma_T, T)}{dT} = \frac{\partial f}{\partial \sigma_T} \frac{\partial \sigma_T}{\partial T} + \frac{\partial f}{\partial T} = \frac{\sigma_T^3}{\xi(3)^3} \left(\frac{\sigma_T}{2} \sigma_T^2 + \frac{\sigma_0^3}{2}\right) + \frac{2}{\xi(3)^3} \sigma_T^2 \ln \left(1 + \exp \left[-\frac{\sigma_T}{T}\right]\right). \]  
(8)

The first term is always non-negative since $\sigma_T \leq \sigma_0$; the second term is always positive.

Finally we describe the effect of the higher order terms in the $1/N_f$ expansion. Through next order, the effective potential is given by
\[ V_{\text{eff}}(\sigma)/N_f = \frac{\sigma_T^2}{2g_0} + i \ln(i\beta - \sigma) + \frac{i}{2N_f} \ln D_\sigma^{-1}, \]  
(9)

where $D_\sigma^{-1}(p^2)$ is the inverse $\sigma$ propagator computed in a background $\sigma$ field at finite temperature $T$. At zero temperature, this propagator is given by
\[ D_\sigma^{-1}(p^2) = N_f \left( \frac{1}{g_0^2} - i \int \frac{d^3 q}{(2\pi)^3} \right) \times \text{Tr} \left[ \frac{1}{\beta - \sigma \beta - \beta - \sigma} \right]. \]  
(10)

Using the leading order gap equation (3) and taking the limit $\Lambda \to \infty$ with $\sigma_0$ fixed, $D_\sigma^{-1}(p^2)$ takes the form
\[ D_\sigma^{-1}(p^2) = N_f \left[ \sigma - \sigma_0 + \frac{p^2 + 4\sigma_0^2}{2\sigma} \tan^{-1} \frac{\sqrt{p^2}}{2\sigma} \right]. \]  
(11)

For nonzero temperature $D_\sigma^{-1}(p^2)$ is given in Ref. [4]. Expressions, similar to Eq. (9) may be written down for higher order terms.

For $T$ well above $T_c$, the $1/N_f$ expansion for $V_{\text{eff}}(\sigma)$ may be seen to converge for large $N_f$. As $T \to T_c$ from above, however, $m^2 \sim T - T_c$. In this limit, higher order terms in $V_{\text{eff}}(\sigma, \pi)$ become singular and trigger the breakdown of the $1/N_f$ expansion. They may be described by an effective 2D, Landau–Ginzburg theory, consisting of the zero mode of
the $\sigma$ field, relevant at scales below $T$. The terms in the Landau–Ginzburg Lagrangian, in addition to the common mass term $m^2\sigma^2/2$, are a kinetic term, and a $\lambda \sigma^4/4!$ interaction term. Each may be computed to any order in $1/N_f$ by integrating out the fermions. To leading order, each arises with coefficient $N_f$.

Using this effective 2D theory and counting infrared powers, it may be seen that the effective dimensionless expansion parameter for $V_{\text{eff}}(\sigma)$, for small $\sigma$, is of the form

$$\frac{1}{N_f} \frac{T^2}{M^2},$$

where $M^2 = m^2 + \lambda \sigma^2$, up to logarithmic corrections. Thus in the neighborhood of the origin, the $1/N_f$ expansion breaks down for $|T - T_c|/T \lesssim 1/N_f$. In particular, it breaks down for the free energy and $f(T)$. This is true also as $T$ approaches $T_c$ from below, as indicated by the absolute value sign [6].

The $1/N_f$ expansion remains convergent as long as $|T_c - T|/T \gg 1/N_f$. In the high temperature limit, it correctly describes the effective potential and free energy in the symmetric phase, leading to small corrections to the leading order estimate $f_{\text{UV}} = 3N_f$. In the low temperature limit, it describes the broken phase, and because of the mass gap, leads to $f_{\text{IR}} = 0$ to all orders. The inequality $f_{\text{UV}} \geq f_{\text{IR}}$ is satisfied.

3. Continuous $U(1) \times U(1)$ chiral symmetry

The NJL model with a continuous $U(1) \times U(1)$ symmetry $\Psi \to \exp(i\gamma_5)\Psi, \Psi \to \exp(i\beta)\Psi$ is described by the Lagrangian [2]:

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi + \frac{g_0^2}{2N_f} \left[ (\bar{\Psi}\gamma^\mu \partial_\mu \Psi)^2 - (\bar{\Psi}\gamma_5 \Psi)^2 \right].$$

The model may be analyzed to leading order in the $1/N_f$ expansion by introducing auxiliary fields $\sigma$ and $\pi$ and integrating out the fermions. The leading-order, finite-temperature effective potential takes the form

$$\frac{V_{\text{eff}}(\sigma, \pi)}{N_f} = \frac{\sigma^2 + \pi^2}{2g_0^2} + i \text{Tr} \ln(i\sigma - i\gamma_5\pi).$$

As in the case of discrete symmetry, the renormalization may be performed by extremizing the effective potential ($\partial V_{\text{eff}}/\partial \sigma = 0$ and $\partial V_{\text{eff}}/\partial \pi = 0$) at zero temperature. The vacuum expectation values may be rotated using the chiral symmetry so that $\pi$ possesses a vanishing expectation value. The zero temperature gap equation then takes the same form as in the discrete-symmetry model:

$$1 = \frac{2}{\pi^2} \left( \frac{g_0^2 A}{\sigma_0} \right) \left( 1 - \frac{a_0}{\Lambda} \tan^{-1} \frac{1}{\Lambda} \right),$$

where $A$ is an ultraviolet cutoff. As before, $\sigma_0$ may be held fixed in the “continuum” limit $A \to \infty$ ($g_0^2 A \to \pi^2/2$) describing the fermion mass gap at zero temperature. The chiral symmetry is broken and the $\pi$ field describes the associated Goldstone boson.

The zero temperature effective potential to leading order in $1/N_f$ now reads:

$$\frac{V_{\text{eff}}(\sigma, \pi)}{N_f} = \frac{1}{3\pi} \left( \frac{\sigma^2 + \pi^2}{\sigma_0^2} \right)^{3/2} - \frac{\sigma_0}{2\pi} \left( \frac{\sigma^2 + \pi^2}{\sigma_0^2} \right),$$

which has degenerate minima at $\sigma^2 + \pi^2 = \sigma_0^2$ and is convex as a function of two variables if $\sigma^2 + \pi^2 \geq \sigma_0^2$.

At finite temperature, the leading order effective potential has the same form as Eq. (5), with the replacement $\sigma^2 \to \sigma^2 + \pi^2$:

$$\frac{V_{\text{eff}}(\sigma, \pi)}{N_f} = \frac{1}{3\pi} \left( \frac{\sigma^2 + \pi^2}{\sigma_0^2} \right)^{3/2} - \frac{\sigma_0}{2\pi} \left( \frac{\sigma^2 + \pi^2}{\sigma_0^2} \right)$$

$$- T^3 \int_0^\infty \frac{dx}{\pi} \ln \left[ 1 + \exp \left( - \frac{\sigma^2 + \pi^2}{T^2} \right) \right],$$

Extremizing it suggests that, as in the case of discrete symmetry, the broken symmetry of the zero temperature theory remains broken at finite temperatures below a critical value $T_c = \sigma_0/2\ln 2$. But unlike the discrete case, this conclusion cannot be correct since the zero-mode of the associated Goldstone boson would describe an effective 2D theory with a spontaneously broken continuous symmetry — in contradiction with the Coleman–Mermin–Wagner theorem.

We explore the resolution of this problem by first noting that as in the case of discrete symmetry, the $1/N_f$ expansion is convergent as long as $T$ is not near the transition temperature $T_c$. $(|T_c - T|/T \gg 1/N_f)$. The next order term in $V_{\text{eff}}(\sigma, \pi)/N_f$, for example, may be written in the form

$$\frac{i}{2N_f} \text{Tr} \ln D^{-1}_\sigma + \frac{i}{2N_f} \text{Tr} \ln D^{-1}_\pi,$$

where $D_{\sigma, \pi}$ are $2 \times 2$ matrices with $\sigma$ and $\pi$ matrices as entries.
where $D^{-1}_T$ and $D^{-1}_Y$ are functions of $T$, $T_c$, momentum, and field strength $\rho$. This term can be seen to be of order $1/N_f$ for $|T - T_c|/T \gg 1/N_f$. But as in the case of discrete symmetry, the $1/N_f$ expansion breaks down due to infrared singularities when $|T - T_c|/T \lesssim 1/N_f$, with the singular terms describable by an effective 2D Landau–Ginzburg Lagrangian.

For $T \gg T_c$, the theory is in the symmetric phase with a convergent $1/N_f$ expansion. For $T \ll T_c$, even though the $1/N_f$ expansion is again convergent, the continuous symmetry model is, unlike the discrete symmetry model, not in the broken phase. The chiral symmetry remains unbroken, although in a very different than in the high temperature range. To see this, it is convenient to use the following parametrization of the auxiliary fields for $T < T_c$ [5]:

$$\sigma + i\pi = \rho e^{i\theta}.$$  \hspace{1cm} (19)

The leading order potential, Eq. (17), then depends only on $\rho$, and extremization leads to a non-zero VEV $\rho_T$, equal to $\rho_0 \equiv \sigma_0$ at $T = 0$ and vanishing like $(T - T_c)^{1/2}$ as $T \to T_c$. The fermion has mass $\rho_T$, the fluctuations of $\rho$ have mass $2\rho_T$ and there is a massless scalar field $\theta$.

This behavior implies symmetry breaking, however, only if $\theta$ takes on some fixed and non-zero vacuum value. As is well known, this does not happen in 2D since quantum effects generate logarithmically infrared divergent fluctuations. In the present model at finite $T$, the same is true since the long distance behavior is effectively 2D. To explore this in detail, we first restrict attention to $T$ sufficiently below $T_c$ $(|T - T_c|/T \gg 1/N_f)$ so that the $1/N_f$ expansion is a reliable tool. The realization of the symmetry may be studied by examining the theory in the infrared, at momentum scales well below $\rho_T$, where the fermions and the $\rho$ fluctuations may be integrated out. The only massless degree of freedom is $\theta$, and if $\rho_T$ and $T$ are of the same order, the non-zero Matsubara frequencies of $\theta$ may also be integrated out. The effective low energy theory describing physics at momentum scales well below $T$ and $\rho_T$ is then a 2D chiral Lagrangian [6,7]:

$$L_{\text{eff}} = \frac{1}{2t} (\theta^2)^2 + \cdots,$$  \hspace{1cm} (20)

where the dots indicate higher derivative terms in $\theta$. The dimensionless coefficient $1/t$ is given to leading order in the $1/N_f$ expansion by

$$\frac{1}{t} = \frac{N_f \rho_T}{4\pi T} \tanh \left( \frac{\rho_T}{2T} \right).$$  \hspace{1cm} (21)

In the very low temperature limit ($T \ll T_c$), physics at momentum scales of order $T$ as well as below it may be described by an effective theory obtained by integrating out the fermions and the $\rho$ fluctuations, but keeping all the Matsubara frequencies of $\theta$. This theory is described the effective chiral Lagrangian (20), but in three dimensions rather than two.

Since the underlying theory is Abelian, this effective Lagrangian describes a free massless scalar field $\theta$. Chiral symmetry breaking depends on the behavior of correlation functions of physical, and therefore single valued functions of $\theta$ ($\cos \theta$, $\sin \theta$, or equivalently $e^{\pm i\theta}$). This behavior is determined the coefficient $t$ in the chiral Lagrangian, which is of order $1/N_f$ or smaller unless $\rho_T/T$ is small. (This only happens if $T$ approaches $T_c$, and we are avoiding that limit now to insure convergence of the $1/N_f$ expansion.) Analysis of this 2D theory, including the role of vortex solutions [7], reveals that for small $t$, it is in the Kosterlitz–Thouless [8] phase where the contribution from vortices is negligible, and where, for example, the correlation function $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle$ has the characteristic power-law behavior $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \sim x^{-1/2\pi}$ at distances $x \gg 1/T$. The power-law fall-off, while corresponding to an infinite correlation length, still indicates an absence of long range order. There is no chiral symmetry breaking.

The transition from broken to unbroken chiral symmetry is at $T = 0$. In the limit $T \to 0$ the parameter $t \to 0$, and the range of relevance for the effective 2D theory and the (weakening) power-law fall off moves off to infinity. In the zero temperature 3D theory, the $\theta$ field develops a fixed VEV and describes a Goldstone boson.

At finite $T$, the analysis of the effective Abelian 2D theory leads to the conclusion [7] that when $t$ becomes
of order unity, the interactions become strong, vortex solutions play an important role, effectively renormalizing $t$, and a finite correlation length develops. This is the normal phase corresponding to in-tact chiral symmetry. For the $U(1)$ model being discussed here, $t$ can become of order unity only when $\rho_T/T \to 0$, that is, when $T \to T_c$ from below. To be more precise, since $\rho_T \sim (T_c - T)^{1/2}$ in this limit, $t$ becomes of order unity only when $(T_c - T) / T \lesssim 1/N_f$. But we have already noted that this is the range where the $1/N_f$ expansion breaks down. The $\rho$-field becomes light and must be included along with $\theta$ in an effective 2D Landau–Ginzburg Lagrangian describing the transition at $T_c$, now interpreted to be the transition from the Kosterlitz–Thouless phase to the symmetric phase with finite correlation length. The infrared singularities associated with the 2D description mean that the $1/N_f$ expansion is not directly useful to describe this transition.

Finally, we comment on the behavior of the thermodynamic free energy $F(T) = V_{\text{eff}}(\rho_T)$ for this continuous-symmetry model, and the proposed inequality constraint [3] using the quantity $f(T) = -2 \pi (F(T) - F(0))/\xi(3) T^3$. In the limit $T \to \infty$, the model is weakly coupled (as was the discrete-symmetry model) with the dynamics governed by a nontrivial UV fixed point of strength $1/N_f$. To leading order in $1/N_f$, $f(T)$ takes the same form as in the discrete-symmetry model:

$$f(T) = -\frac{2N_f}{\xi(3) T^3} \left( \frac{\rho_T^3}{3} - \frac{\rho_0^2}{2} \rho_T^2 + \frac{\rho_0^3}{6} \right) + 2 T^3 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3} \left( 1 + k T \rho_T \right) \exp \left[ -k T \rho_T \right].$$

Thus to this order, $f_{\text{UV}} = 3N_f$. There will be $O(1)$ corrections to this result due to the weakly interacting UV fixed point.

In the low temperature limit ($T \ll T_c$), the discrete- and continuous-symmetry models behave differently. In both cases, the $1/N_f$ expansion for the free energy is convergent, describing in the discrete case a mass gap and leading to $f_{\text{IR}} = 0$ to all orders. In the continuous case, there is one massless degree of freedom described by the $\theta$ field. At momentum scales on the order of $T$ which determine the free energy, its behaviour is fully 3D, governed by an effective 3D chiral Lagrangian of the form of Eq. (20). Since this 3D Lagrangian is non-interacting, $f_{\text{IR}}$ may be computed exactly to give $f_{\text{IR}} = 1$ corresponding to the single massless degree of freedom. This computation of $f_{\text{IR}}$ is, in effect, an evaluation of the next to leading term in the $V_{\text{eff}}$ given by Eq. (18), for $T \ll T_c$. For this model as for the discrete symmetry model, $f_{\text{IR}} < f_{\text{UV}}$.

4. Summary and discussion

We have described the finite temperature phase structure of two 3D NJL models analyzed in a $1/N_f$ expansion where $N_f$ is the (large) number of four-component fermions. In the more familiar case of a discrete chiral symmetry, broken at $T = 0$, there is a transition from broken symmetry to unbroken symmetry at a non-zero temperature $T_c$. The low temperature phase is characterized by a mass gap; since there are no massless degrees of freedom, the quantity $f_{\text{IR}}$ defined in the Section 1 takes the value $f_{\text{IR}} = 0$. The $1/N_f$ expansion breaks down as $T \to T_c$ due to infrared singularities describable by an effective 2D Landau–Ginzburg theory. At high temperatures, the $1/N_f$ expansion is again convergent for large $N_f$ and the theory is governed by a weak $O(1/N_f)$ UV fixed point. One finds $f_{\text{UV}} = 3N_f$ up to corrections of $O(1/N_f)$.

In the case of a continuous $U(1) \times U(1)$ symmetry broken at $T = 0$, there is no spontaneous breaking of the chiral symmetry at $T \neq 0$, although a phase transition still exists at a non-zero temperature $T_c$. The normal high-temperature symmetric phase changes at $T_c$ to a low-temperature phase with its low momentum components ($p \ll T$) described by an effective 2D theory in the Kosterlitz–Thouless (KT) phase and with a power-law behaviour of correlation functions for $x \gg 1/T$. In both finite-$T$ phases, the continuous chiral symmetry is unbroken. The transition from the KT phase to the broken phase is at $T = 0$. There is a single massless degree of freedom for $T < T_c$ which becomes a conventional Goldstone boson in the limit $T \to 0$. As in the case of discrete symmetry, the $1/N_f$ expansion breaks down at the phase transition $T \sim T_c$ due to 2D infrared singularities, but is well behaved away from the transition for large $N_f$. We have nothing to say in this paper about the phase structure of NJL models at small $N_f$ [11].
The quantity $f_{IR}$ defined in the Section 1 takes the value $f_{IR} = 1$, reflecting the presence of a single massless degree of freedom below $T_c$. The UV fixed point leads to the value for $f_{UV} = 3N_f$ up to corrections of $O(1/N_f)$ as in the discrete case.

For both models, the exploration of the transition at $T = T_c$ requires methods that are non-perturbative in $1/N_f$. For the discrete case, $\epsilon$-expansion methods ($4 - \epsilon$ dimensions) may be brought to bear although convergence is problematic since $\epsilon = 2$. In the continuous case, the conventional $\epsilon$ expansion is not useful since the model is in the broken phase for $T < T_c$ for any $\epsilon < 2$. In both cases, the breakdown of the $1/N_f$ expansion is associated with finite-temperature infrared divergences characteristic of a theory in less than four spacetime dimensions, and has no direct counterpart in four-dimensional theories.

It is interesting to generalize our discussion to NJL models with larger continuous chiral symmetries. Suppose, for example, that the continuous $U(1) \otimes U(1)$ symmetry of (13) is extended to a $U(n) \otimes U(n)$ symmetry which at zero temperature breaks to the diagonal $U(n)$ producing $n^2$ Goldstone modes. For the case $n^2 \ll N_f$, the $1/N_f$ expansion may be used to analyze the symmetry breaking pattern at zero and finite temperatures. An order by order analysis concludes that for $T > T_c \sim \rho_0$ (where $\rho_0$ is a zero temperature VEV of an auxiliary field) the model will be in the symmetric phase. As in the Abelian case the $1/N_f$ expansion breaks down when $T \sim T_c$ due to 2D infrared singularities. For $T < T_c$, the fermions are massive, as in the Abelian model, and may be integrated out to describe lower energy physics. At momentum scales below $T$, an effective 2D chiral Lagrangian emerges, which naturally breaks into two parts. One is the free Abelian model we considered previously and the other is an $SU(n) \times SU(n)$ chiral Lagrangian. The latter is interacting and asymptotically free [6,12,13].

The Abelian model will be in the KT phase as long as $T$ is not close to $T_c$, ensuring that the coupling strength is small — of order $1/N_f$. At intermediate momentum scales, $p < T$, the coupling strength of the non-Abelian part is also small — of order $1/N_f$. The $\beta$-function for this coupling $\tilde{\beta}$ is $\tilde{\beta} \sim -b\tilde{\beta}^2$ where the constant $b$ is positive and of order $n^2/(4\pi)$ for $n > 1$. Thus the effective $\tilde{\beta}$ coupling runs, becoming of order unity at ultra-low scales $\mu_{IR} \sim T \exp(-2\pi N_f/n^2 \times \rho_T/T)$. As in the case of the $U(1) \otimes U(1)$ model, unit strength is expected to disorder the system leading to a finite correlation length, now of order $1/\mu_{IR}$. But since this happens for any finite $T < T_c$, the non-Abelian theory is in the ordinary symmetric phase, not the KT phase. For the Abelian part only, there is a KT transition as $T \rightarrow T_c$. The value of the $f_{IR}$ is $n^2$ because although the light scalars of the non-Abelian part of the effective theory are massive, their mass $m_{SU(N)} \sim \mu_{IR}$ vanishes exponentially as $T \rightarrow 0$. Thus they are effectively massless in this limit. Clearly, for generic $n^2 \ll N_f$ the inequality $f_{IR} \ll f_{UV}$ will always be satisfied since $f_{UV} = 3N_f$.

If the $U(n) \otimes U(n)$ model is arranged so that $n^2$ becomes of order $N_f$, the $1/N_f$ expansion breaks down at all $T$ due to the large number of scalar degrees of freedom ($O(n^2)$) that are formed. It is beyond the scope of this paper to analyze the model when $n^2 \sim N_f$, but we note that it may well be accessible to a combined $1/N_f$, $1/n^2$ expansion. In the limit $T \ll \rho_T$, the model is described by an effective, 3D chiral Lagrangian at the scales of $T$, and the $O(N)$ models of this sort have been analyzed using the $1/N$ expansion [15].

Acknowledgements

We are grateful for several helpful discussions with Nick Read, Subir Sachdev and L.C.R. Wijewardhana. We also thank Nick Read for a careful reading of the manuscript.

References

[5] This conclusion is modified in the presence of the Wess–Zumino–Witten term. However, because the underlying 3D fermionic theory does not carry anomalies, the effective low energy scalar theory is also anomaly free and no WZW term is generated [14].
[11] For some speculations along these lines, see, e.g., E. Babaev, hep-th/9907089.
Erratum to: "Constraining the free parameter of the high parton density effects"

M.B. Gay Ducati a, V.P. Gonçalves a,b,*

a Instituto de Física, Univ. Federal do Rio Grande do Sul, Caixa Postal 15051, 91501-970, Porto Alegre, RS, Brazil
b Departamento de Ciências Exatas e da Terra, Univ. Regional Integrada do Alto Uruguai e das Missões, CEP 98400-000, Frederico Westphalen, RS, Brazil

Received 19 September 2000

Due to an Editorial error all figures of our paper were misplaced and one figure is not present. Below follow the correct figures and respective captions (see Figs. 1–5).

Fig. 1. First order contribution to the unitarity corrections. In (a) these corrections are controlled by the proton radius, while in (b) by the constituent radius.

Fig. 2. The $F_2$ structure function as a function of the variable $\ln(1/s)$ for different virtualities and radii. Only the unitarity corrections in the quark sector are considered. Data from H1 [1].

PII of original article: S0370-2693(00)00813-3.
* Corresponding author.
E-mail addresses: gay@if.ufrgs.br (M.B. Gay Ducati), barros@if.ufrgs.br, victor@inf.uri.br (V.P. Gonçalves).

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.
PII: S0370-2693(00)01038-8
Fig. 3. The $F_2$ slope as a function of the variable $x$ for different radii. (a) Only the unitarity corrections in the quark sector are considered. (b) The unitarity corrections in the gluon–quark sector are considered. Data from ZEUS [2]. The data points correspond to a different $x$ and $Q^2$ value.

Fig. 4. Comparison between the DGLAP and Glauber–Mueller (GM) predictions for the behavior of the $F_2$ slope using as input in the calculations the GRV(94) or GRV(98) parameterizations. Data from ZEUS [2]. The data points correspond to a different $x$ and $Q^2$ value.

Fig. 5. Comparison between the Glauber–Mueller (GM) prediction and DGLAP using as input in the calculations the GRV(94) or GRV(98) parameterizations, for the behavior of the $F_2$ slope. Preliminary data from H1 [3].

References

Erratum

Erratum to: “Oscillatory behaviour in homogeneous string cosmology models”

T. Damour\textsuperscript{a,∗}, M. Henneaux\textsuperscript{b,c}

\textsuperscript{a} Institut Hautes Etudes Scientifiques — IHES, 35, Route de Chartres, F-91440 Bures-sur-Yvette, France
\textsuperscript{b} Physique Théorique et Mathématique, Université Libre de Bruxelles, C.P. 231, B-1050, Bruxelles, Belgium
\textsuperscript{c} Centro de Estudios Científicos, Casilla 1469, Valdivia, Chile

Received 19 September 2000

The production process introduced some misprints in the published version of this paper. The matrices $U$ and $\eta$ occurring in the text between Eqs. (10) and (11) should read

$$U = \begin{pmatrix} W & X \\ Y & Z \end{pmatrix},$$

and

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

respectively. Similarly, the matrices $u$ and $U$ occurring in the text between Eqs. (11) and (12) should read

$$u = \begin{pmatrix} w & x \\ y & -w^T \end{pmatrix},$$

and

$$U = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix},$$

respectively.

E. Stedile

*PUCPR, Department of Mathematics, 81611-970 Curitiba, Brazil*

Received 19 September 2000

The paragraph below Eq. (2.6) should be:

Finally, taking into account Eq. (2.2) and considering that the eigenvalues of \(K\) are \(\pm 1\), we easily see that \(\Omega^2 \Phi = \ell (\ell + 1) \Phi\), where \(\ell = 0, 1\). Hence, if we define the dimensionless parameter \(k = 4 \xi / \gamma\) and write the wave function in the separate form \(\Phi(\rho, \phi, \psi) = \Psi(\rho) F(\phi, \psi)\), we obtain from Eq. (2.6) the radial wave equation

\[
\nabla_\rho^2 \Psi(\rho) + \frac{\gamma^2}{16} \left( k - \frac{2 \lambda}{\rho} \right)^2 - \frac{16 \xi (\ell + 1)}{\gamma^2 \rho^2} - \rho^4 \Psi(\rho) = 0.
\]

Eq. (3.1) should be:

\[
\frac{d^2 \Phi(\rho)}{d \rho^2} + \left[ \lambda - \frac{\xi (\ell + 1)}{\rho^2} - \frac{\delta \rho^4}{\rho^2} \right] \Phi(\rho) = 0.
\]

Table 2 should be:

<table>
<thead>
<tr>
<th>(\ell = 1)</th>
<th>(n)</th>
<th>(\lambda_n)</th>
<th>(k_n)</th>
<th>(E_n) (MeV)</th>
<th>(\Delta E) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.106150</td>
<td>178.542</td>
<td>30.411</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.239415</td>
<td>268.136</td>
<td>45.672</td>
<td>15.26</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.393510</td>
<td>343.762</td>
<td>58.554</td>
<td>12.88</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.564074</td>
<td>411.574</td>
<td>70.104</td>
<td>11.55</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.748542</td>
<td>474.120</td>
<td>80.758</td>
<td>10.65</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.945078</td>
<td>532.739</td>
<td>90.743</td>
<td>9.98</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.152368</td>
<td>588.269</td>
<td>100.202</td>
<td>9.46</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.369392</td>
<td>641.275</td>
<td>109.230</td>
<td>9.03</td>
<td></td>
</tr>
</tbody>
</table>

\(\dagger\) PI of original article: S0370-2693(00)00918-7.
E-mail address: stedile@rla01.pucpr.br (E. Stedile).

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.
PIE: S0370-2693(00)01036-4
## Author index to volume 491

<table>
<thead>
<tr>
<th>Name</th>
<th>First Page</th>
<th>Last Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abreu, P.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Adam, W.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Adye, T.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Adzic, P.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Afanasiev, S.V.</td>
<td>491, 59</td>
<td></td>
</tr>
<tr>
<td>Akhmetshin, R.R.</td>
<td>491, 81</td>
<td></td>
</tr>
<tr>
<td>Akiyoshi, H.</td>
<td>491, 8</td>
<td></td>
</tr>
<tr>
<td>Albrecht, Z.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Alderweireld, T.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Alemany, R.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Allmendinger, T.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Allport, P.P.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Almehed, S.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Amaldi, U.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Amapane, N.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Amato, S.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Anisovich, A.V.</td>
<td>491, 47, 40, 47</td>
<td></td>
</tr>
<tr>
<td>Antilogus, P.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Apel, W.-D.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Appelquist, T.</td>
<td>491, 367</td>
<td></td>
</tr>
<tr>
<td>Aprahamian, A.</td>
<td>491, 225</td>
<td></td>
</tr>
<tr>
<td>Arena, V.</td>
<td>491, 232</td>
<td></td>
</tr>
<tr>
<td>Armoud, Y.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Arpagaus, M.</td>
<td>491, 81</td>
<td></td>
</tr>
<tr>
<td>Arrizabalaga, A.</td>
<td>491, 214</td>
<td></td>
</tr>
<tr>
<td>Åsman, B.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Auger, G.</td>
<td>491, 15</td>
<td></td>
</tr>
<tr>
<td>Augustin, J.-E.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Augustinus, A.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Aubchenko, V.M.</td>
<td>491, 81</td>
<td></td>
</tr>
<tr>
<td>Babu, K.S.</td>
<td>491, 148</td>
<td></td>
</tr>
<tr>
<td>Bächler, J.</td>
<td>491, 59</td>
<td></td>
</tr>
<tr>
<td>Bacri, Ch.O.</td>
<td>491, 15</td>
<td></td>
</tr>
<tr>
<td>Bailin, D.</td>
<td>491, 161</td>
<td></td>
</tr>
<tr>
<td>Bailaron, P.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Baker, C.A.</td>
<td>491, 40, 47</td>
<td></td>
</tr>
<tr>
<td>Baldo, M.</td>
<td>491, 240</td>
<td></td>
</tr>
<tr>
<td>Balestra, F.</td>
<td>491, 29</td>
<td></td>
</tr>
<tr>
<td>Ballestero, A.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Bamade, P.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Banzarov, V.Sh.</td>
<td>491, 81</td>
<td></td>
</tr>
<tr>
<td>Baranov, S.P.</td>
<td>491, 111</td>
<td></td>
</tr>
<tr>
<td>Barao, F.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Barbieri, G.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Barhier, R.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Bardin, D.Y.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Barker, G.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Barkov, L.M.</td>
<td>491, 81</td>
<td></td>
</tr>
<tr>
<td>Barna, D.</td>
<td>491, 59</td>
<td></td>
</tr>
<tr>
<td>Barnby, L.S.</td>
<td>491, 59</td>
<td></td>
</tr>
<tr>
<td>Baroncelli, A.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Bartke, J.</td>
<td>491, 59</td>
<td></td>
</tr>
<tr>
<td>Barton, R.A.</td>
<td>491, 59</td>
<td></td>
</tr>
<tr>
<td>Bashir, A.</td>
<td>491, 280</td>
<td></td>
</tr>
<tr>
<td>Bashhtovoy, N.S.</td>
<td>491, 81</td>
<td></td>
</tr>
<tr>
<td>Battaglia, M.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Batty, C.J.</td>
<td>491, 40, 47, 219</td>
<td></td>
</tr>
<tr>
<td>Baubillier, M.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Beck, K.-H.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Bedfere, Y.</td>
<td>491, 29</td>
<td></td>
</tr>
<tr>
<td>Bediaga, J.</td>
<td>491, 232</td>
<td></td>
</tr>
<tr>
<td>Beeene, J.R.</td>
<td>491, 23</td>
<td></td>
</tr>
<tr>
<td>Begalli, M.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Behrmann, A.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Beillicere, P.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Bellez, N.</td>
<td>491, 15</td>
<td></td>
</tr>
<tr>
<td>Belokopytov, Yu.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Belous, K.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Benekos, N.C.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Benlliure, J.</td>
<td>491, 225</td>
<td></td>
</tr>
<tr>
<td>Benvenuti, A.C.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Berat, C.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Berggren, M.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Berntzon, L.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Bertini, R.</td>
<td>491, 29</td>
<td></td>
</tr>
<tr>
<td>Bertrand, D.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Besancon, M.</td>
<td>491, 67</td>
<td></td>
</tr>
<tr>
<td>Betev, L.</td>
<td>491, 59</td>
<td></td>
</tr>
</tbody>
</table>

0370-2693/00/$ – see front matter © 2000 Elsevier Science B.V. All rights reserved.

PII: S0370-2693(00)01131-X
Białkowska, H., 491, 59
Bianco, S., 491, 232
Bilenky, M.S., 491, 67
Billmeier, A., 491, 59
Bizouard, M.-A., 491, 67
Bland, L.C., 491, 29
Bloch, D., 491, 67
Blom, H.M., 491, 67
Blume, C., 491, 59
Blyth, C.O., 491, 59
Boca, G., 491, 232
Bocage, F., 491, 15
Boimska, B., 491, 59
Bondar, A.E., 491, 81
Bondarev, D.V., 491, 81
Bonesini, M., 491, 67
Bonomi, G., 491, 232
Boonekamp, M., 491, 67
Booth, P.S.L., 491, 67
Borisov, G., 491, 67
Boschini, M., 491, 232
Bosio, C., 491, 67
Botner, O., 491, 67
Boudinov, E., 491, 67
Bougault, R., 491, 15
Bouquet, B., 491, 67
Bourdarios, C., 491, 67
Bouriquet, B., 491, 15
Bowcock, T.J.V., 491, 67
Boyko, I., 491, 67
Bozovic, I., 491, 67
Bozzo, M., 491, 67
Bracinik, J., 491, 59
Bracko, M., 491, 67
Brady, F.P., 491, 59
Bragin, A.V., 491, 81
Brambilla, D., 491, 232
Branchini, P., 491, 67
Brenner, R.A., 491, 67
Brenschede, A., 491, 29
Brochard, F., 491, 29
Brou, R., 491, 15
Bruce, A.M., 491, 225
Bruckman, P., 491, 67
Brun, R., 491, 59
Brunet, J.-M., 491, 67
Buchet, Ph., 491, 15
Buchmüller, W., 491, 183
Bugg, D.V., 491, 40, 47
Bugge, L., 491, 67
Bunčić, П., 491, 59
Buran, T., 491, 67
Buschbeck, B., 491, 67
Buschmann, P., 491, 67
Busa, M.P., 491, 29
Butler, J.N., 491, 232
Butler, P.A., 491, 225
Caamaño, M., 491, 225
Cabrera, S., 491, 67
Caccia, M., 491, 67
Caccianiga, B., 491, 232
Calandrinio, A., 491, 232
Calvi, M., 491, 67
Camporesi, T., 491, 67
Canale, V., 491, 67
Carena, F., 491, 67
Carr, L., 491, 59
Carrillo, S., 491, 232
Carroll, L., 491, 67
Carstoiu, F., 491, 1
Casimiro, E., 491, 232
Castillo Gimenez, M.V., 491, 67
Catford, W.N., 491, 1
Cattai, A., 491, 67
Cavallo, F.R., 491, 67
Cawfield, C., 491, 232
Cebra, D., 491, 59
Chapkin, M., 491, 67
Charpentier, Ph., 491, 67
Charvet, J.L., 491, 15
Chibbi, A., 491, 15
Checchia, P., 491, 67
Chelkov, G.A., 491, 67
Chernyak, D.V., 491, 81
Cheung, H.W.K., 491, 232
Chierici, R., 491, 67
Chliapnikov, P., 491, 67
Cho, K., 491, 232
Chochula, P., 491, 67
Choi, S., 491, 29
Chorowicz, V., 491, 67
Chudoba, J., 491, 67
Chung, Y.S., 491, 232
Cieslik, K., 491, 67
Cinquini, L., 491, 232
Clarke, N.M., 491, 1
Colin, J., 491, 15
Collins, P., 491, 67
Contri, R., 491, 67
Cooper, G.E., 491, 59
Copty, N., 491, 232
Cortina, F., 491, 67
Cortina Gil, D., 491, 225
Cosme, G., 491, 67
Cossutti, F., 491, 67
Costa, M., 491, 67
Covi, L., 491, 183
Cramer, J.G., 491, 59
Crawley, H.B., 491, 67
Crennell, D., 491, 67
Crosetti, G., 491, 67
Csaató, P., 491, 59
Cuevas Maestro, J., 491, 67
Cullen, D.M., 491, 225
Cumalat, J.P., 491, 232
Curtis, N., 491, 1
Cussol, D., 491, 15
Czellar, S., 491, 67
D’Angelo, P., 491, 232
D’Hondt, J., 491, 67
Dalmau, J., 491, 67
Damour, T., 491, 377
Da Silva, W., 491, 67
Davenport, M., 491, 67
Davenport III, T.F., 491, 232
Dayras, R., 491, 15
De Angelis, A., 491, 67
De Boer, W., 491, 67
Debowski, M., 491, 29
De Clercq, C., 491, 67
De Deus, J.D., 491, 253
Delépine, D., 491, 183
Della Ricca, G., 491, 67
De Lotto, B., 491, 67
DELPHI Collaboration, 491, 67
Delpierre, P., 491, 67
Demaria, N., 491, 67
Demeyer, A., 491, 15
De Min, A., 491, 67
De Miranda, J.M., 491, 232
De Oliveira, F., 491, 225
De Paula, L., 491, 67
De S. Pires, C.A., 491, 143
Di Ciaccio, L., 491, 67
Dick, R., 491, 333
DiCorato, M., 491, 232
Dijkstra, H., 491, 67
Dini, P., 491, 232
DISTO Collaboration, 491, 29
Dolbeau, J., 491, 67
Dracos, M., 491, 67
Dris, M., 491, 67
Durand, D., 491, 15
Dzemidzic, M., 491, 29
Eckardt, V., 491, 59
Eckhardt, F., 491, 59
Egorov, A.M., 491, 137
Eidelman, S.I., 491, 81
Eigen, G., 491, 67
Eiras, D., 491, 101
Ekelof, T., 491, 67
Ellert, M., 491, 67
Elsing, M., 491, 67
Engel, J.-P., 491, 67
Engh, D., 491, 232
Enqvist, T., 491, 225
Esbensen, H., 491, 23
Espírito Santo, M., 491, 67
Fabbri, F.L., 491, 232
Faivre, J.-C., 491, 29
Falkowski, A., 491, 172
Falomkin, I.V., 491, 29
Fanourakis, G., 491, 67
Fassoulatis, D., 491, 67
Fava, L., 491, 29
Fayans, S.A., 491, 245
Fedotovitch, G.V., 491, 81
Feindt, M., 491, 67
Feinbien, A., 491, 190
Feren, D., 491, 59
Fernandez, J., 491, 67
Ferrer, A., 491, 67
Ferrer-Ribas, E., 491, 67
Ferrero, L., 491, 29
Ferro, F., 491, 67
Fiasconaro, A., 491, 240
Firestone, A., 491, 67
Fischer, H.G., 491, 59
Flachi, A., 491, 157
Flagmeyer, U., 491, 67
FOCUS Collaboration, 491, 232
Fodor, Z., 491, 59
Foeth, H., 491, 67
Foka, P., 491, 59
Fokitis, E., 491, 67
Fontanelli, F., 491, 67
Foot, R., 491, 291
Foy, J., 491, 29
Fox, C., 491, 225
Franek, B., 491, 67
Frankland, J.D., 491, 15
Freer, M., 491, 1
Freund, P., 491, 59
Friedman, E., 491, 219
Friese, V., 491, 59
Frodoeis, A.G., 491, 67
Frohlich, J., 491, 29
Frolov, V., 491, 29
Fruhwirth, R., 491, 67
Fucnik, J., 491, 59
Fujiwara, H., 491, 8
Fukuda, N., 491, 8
Fulda-Quenzer, F., 491, 67
Fulfop, Zs., 491, 8
Fuster, J., 491, 67
Gabyshev, N.I., 491, 81
Gaines, I., 491, 232
Gal, A., 491, 219
Gál, J., 491, 59
Galichet, E., 491, 15
Galindo-Uribarri, A., 491, 23
<table>
<thead>
<tr>
<th>Author</th>
<th>Volume</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galloni, A.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gamba, D.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gamblin, S.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gandelman, M.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Ganz, R.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Garbincius, Ph.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Garcés Narro, J.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Garcia, C.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gardner, R.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Garfagnini, R.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Garren, L.A.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Gaspar, C.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gaspar, M.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gasparini, U.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gavillet, Ph.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gay Ducati, M.B.</td>
<td>491</td>
<td>375</td>
</tr>
<tr>
<td>Gázdzicki, M.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Gazis, E.N.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Geissel, H.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Gele, D.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gellerby, W.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Genoun-Duhamel, E.</td>
<td>491</td>
<td>15</td>
</tr>
<tr>
<td>Geralis, T.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gerl, J.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Gerlic, E.</td>
<td>491</td>
<td>15</td>
</tr>
<tr>
<td>Gherghetta, T.</td>
<td>491</td>
<td>353</td>
</tr>
<tr>
<td>Ghodbane, N.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gianmarchi, M.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Gianauro, G.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Gil, I.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Giovinazzo, J.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Gladysz, E.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Glege, F.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gobel, C.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Gokeli, R.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Goldberger, W.D.</td>
<td>491</td>
<td>339</td>
</tr>
<tr>
<td>Golob, B.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gomez-Ceballos, G.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gomez del Campo, J.</td>
<td>491</td>
<td>23</td>
</tr>
<tr>
<td>Goncalves, P.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gonçalves, V.P.</td>
<td>491</td>
<td>375</td>
</tr>
<tr>
<td>Gonzalez Caballero, I.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gopal, G.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gorbar, E.V.</td>
<td>491</td>
<td>305</td>
</tr>
<tr>
<td>Gorn, L.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gorska, M.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Gottschalk, E.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Gourlay, S.A.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Gouz, Yu.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gracco, V.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Grah, J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Grasso, A.</td>
<td>491</td>
<td>29</td>
</tr>
<tr>
<td>Grawe, H.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Graziani, E.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Grebenjak, A.A.</td>
<td>491</td>
<td>81</td>
</tr>
<tr>
<td>Grebieszkow, J.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Grévy, S.</td>
<td>491</td>
<td>1</td>
</tr>
<tr>
<td>Grigoriev, D.N.</td>
<td>491</td>
<td>81</td>
</tr>
<tr>
<td>Gris, P.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Grosdidier, G.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Gross, C.J.</td>
<td>491</td>
<td>23</td>
</tr>
<tr>
<td>Grzelak, K.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Grzywacz, R.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Guinet, D.</td>
<td>491</td>
<td>15</td>
</tr>
<tr>
<td>Guy, J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Haag, C.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hahn, F.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hahn, K.I.</td>
<td>491</td>
<td>8</td>
</tr>
<tr>
<td>Haider, S.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hajduk, Z.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hallbert, M.L.</td>
<td>491</td>
<td>23</td>
</tr>
<tr>
<td>Hallgren, A.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hamacher, K.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Handler, T.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Hansen, J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Harris, F.J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Harris, J.W.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Hauler, F.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hedberg, V.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hegyi, S.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Heinz, S.</td>
<td>491</td>
<td>29</td>
</tr>
<tr>
<td>Heissing, S.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hellström, M.</td>
<td>491</td>
<td>225</td>
</tr>
<tr>
<td>Hennemaux, M.</td>
<td>491</td>
<td>377</td>
</tr>
<tr>
<td>Hernandez, H.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Hernandez, J.J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Herquet, P.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Herr, H.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Higon, E.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Higurashi, Y.</td>
<td>491</td>
<td>8</td>
</tr>
<tr>
<td>Hirai, M.</td>
<td>491</td>
<td>8</td>
</tr>
<tr>
<td>Hisanaga, I.</td>
<td>491</td>
<td>8</td>
</tr>
<tr>
<td>Hlinka, V.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Hodd, C.</td>
<td>491</td>
<td>40, 47</td>
</tr>
<tr>
<td>Höhne, C.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>Holm gren, S.-O.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Holt, P.J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hoorelbeke, S.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hori, M.</td>
<td>491</td>
<td>275</td>
</tr>
<tr>
<td>Hosack, M.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Houlden, M.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hrubec, J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Huber, M.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hudan, S.</td>
<td>491</td>
<td>15</td>
</tr>
<tr>
<td>Hughes, G.J.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Hughes, V.W.</td>
<td>491</td>
<td>81</td>
</tr>
<tr>
<td>Hulqvist, K.</td>
<td>491</td>
<td>67</td>
</tr>
<tr>
<td>Ignatov, F.V.</td>
<td>491</td>
<td>81</td>
</tr>
<tr>
<td>Igo, G.</td>
<td>491</td>
<td>59</td>
</tr>
<tr>
<td>IN DRA Collaboration.</td>
<td>491</td>
<td>15</td>
</tr>
<tr>
<td>Inzani, P.</td>
<td>491</td>
<td>232</td>
</tr>
<tr>
<td>Ishihara, M.</td>
<td>491</td>
<td>8</td>
</tr>
</tbody>
</table>
Itoh, T., 491, 362
Ivanov, M., 491, 59
Ivanov, P.M., 491, 81
Ivanov, V.V., 491, 29
Iwasa, N., 491, 8
Iwasaki, H., 491, 8
Jack, I., 491, 151
Jack, J.N., 491, 67
Jacobs, P., 491, 59
Jacobs, W.W., 491, 29
Jacobsson, R., 491, 67
Jalocha, P., 491, 67
Janik, R., 491, 59, 47
Jarlskog, Ch., 491, 67
Jarlskog, G., 491, 67
Jarry, P., 491, 67
Jean-Marie, B., 491, 67
Jeans, D., 491, 67
Johansson, E.K., 491, 67
Johnson, W.E., 491, 232
Jones, D.R.T., 491, 151
Jones, P.G., 491, 59
Jonsson, P., 491, 67
Joram, C., 491, 67
Juillot, P., 491, 67
Jungermann, L., 491, 67
Kadija, K., 491, 59
Kakushadze, Z., 491, 317
Kang, J.S., 491, 232
Kapusta, F., 491, 67
Karafasoulis, K., 491, 67
Kasper, P.H., 491, 67
Katsanevas, S., 491, 67
Katsoufis, E.C., 491, 67
Kawamura, H., 491, 275
Kazanin, V.F., 491, 81
Keranen, R., 491, 67
Kernel, G., 491, 67
Kersevan, B.P., 491, 67
Ketov, S.J., 491, 207
Khzazin, B.I., 491, 81
Khokhlov, Yu., 491, 67
Khomenko, B.A., 491, 67
Khovanski, N.N., 491, 67
Kikin, A., 491, 67
Kim, D.Y., 491, 232
King, B., 491, 67
Kinvig, A., 491, 67
Kjaer, N.J., 491, 67
Klapp, O., 491, 67
Klemböhl, A., 491, 225
Kluft, P., 491, 67
Ko, B.R., 491, 232
Kodaira, J., 491, 275
Kokkinias, P., 491, 67
Kolesnikov, V.I., 491, 59
Kondo, K.-I., 491, 263
Koop, J.A., 491, 81
Korostelev, M.S., 491, 81
Korten, W., 491, 225
Kostioukhine, V., 491, 67
Kourkoumelis, C., 491, 67
Kouznetsov, O., 491, 67
Kowalski, M., 491, 59
Krammer, M., 491, 67
Kraniotis, G.V., 491, 161
Kreymer, A.E., 491, 232
Kriznic, E., 491, 67
Krokovny, P.P., 491, 81
Krumstein, Z., 491, 67
Kubinec, P., 491, 67
Kucewicz, W., 491, 67
Kühn, W., 491, 29
Kunze, K.E., 491, 190
Kurdadze, L.M., 491, 81
Kurvska, J., 491, 67
Kurtinien, K., 491, 67
Kutschke, R., 491, 232
Kuzmin, A.S., 491, 81
Kwak, J.W., 491, 232
Kwong, Y., 491, 232
Lalak, Z., 491, 172
Lamsa, J.W., 491, 67
Lane, D.W., 491, 67
Lapin, V., 491, 67
Lassuk, B., 491, 59
Laugier, J.-P., 491, 67
Lauhakangas, R., 491, 67
Lautesse, P., 491, 15
Lavaud, F., 491, 15
Laville, J.L., 491, 15
Le Brun, C., 491, 1
Lecolley, I.E., 491, 15
Leder, G., 491, 67
Ledroit, F., 491, 67
Leduc, C., 491, 15
Lee, H.-J., 491, 257
Lee, K.B., 491, 232
Lefort, T., 491, 15
Legrain, R., 491, 15
Lenzen, L., 491, 67
Leisos, A., 491, 67
Leitner, R., 491, 67
Le Neindre, N., 491, 15
Lemasse, N., 491, 15
Lenzen, G., 491, 67
Lepeltier, V., 491, 67
Lehuaultier, M., 491, 67
Lévai, P., 491, 59
Leveraro, F., 491, 232
Lewitowicz, M., 491, 1, 225
Liang, J.F., 491, 23
Libby, J., 491, 67
Liebig, W., 491, 67
Liégard, E., 491, 1
Liguori, G., 491, 232
Liko, D., 491, 67
Link, J.M., 491, 232
Lipniacka, A., 491, 67
Lippi, I., 491, 67
Littenberg, L.S., 491, 285
Lobanov, A.E., 491, 137
Loerstad, B., 491, 67
Logashenko, I.B., 491, 81
Loken, J.G., 491, 67
Lopes, J.H., 491, 67
Lopez, A.M., 491, 232
Lopez, J.M., 491, 67
Lopez, O., 491, 15
Lopez-Fernandez, R., 491, 67
Loukas, D., 491, 67
Louvel, M., 491, 15
Love, A., 491, 161
Lu, H.C., 491, 47
Lucas, R., 491, 225
Lukin, P.A., 491, 81
Lutz, P., 491, 67
Lyons, L., 491, 67
Ma, E., 491, 297
Mac Cormick, M., 491, 1
Mach, H., 491, 225
MacNaughton, J., 491, 67
Maggiora, A., 491, 29
Maggiora, M., 491, 29
Magnin, J., 491, 232
Mahon, J.R., 491, 67
Maio, A., 491, 67
Majumdar, P., 491, 199
Malakhov, A.I., 491, 59
Malek, A., 491, 67
Maletezos, S., 491, 67
Malvezzi, C., 491, 67
Manara, A., 491, 29
Mandl, F., 491, 67
Marco, J., 491, 67
Marco, R., 491, 67
Maréchal, B., 491, 67
Margetis, S., 491, 59
Margoni, M., 491, 67
Marin, J.-C., 491, 67
Mariotti, C., 491, 67
Markert, C., 491, 59
Markou, A., 491, 67
Marquès, F.M., 491, 1
Marti i García, S., 491, 67
Martinez-Rivero, C., 491, 67
Masik, J., 491, 67
Maskay, A.M., 491, 15
Mastroianniopoulos, N., 491, 67
Matorras, F., 491, 67
Matteuzzi, C., 491, 67
Matthiae, G., 491, 67
Mayes, B.W., 491, 59
Mayet, P., 491, 225
Mazzucato, F., 491, 67
Mazzucato, M., 491, 67
Mc Cubbin, M., 491, 67
Mc Kay, R., 491, 67
Mc Nulty, R., 491, 67
Mc Pherson, G., 491, 67
Meldrum, G.L., 491, 59
Menasce, D., 491, 232
Mendez, H., 491, 232
Mendez, L., 491, 232
Mengoni, A., 491, 8
Meze, E., 491, 67
Merlo, M., 491, 232
Meroni, C., 491, 67
Meyer, W.T., 491, 67
Mezzadri, M., 491, 232
Miagkov, A., 491, 67
Migliore, E., 491, 67
Mikhailov, K.Yu., 491, 81
Milazzo, L., 491, 232
Milstein, A.L., 491, 81
Min, D.-P., 491, 257
Minemura, T., 491, 8
Mineva, M., 491, 225
Mirabito, L., 491, 67
Mirsles, A., 491, 232
Mischke, A., 491, 59
Mitroff, W.A., 491, 67
Mjöenmark, U., 491, 67
Moa, T., 491, 67
Moch, M., 491, 67
Moeller, R., 491, 67
Moening, K., 491, 67
Moffat, J.W., 491, 345
Mohapatra, R.N., 491, 143
Mohr, J., 491, 59
Monge, M.R., 491, 67
Montiel, E., 491, 232
Moraes, D., 491, 67
Morettini, P., 491, 67
Morii, T., 491, 117
Moroni, L., 491, 232
Morton, G., 491, 67
Moss, I.G., 491, 203
Motobayashi, T., 491, 8
Mueller, P.E., 491, 23
Mueller, U., 491, 67
Muenich, K., 491, 67
Mulders, M., 491, 67
Mulet-Marquis, C., 491, 67
Mundim, L.M., 491, 67
Muresan, R., 491, 67
Murray, W.J., 491, 67
Muryn, B., 491, 67
Myatt, G., 491, 67
Myklebust, T., 491, 67
Myung, S.S., 491, 232
NA49 Collaboration, 491, 59
Nakamura, T., 491, 8
Nalpas, L., 491, 15
Naraghi, F., 491, 67
Narz, F.S., 491, 67
Narz, F.S., 491, 67
Navarrina, F.L., 491, 67
Navrocki, K., 491, 67
Negri, P., 491, 67
Nehring, M.S., 491, 232
Nelson, J.M., 491, 59
Nesterenko, I.N., 491, 81
Neufeld, N., 491, 67
Nicolaidou, R., 491, 67
Nielsen, B.S., 491, 67
Niezurawski, P., 491, 67
Nikolenko, M., 491, 67
Nikonov, V.A., 491, 40, 47
Nomokonov, V., 491, 67
Normand, J., 491, 15
Notani, M., 491, 8
Nygren, A., 491, 67
O’Leary, C.D., 491, 225
O’Reilly, B., 491, 232
Oblakowska Mucha, A., 491, 67
Obraztsov, V., 491, 67
Oldenburg, M.D., 491, 67
Olshevsky, A.G., 491, 67
Onofre, A., 491, 67
Orazi, G., 491, 67
Orr, N.A., 491, 1
Osterberg, K., 491, 67
Ott, W., 491, 67
Ouyang, H., 491, 67
Park, B.-Y., 491, 257
Park, H., 491, 232
Park, K.S., 491, 232
Parkes, C., 491, 67
Parlog, M., 491, 15
Parodi, F., 491, 67
Parzefall, U., 491, 67
Passeri, A., 491, 67
Passon, O., 491, 67
Pavel, T., 491, 67
Pawlowski, T., 491, 15
Pearson, C.J., 491, 232
Pedrini, D., 491, 232
Pegoraro, M., 491, 67
Penttinen, M., 491, 96
Pepe, I.M., 491, 232
Peralta, L., 491, 67
Perevedentsev, E.A., 491, 81
Pérez-Lorenzana, A., 491, 143
Pernicka, M., 491, 67
Petr, J., 491, 67
Petrakis, A., 491, 59
Petr, J., 491, 67
Petr, J., 491, 67
Petrov, A., 491, 67
Petrov, A., 491, 59
Petrov, A., 491, 67
Petr, J., 491, 67
Petr, J., 491, 67
Petr, J., 491, 67
Petr, J., 491, 67
Pfeffer, H., 491, 29
Pflüger, M., 491, 225
Philipps, H.T., 491, 67
Pine, F., 491, 67
Pillna, M., 491, 67
Pimenta, M., 491, 67
Pimschenko, L., 491, 59
Pirotta, E., 491, 67
Piragnolo, G., 491, 29
Pis, A., 491, 67
Podobnik, T., 491, 67
Podolyak, Zs., 491, 225
Poirier, V., 491, 67
Pokorski, S., 491, 172
Pol, M.E., 491, 67
Polok, G., 491, 67
Polyakov, M.V., 491, 96
Pontecorvo, G.B., 491, 29
Popov, A., 491, 29
Popov, A.S., 491, 81
Poropat, P., 491, 67
Poskanzer, A.M., 491, 59
Pozdnyakov, V., 491, 67
Prezelz, F., 491, 232
Prindl, D.J., 491, 59
Palacios, J., 491, 67
Palla, G., 491, 59
Panagiotou, A.D., 491, 59
Panitz, A., 491, 232
Panizier, D., 491, 29
Paolone, V.S., 491, 232
Papadopoulou, Th.D., 491, 67
Pape, L., 491, 67
Park, B.-Y., 491, 257
Park, H., 491, 232
Park, K.S., 491, 232
Parlog, M., 491, 15
Parodi, F., 491, 67
Parzefall, U., 491, 67
Passeri, A., 491, 67
Passon, O., 491, 67
Pavel, T., 491, 67
Pawlicki, P., 491, 15
Pearson, C.J., 491, 232
Podrini, D., 491, 232
Pegoraro, M., 491, 67
Penttinen, M., 491, 96
Pepe, I.M., 491, 232
Peralta, L., 491, 67
Perevedentsev, E.A., 491, 81
Perez-Lorenzana, A., 491, 143
Pernicka, M., 491, 67
Petr, J., 491, 67
Petrakis, A., 491, 59
Petr, J., 491, 67
Petr, J., 491, 67
Petr, J., 491, 67
Petr, J., 491, 67
Pfeffer, H., 491, 29
Pflueger, M., 491, 225
Philipps, H.T., 491, 67
Pine, F., 491, 67
Pillna, M., 491, 67
Pimenta, M., 491, 67
Pimschenko, L., 491, 59
Pirotta, E., 491, 67
Piragnolo, G., 491, 29
Pis, A., 491, 67
Podobnik, T., 491, 67
Podolyak, Zs., 491, 225
Poirier, V., 491, 67
Pokorski, S., 491, 172
Pol, M.E., 491, 67
Polok, G., 491, 67
Polyakov, M.V., 491, 96
Pontecorvo, G.B., 491, 29
Popov, A., 491, 29
Popov, A.S., 491, 81
Poropat, P., 491, 67
Poskanzer, A.M., 491, 59
Pozdnyakov, V., 491, 67
Prezelz, F., 491, 232
Prindl, D.J., 491, 59
Privitera, P., 491, 67
Pühlhofer, F., 491, 59
Pukhava, N., 491, 67
Pulia, A., 491, 67
Purlatz, T.A., 491, 81
Purohit, M., 491, 232
Qian, W.L., 491, 90
Quinones, J., 491, 232
Radojicic, D., 491, 67
Ragazzi, S., 491, 67
Rahimi, A., 491, 67
Rahmani, H., 491, 67
Raidal, M., 491, 297
Rames, J., 491, 67
Rebecchi, P., 491, 67
Redaelli, N.G., 491, 67
Redin, S.I., 491, 81
Regan, P.H., 491, 225
Regler, M., 491, 67
Rehn, J., 491, 67
Reid, D., 491, 67
Reid, J.G., 491, 59
Reinertsen, P., 491, 67
Reinhardt, R., 491, 67
Rejmund, F., 491, 225
Rejmund, M., 491, 225
Renton, P.B., 491, 67
Resvanis, L.K., 491, 67
Retyk, W., 491, 59
Reyes, M., 491, 232
Rips-Baudot, I., 491, 67
Ritter, H.G., 491, 59
Rivera, C., 491, 232
Rivet, M.F., 491, 15
Roessl, E., 491, 353
Röhrich, D., 491, 59
Roland, C., 491, 59
Roland, G., 491, 59
Romero, A., 491, 67
Ronchese, P., 491, 67
Root, N.I., 491, 81
Rosato, E., 491, 15
Rosenberg, E.I., 491, 67
Rosinsky, P., 491, 67
Rothstein, I.Z., 491, 339
Roudeau, P., 491, 67
Roussel-Chomaz, P., 491, 1
Rovere, T., 491, 67
Rovere, M., 491, 232
Ruban, A.A., 491, 81
Ruhlmann-Kleider, V., 491, 67
Ruiz, A., 491, 67
Rybicki, A., 491, 59
Ryskulov, N.M., 491, 81
Saarikko, H., 491, 67
Saccoun, Y., 491, 67
Sadovsky, A., 491, 67
Sagawa, H., 491, 8
Saint-Laurent, F., 491, 15
Saint Laurent, M.G., 491, 1
Sajot, G., 491, 67
Sakurai, H., 491, 8
Sala, A., 491, 232
Sala, S., 491, 232
Salabura, P., 491, 29
Sales, M., 491, 232
Salt, J., 491, 67
Sammer, T., 491, 59
Sampsonidis, D., 491, 67
Sánchez-Hernández, A., 491, 232
Sandoval, A., 491, 59
Sannino, M., 491, 67
Sarantsev, A.V., 491, 40, 47
Sarantsev, V.V., 491, 40, 47
Sarwar, S., 491, 232
Sauvan, E., 491, 1
Savoy-Navarro, A., 491, 67
Sawicka, M., 491, 225
Schäfer, E., 491, 59
Schaffner, H., 491, 225
Schlegel, Ch., 491, 225
Schmidt, K., 491, 225
Schmitz, N., 491, 59
Schwemling, Ph., 491, 67
Schwering, B., 491, 67
Schwetz, M., 491, 367
Schwickerath, U., 491, 67
Seeger, E., 491, 67
Sedýk, Y., 491, 57
Segar, A.M., 491, 67
Seibert, N., 491, 67
Sekulin, R., 491, 67
Semenov, A.Yu., 491, 59
Sette, G., 491, 67
Seyboth, P., 491, 59
Shamov, A.G., 491, 81
Shapara, D., 491, 23
Shaposhnikov, M., 491, 329, 353
Sharatchandra, H.S., 491, 199
Shatunov, Yu.M., 491, 81
Shawcross, M., 491, 1
Sheaff, M., 491, 232
Shellard, R.C., 491, 67
Shimoura, S., 491, 8
Shinohara, T., 491, 263
Shrock, R., 491, 285
Shuvaev, A.G., 491, 96
Shwartz, B.A., 491, 81
Sibidanov, A.L., 491, 81
Sidorov, V.A., 491, 81
Siebel, M., 491, 67
Siklér, F., 491, 59
Simão, F.R.A., 491, 232
Simard, L., 491, 67
Simonetto, F., 491, 67
Sisakian, A.N., 491, 67
Sitar, B., 491, 59
Skrinsky, A.N., 491, 81
Skrzypczak, E., 491, 59
Smadja, G., 491, 67
Smakhtin, V.P., 491, 67
Smirnov, V.A., 491, 130
Smirnova, O., 491, 67
Smith, G.R., 491, 67
Solovianov, O., 491, 67
Sopczak, A., 491, 67
Sosnowski, R., 491, 67
Soto, S., 491, 101
Spassov, F., 491, 67
Spendiari, B., 491, 67
Squares, S., 491, 67
Squier, G.T.A., 491, 59
Stanescu, C., 491, 67
Stanitzki, M., 491, 67
Steckmeyer, J.C., 491, 15
Soldatow, P.Yu., 491, 81
Sten, M., 491, 15
Stefanescu, C., 491, 67
Stocchino, L., 491, 67
Strock, R., 491, 67
Stracener, D.W., 491, 23
Strasburg, J., 491, 67
Strikman, M., 491, 96
Strine, P., 491, 59
Srostele, H., 491, 59
Srub, R., 491, 67
Studentik, A.I., 491, 137
Stugu, B., 491, 67
Su, R.K., 491, 90
Suh, K., 491, 117
Sukhanov, A.I., 491, 81
Susa, T., 491, 59
Szarka, L., 491, 59
Szczekowski, M., 491, 67
Szentpétery, L., 491, 59
Szeptycka, M., 491, 67
Sziklai, J., 491, 59
Tabacaru, G., 491, 15
Tabarelli, T., 491, 67
Taffard, A., 491, 67
Takanuchi, S., 491, 8
Tamain, B., 491, 15
Tassan-Got, L., 491, 15
Tchalychev, V., 491, 29
Tchikilev, O.G., 491, 36
Tegenfeldt, F., 491, 67
Teranishi, T., 491, 8
Terranova, F., 491, 67
Theisen, Ch., 491, 225
Thompson, J.A., 491, 81
Timmermans, J., 491, 67
Tinti, N., 491, 67
Tirel, O., 491, 15
Titov, V.M., 491, 81
Tkatchev, L.G., 491, 67
Tobin, M., 491, 67
Todorova, S., 491, 67
Tolokonnikov, S.V., 491, 245
Tomé, B., 491, 67
Toms, D.J., 491, 157
Tonazzo, A., 491, 67
Torre, P., 491, 232
Tortora, L., 491, 67
Tortosa, P., 491, 67
Tosello, F., 491, 29
Toy, M., 491, 59
Trainor, T.A., 491, 59
Transtromer, G., 491, 67
Treille, D., 491, 67
Trentalange, S., 491, 59
Tristram, G., 491, 67
Truchimczuk, M., 491, 67
Troncon, C., 491, 67
Turluer, M.-L., 491, 67
Tyapkin, L.A., 491, 67
Tyapkin, P., 491, 67
Tzamarias, S., 491, 67
Ugoccioni, R., 491, 253
Ullaland, O., 491, 67
Ullrich, T., 491, 59
Uribe, C., 491, 232
Uvarov, V., 491, 67
Vaandering, E.W., 491, 232
Valenti, G., 491, 67
<table>
<thead>
<tr>
<th>Author Name</th>
<th>Page Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valishev, A.A.</td>
<td>491, 81</td>
</tr>
<tr>
<td>Vallazza, E.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Van Dam, P.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Van den Boeck, W.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Van Eldik, J.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Van Lysebetten, A.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Van Remortel, N.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Van Vulpen, I.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Vargas, D.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Varnes, R.L.</td>
<td>491, 23</td>
</tr>
<tr>
<td>Vassiliou, M.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Vazquez, F.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Vázquez-Mozo, M.A.</td>
<td>491, 190</td>
</tr>
<tr>
<td>Venug, G.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Vento, V.</td>
<td>491, 257</td>
</tr>
<tr>
<td>Ventura, L.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Venus, W.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Verbeure, F.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Verdier, P.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Veres, G.I.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Verlato, M.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Vertogradov, L.S.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Verzi, V.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Vesztergombi, G.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Vient, E.</td>
<td>491, 15</td>
</tr>
<tr>
<td>Vigdor, S.E.</td>
<td>491, 29</td>
</tr>
<tr>
<td>Vilanova, D.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Viola, L.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Vitale, L.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Vitolu, P.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Vlasov, E.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Vodopyanov, A.S.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Volant, C.</td>
<td>491, 15</td>
</tr>
<tr>
<td>Voloshin, M.B.</td>
<td>491, 311</td>
</tr>
<tr>
<td>Voloshin, S.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Voulgaris, G.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Vranic, D.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Vrba, V.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Wahlen, H.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Walker, P.M.</td>
<td>491, 225</td>
</tr>
<tr>
<td>Wang, F.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Wang, P.</td>
<td>491, 90</td>
</tr>
<tr>
<td>Warner, D.D.</td>
<td>491, 225</td>
</tr>
<tr>
<td>Washbrook, A.J.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Webster, M.S.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Weerasundara, D.D.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Weiser, C.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Weng, S.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Wheldon, C.</td>
<td>491, 225</td>
</tr>
<tr>
<td>Whitten, C.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Wicke, D.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Wickens, J.H.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Wieleczko, J.P.</td>
<td>491, 15</td>
</tr>
<tr>
<td>Wilkinson, G.R.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Wilson, J.R.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Winfield, J.S.</td>
<td>491, 1</td>
</tr>
<tr>
<td>Winter, M.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Wiss, J.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Wolf, G.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Wollersheim, H.J.</td>
<td>491, 225</td>
</tr>
<tr>
<td>Wooding, S.C.</td>
<td>491, 225</td>
</tr>
<tr>
<td>Xu, F.R.</td>
<td>491, 225</td>
</tr>
<tr>
<td>Xu, N.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Yager, P.M.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Yanagida, T.</td>
<td>491, 148</td>
</tr>
<tr>
<td>Yanagisawa, Y.</td>
<td>491, 8</td>
</tr>
<tr>
<td>Yates, T.A.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Yi, J.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Yoo, I.K.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Yoon, T.L.</td>
<td>491, 291</td>
</tr>
<tr>
<td>Yudin, Yu.V.</td>
<td>491, 81</td>
</tr>
<tr>
<td>Yushchenko, O.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zalewska, A.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zalewski, P.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zallo, A.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Zavrtanik, D.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zawischa, D.</td>
<td>491, 245</td>
</tr>
<tr>
<td>Zegolatakos, E.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zhang, Y.</td>
<td>491, 232</td>
</tr>
<tr>
<td>Zimányi, J.</td>
<td>491, 59</td>
</tr>
<tr>
<td>Zimin, N.I.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zintchenko, A.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zoiller, Ph.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zosi, G.</td>
<td>491, 29</td>
</tr>
<tr>
<td>Zotov, N.P.</td>
<td>491, 111</td>
</tr>
<tr>
<td>Zou, B.S.</td>
<td>491, 40, 47</td>
</tr>
<tr>
<td>Zamerle, G.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zupan, M.</td>
<td>491, 67</td>
</tr>
<tr>
<td>Zverev, S.G.</td>
<td>491, 81</td>
</tr>
</tbody>
</table>